Seskio Loosveldt Tomàs

Ex 1 PDEs

1 -

$f(x) = x^3 + 2x^2 + 10x - 20 = 0$

$f'(x) = 3x^2 + 4x + 10$

$x^{k+1} = x^k - \frac{f(x^k)}{f'(x^k)}$

$x^0 = \sqrt[3]{20}$

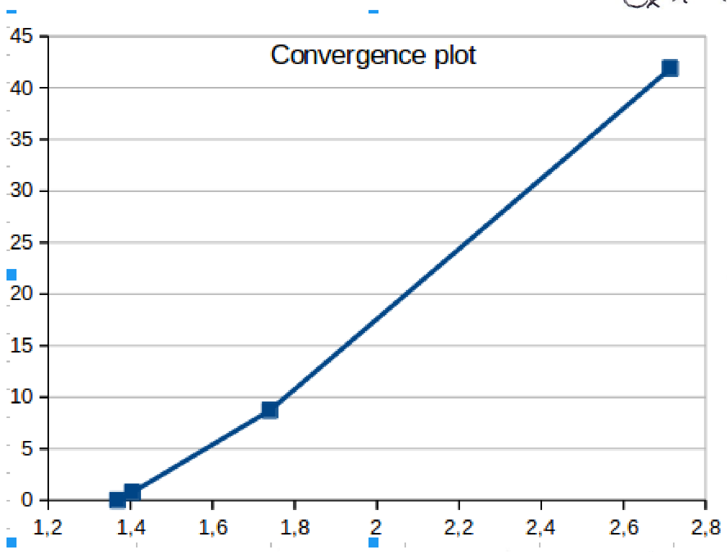
$x^1 = 1{,}73959$

$x^2 = 1{,}40497$

$x^3 = 1{,}36918$

$x^4 = 1{,}36881$

~~Exact~~ solution $\simeq 1{,}388$



5 - ↳ Analitic

a) A 3rd order quadrature will need a minimum of 2 points if using a Gauss quadrature:

$3 = 2n+1 \Rightarrow n = 1 \Rightarrow$ z_0 and z_1 are the points needed

$$\left.\begin{aligned} \int_0^1 1\,dx &= 1 = w_0 + w_1 \\ \int_0^1 x\,dx &= \frac{1}{2} = w_0 z_0 + w_1 z_1 \\ \int_0^1 x^2 dx &= \frac{1}{3} = w_0 z_0^2 + w_1 z_1^2 \\ \int_0^1 x^3 dx &= \frac{1}{4} = w_0 z_0^3 + w_1 z_1^3 \end{aligned}\right\}$$

Solving the system:

$$\boxed{\begin{aligned} w_0 &= \frac{1}{2} \\ w_1 &= \frac{1}{2} \\ z_0 &= \frac{1}{6}(3+\sqrt{3}) \\ z_1 &= \frac{1}{6}(3-\sqrt{3}) \end{aligned}}$$

b) Given the 4 equally spaced points it is possible to obtain a 3rd order quadrature because Newton-Cotes only requires 3 points

$$\left.\begin{aligned} \int_0^1 1\,dx &= 1 = w_0 + w_1 + w_2 + w_3 \\ \int_0^1 x\,dx &= \frac{1}{2} = \frac{w_0}{4} + \frac{w_1}{2} + \frac{3w_2}{4} + w_3 \\ \int_0^1 x^2 dx &= \frac{1}{3} = \frac{w_0}{16} + \frac{w_1}{4} + \frac{9w_2}{4} + w_3 \\ \int_0^1 x^3 dx &= \frac{1}{4} = \frac{w_0}{64} + \frac{w_1}{8} + \frac{27w_2}{64} + w_3 \end{aligned}\right\}$$

Solving the system:

$w_0 = \frac{2}{3}$

$w_1 = -\frac{1}{3}$

$w_2 = \frac{2}{3}$

$w_3 = 0 \rightarrow$ the 4th point is not required

6.

a) n + 1 ~~para~~ points that go from z_0 to z_n $\Rightarrow$ Order of the polynomial integrated exactly will be $2n+1$

b) $n = 2 \Rightarrow$ Order of the polynomials integrated exactly $\leq 2n+1 = 5$

i) $\int_0^{\pi} \sin x \, dx \rightarrow$ Can't be ~~written~~ computed exactly because the Taylor series only will be integrated exactly until order 5; the next terms will have an error

ii) $\int_0^1 x^3 \, dx \rightarrow$ Order $< 5 \Rightarrow$ It can be computed exactly

iii) $\int_0^1 x^4 \, dx \rightarrow$ Order $< 4 \Rightarrow$ It can be computed exactly

iv) $\int_0^1 x^{5.5} \, dx = \int x^5 dx \cdot \int \sqrt{x} \, dx \Rightarrow$ It can't be computed exactly

7-

$I_1 = \int_0^1 12x\,dx$, $I_2 = \int_0^1 (5x^3 + 2x)\,dx$

a) Trapezoidal rule over 2 intervals

$$\int_a^b f(x)\,dx \approx \frac{h}{2}\left[f(a) + 2f(a+h) + f(b)\right] \quad \text{with} \begin{cases} a = 0 \\ h = \frac{1}{2} \\ b = 1 \end{cases}$$

$I_1 \approx \frac{12}{4}\left[0 + 1 + 1\right] = 6$; $Err_1 = 0$

$I_2 \approx \frac{1}{4}\left[0 + 2 \cdot \frac{13}{8} + 7\right] = 2{,}5625$, $Err_2 = 0{,}3125$

The trapezoidal rule integrates exactly ~~polinomials~~ 1st order - polynomials

b) Simpsons rule over 2 intervals

$$\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f(a) + 4f(a+h) + f(b)\right] \quad \text{with} \begin{cases} a = 0 \\ h = 1/2 \\ b = 1 \end{cases}$$

$I_1 \approx \frac{12}{6}\left[0 + 2 + 1\right] = 6$; $Err_1 = 0$

$I_2 \approx \frac{1}{6}\left(0 + \frac{13}{2} + 7\right) = 2{,}25$; $Err_2 = 0$

Simpsons' rule integrantes exactly up to 3rd order polynomials

10-

$$\underbrace{\int_0^1 9x^3 + 8x^2\,dx}_{I_1} \underbrace{\int_0^1 y^3 + y\,dy}_{I_2} = I$$

Simpsons rule : $\int_a^b f(x)\,dx \approx \frac{h}{3}\left[f(a) + 4f(a+h) + f(b)\right]$ with $\begin{cases} a = 0 \\ h = 1/2 \\ b = 1 \end{cases}$

$I_1 \approx 4{,}91666$
$I_2 \approx \frac{3}{4}$ $\Bigg\}$ $I = I_1 \cdot I_2 = 3{,}6875$, $Err_I = 0$ $\Rightarrow$ The integral is exect because in each direction the polynomials can be integrated exactly.