PDEs: HW 1

Saskia Loosveldt
Alba Ruiz de Alegría
Gabriel Valdés

1.

Dekker's method is an itetarive hybrid method for root finding that combines the bisection method with the secant method.

For an initial interval [a,b] , where b is the best guess for the root at each iteration, Dekker's method does the following steps

1 Computes the next point using the secant method with points a and b, where a is the previous iterate of the root
2 Computes the next point using the bisection method using point c and b, where c is initally set to a and brackets the interval so that $f(c)\cdot f(b)<0$.
3
   - If when computing the secant, the denominator is much smaller than the numerator, the next value of $b_{k+1}=b_k+\varepsilon(b_k)$
   - If the value computed by the secant method is between the current value of b and the value computed by the bisection method, then $b_{k+1}=b_{secant}$
   - Otherwise, $b_{k+1}=b_{bisection}$

4 In each iteration the method reorders and swaps the points to ensure that b is the best guess for the root and the interval that brackets the solution still meets the condition $f(c)\cdot f(b)<0$.

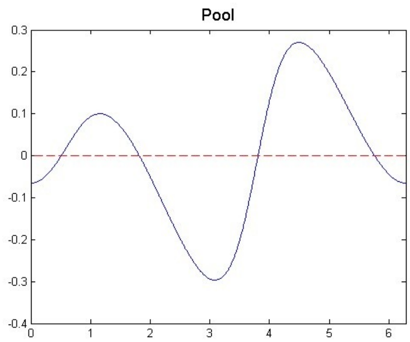


Figure 1. Pool function plot.

For the pool function, which is continuos and well behaved, each of the 4 roots has been computed using the three methods: Dekker, Bisection and Secant.

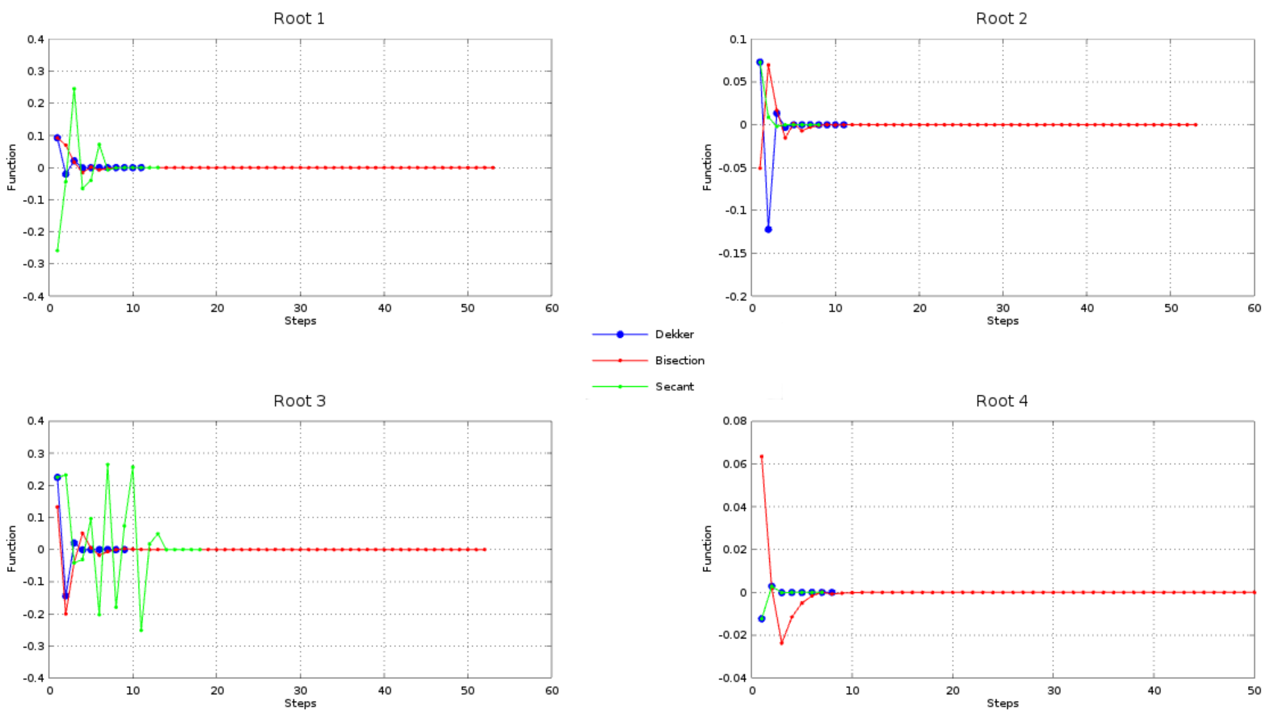


Figure 2. Convergence of the solution for each method and each root for the pool function

From **Figure 2** it can be seen that althought the three methods converge, Dekker's method is the faster for Root 1 and Root 3; whereas for Root 2 and Root 4 the solution converges faster with the secant method. This is due to the fact that Dekker's method can get stuck computing the next iterations with either the error or bisection steps, which ensure convergence but are much slower. Therefore, in these cases, a way to improve Dekker's would be to go back to applying the secant method.

The same procedure has been done for the function $f(x)=\frac{1}{x-3}-6$, which has an assymptote at x=3 and a root at x=19/6. The interval selected to start the computation is [3.05,4] in order to avoid the assymptote.

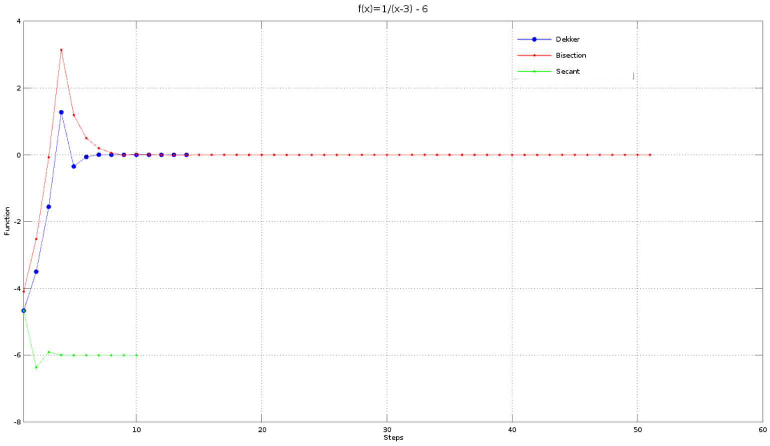


Figure 3. Convergence of the solution for each method for the f(x) function

Figure 3. shows how while Dekker's and the bisection method converge, the first much faster, the secant method stops before finding the root of the function. The secant method starts evaluating points outside the initial interval, and since the function has an assymptote, the method can't evaluate it at that point.
On the other hand, applying the bisection method, the solution converges as expected but it requires much more iterations than Dekker's method. In this case the combination applied by Dekker's method is the following:

Sec-Bis-Bis-Bis-Sec-Sec-Sec-Sec-Sec-Sec-Sec-Sec-Error-Error

This combination converges fast because most of the steps compute the secant method and it's also robust because at the points where the secant method could diverge, the other two methods are applied.

In conclusion, from these two examples it can be seen that Dekker's method is a fast and robust method, but it can become slower if it implements bisection or the error step too often.

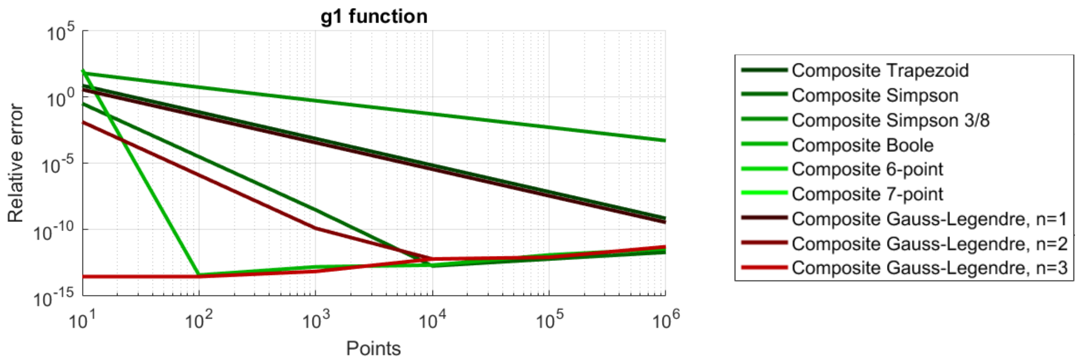
2.

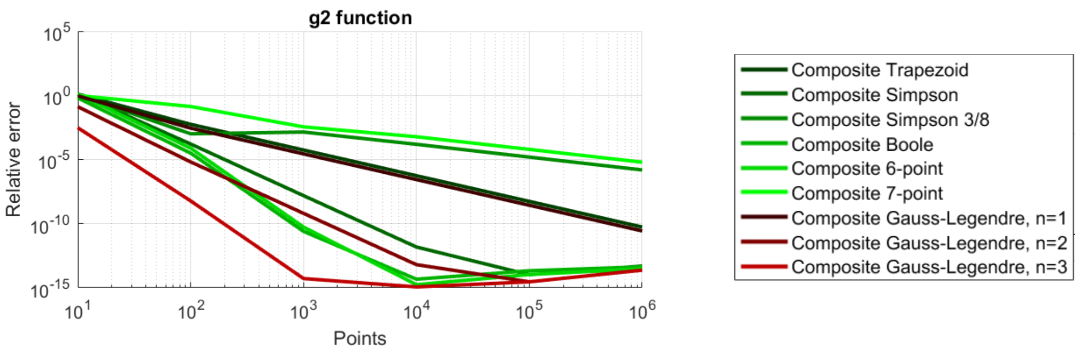
The values obtained for the 5 integrals are

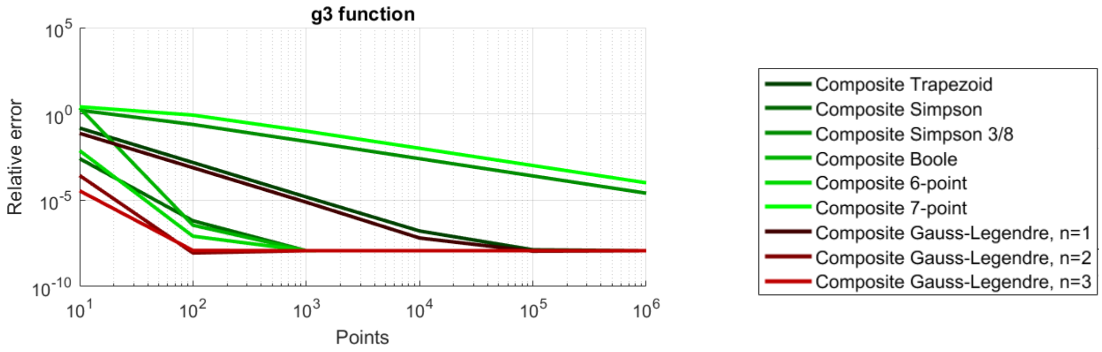
Function	Value	Method
I1	0.043662222222213	Gauss-Legendre Quadrature (n=3)
I2	1.49426768929622**7**	Composite Gauss-Legendre Quadrature (n=3)
I3	12.162401**687576580**	Composite Gauss-Legendre Quadrature (n=2)
I4	31.8170258333333**29**	Composite Gauss-Legendre Quadrature (n=3)
I5	0.820015034240487	Composite Trapezoid Method

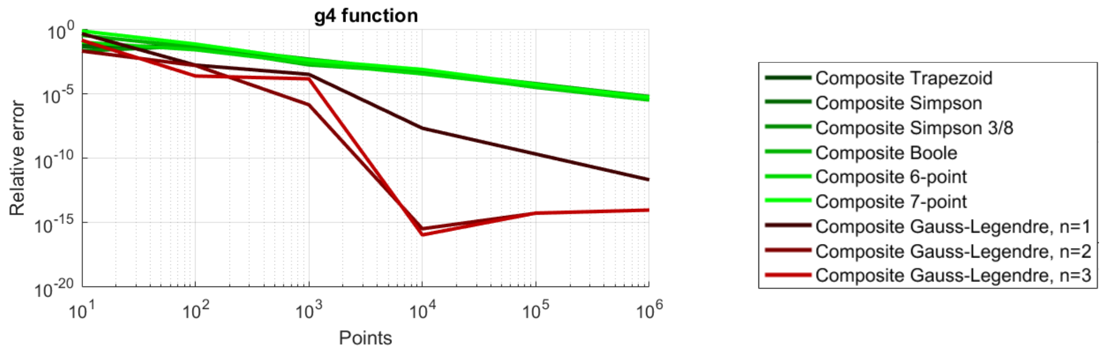
(Black numbers shows the difference between the values obtained and exact values).

The integral for the first function is exact because it's a polynomial. This could be determined by the fact that it was not needed to have more than 5 points to obtain a 14 decimal precision for the integral value, what suggest that a polynomial of order 6 or less. The Gauss-Legendre Quadrature with n=3 gives an exact solution (compared to the value given), which reinforces the idea of the polynomial.
In the case of the 5th integral, the value is computed with trapezoid method assuming straight lines between points, which gives an exact result. Another approach to the problem is to assume that the plot is a strain-stress curve for a linear-elastic material. In that case, doing a linear fitting of the curve passing through the origin gives a curve of the form y=1.71x, which integrated from the first strain point to the last one gives a value of 0.776903159494345, which correspond to the toughness of the material.
To obtain the best value, an error analysis was done, iterating several Newton-Cotes and Gauss-Legendre quadratures with different amounts of points. A convergence plot was made, which shows the following.









For the 4th integral the methods where used over the whole interval and between discontinuities, the first giving better accuracy respect the values given as solutions.