UNIVERSITAT POLITÈCNICA
DE CATALUNYA
BARCELONATECH

Numerical Methods for Partial Differential Equations

Basics Exercises

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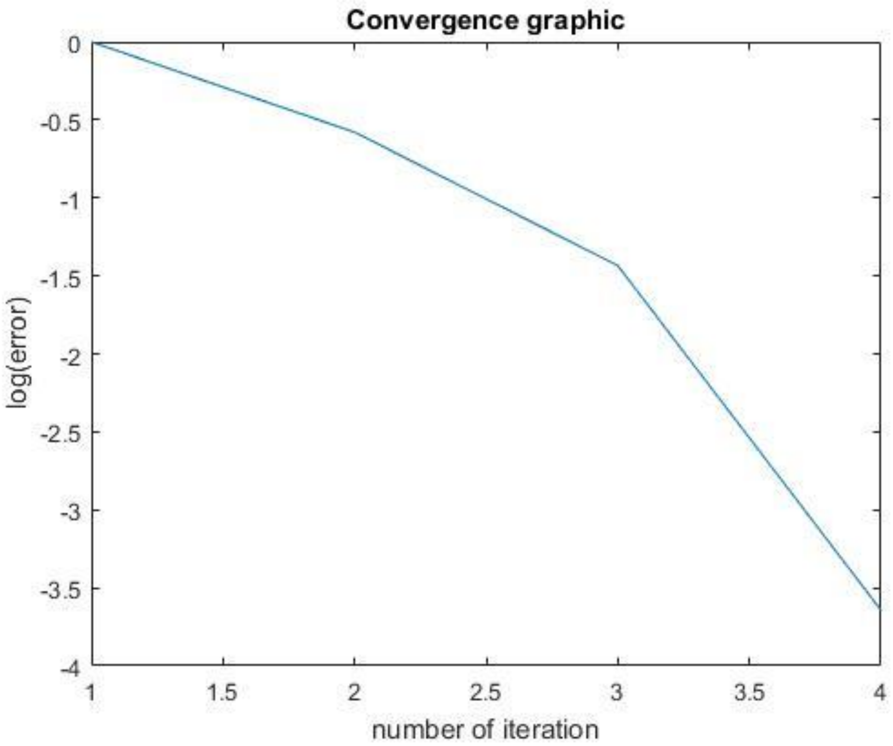
Exercise 1.

Root= 1.3692;




```
clear all; clc; close all;

x=zeros(1,4);
x(1)=20^(1/3);
n=4;

f=@(x) x^3+2*x^2+10*x-20;
df=@(x) 3*x^2+4*x+10;

for i=1:n
    x(i+1)=x(i)-f(x(i))/df(x(i));
end

root=x(n)

err=ones(1,4);
for i=1:n-1
    err(i+1)=abs((x(i)-x(i+1))/x(i+1));
end
l=[1:4];
k=log(err);
plot(l,k)

title('Convergence graphic')
xlabel('number of iteration')
ylabel('log(error)')
```

Exercise 5*

a) $$\int_{x_0}^{x_3} f(x)\,dx = \frac{3h}{8}\Big(f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)\Big) - \frac{3h^5}{80} f^{4'}(\mu)$$

$$h = \frac{x_3 - x_0}{3}$$

$x_0 < \mu <_3$

minimum 4 interogation points

$h =$

	x_i	w_i
0	x_0	$3h/8$
1	x_1	$9h/8$
2	x_2	$9h/8$
3	x_3	$3h/8$

b) 

$$h = \frac{1 - 1/4}{3} = 0{,}25$$

$$I = \frac{0{,}75}{8}\Big(f(1/4) + 3\cdot f(1/2) + 3\cdot f(3/4) + f(1)\Big)$$

$f^{4'} = \emptyset$

It is possible to compute.

$\boxed{\text{Exercise } 6^*.}$

a) $n+1$ points Gaussian quadrature.

we can integrate an $2n+1$ degree polynomial

b) $n = 2$

$\hookrightarrow 2 \times 2 + 1 = 5$

So the following ~~integrated~~ integrals will be integrated exactly:

i) $\int_0^1 \sin(x)\,dx$

ii) $\int_0^1 x^3\,dx$

iii) $\int_0^1 x^4\,dx$

$\boxed{\text{Exercise } 7^*}$

$$\boxed{\int_0^1 12x\,dx}$$

a) trapezoidal rule:

$$I = \frac{h}{2}\left(f(x_0) + f(x_1)\right) - \frac{h^3}{12} f''(\mu)$$

$f(x_0) = 0$ $\qquad I \approx \frac{0{,}5}{2}\left(0 + 12\right) = 3$

$f(x_1) = 12$

$h = 0{,}5$

$$f^*(\mu) = \frac{1}{1-0}\int_0^1 12x\,dx = \int_0^1 12x\,dx = \left[\frac{x^{12}}{12}\right]_0^1 =$$

$$12\mu = \left[\frac{x^{12}}{12}\right]_0^1 = \left[\frac{1^{12}}{12} - \frac{0^{12}}{12}\right] = \frac{1}{12}$$

$$\mu = \frac{1}{144}$$

$f'' = \emptyset$

$$\boxed{I = 3 + 0 = 3}$$

$\boxed{\int_0^1 12x\,dx}$

ii) simpson's rule

$$I \approx \frac{h}{3}\left(f(x_0) + 4f(x_1) + f(x_2)\right)$$

$X_0 = 0$ $f(x_0) = 0$

$X_1 = 0{,}5$ $f(x_1) = 6$

$X_2 = 1$ $f(x_2) = 12$

$$I \approx \frac{0{,}5}{3}\left(0 + 4\times 6 + 12\right) = 6$$

$$E = -\frac{h^5}{90} \cdot f^{4'}(\mu) \qquad I = \tilde{I} + E$$

$$f^{4'} = \varnothing \qquad \boxed{I = 6}$$

$\boxed{\int_0^1 (5x^3 + 2x)\,dx}$

i) trapezoidal rule:

$$I \approx \frac{h}{2}\left(f(x_0) + f(x_1)\right)$$

$f(x_0) = 0$ $\tilde{I} \simeq 1{,}75$

$f(x_1) = 7$

$h = 0{,}5$

$$E = -\frac{h^3}{12} f''(\mu)$$

$$f(\mu) = \frac{1}{1-0}\int_0^1 (5x^3 + 2x)\,dx = \left[\frac{5}{4}\cdot x^4 + x^2\right]_0^1$$

$$5\mu^3 + 2\mu = \left[\frac{5}{4} + 1\right] = 2{,}25$$

$$\mu^3 + 2\mu - 0{,}45 = 0$$

$f' = 15x^2 + 2$

$f'' = 30x$

$\mu = 0{,}5959$ [result obtained by matlab's root finder]

$$E = -\frac{0{,}5^3}{12} \cdot 30 \cdot 0{,}5959 = 0{,}1862$$

$$\boxed{I = \tilde{I} - E = 1{,}75 - 0{,}1862 = 1{,}5638}$$

$$\int_0^1 (5x^3 + 2x)\,dx$$

ii) Simpson's rule :

$$\tilde{I} = \frac{h}{3}\left(f(x_0) + 4f(x_1) + f(x_2)\right) = \frac{0{,}5}{3}\left(0 + 4 \times 1{,}6250 + 7\right) = 2{,}25$$

$f(x_0) = 0$

$f(x_1) = 1{,}6250$

$f(x_2) = 7$

$h = 0{,}5$

$$E = -\frac{h^5}{90} f^{4'}(\mu) = 0 \qquad \boxed{I = \tilde{I} - E = 1{,}6250}$$

$f' = 15 \cdot x^2 + 2$

$f'' = 30x$

$f''' = 30$

$f^{4'} = \varnothing$

Exercise 10*

$$\int_0^1\int_0^1 (9x^3+8x^2)(y^3+y)\,dx\,dy$$

$$\int_0^1 g(y)\,dy$$

$$g(y) = \int_0^1 (9x^3+8x^2)(y^3+y)\,dx$$

Simpson's rule:

$$I = \int_0^1 g(y)\,dy = \frac{h}{3}\left[g(a) + 4 * g\left(\frac{a+b}{2}\right) + g(b)\right] ; \quad h = \frac{b-a}{n}$$

$$g(0) = \int_0^1 (9x^3+8x^2)(0^3+0)\,dx = 0$$

$$g(1) = \int_0^1 (9x^3+8x^2)(1^3+1)\,dx = \int_0^1 18x^3+16x^2\,dx = \left[\frac{18}{4}\cdot x^4 + \frac{16}{3}x^3\right]_0^1$$

$$= \frac{18}{4} + \frac{16}{3} = 9{,}833\dot{3}$$

$$g(0{,}5) = \int_0^1 (9x^3+8x^2)(0{,}5^3+0{,}5)\,dx = \int_0^1 5{,}625x^3+5x^2\,dx = \left[\frac{5{,}625}{4}\cdot x^4 + \frac{5}{3}x^3\right]_0^1 =$$

$$= \frac{5{,}625}{4} + \frac{5}{3} = 3{,}0729$$

$n = 2$

$$I \approx \frac{0{,}5}{3}\left[0 + 4 \times 3{,}0729 + 9{,}8333\right] = 3{,}6875$$

$f = 9x^3y^3 + 9x^3y + 8x^2y^3 + 8x^2y$

$f'_y = 27y^2 + 9 + 24y^2 + 8$

$f''_y = 54y + 48y$

$f'''_y = 54 + 48$

$f^{4'} = \varnothing$

$E = 0$

$$\boxed{I = 3{,}6875}$$