1 $f(x) := x^3 + 2x^2 + 10x - 20 = 0$ $x^0 = \sqrt[3]{20}$

$$f(x) = x^3 + 2x^2 + 10x - 20 = 0$$

$$f'(x) = 3x^2 + 4x + 10$$

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$$x_{k+1} = x_k - \frac{x^3 + 2x^2 + 10x - 20}{3x^2 + 4x + 10}$$

$$x_1 = x_0 - \frac{x^3 + 2x^2 + 10x - 20}{3x^2 + 4x + 10}$$

$$x_1 = 2.7141 - \frac{x^3 + 2x^2 + 10x - 20}{3x^2 + 4x + 10} = 1.7394$$

$$x_2 = 1.7394 - \frac{x^3 + 2x^2 + 10x - 20}{3x^2 + 4x + 10} = 1.3885$$

$$x_3 = 1.3885 - \frac{x^3 + 2x^2 + 10x - 20}{3x^2 + 4x + 10} = 1.3680$$

x_0	$f(x)$	$f'(x)$
2.7141	41.8666	42.9554
1.7394	8.7076	22.1758
1.3885	0.4177	19.7195
1.3680	-0.0170	21.0862

Yes Newton's method is good because we acheive good amount in 4 ittretion and we can use this method

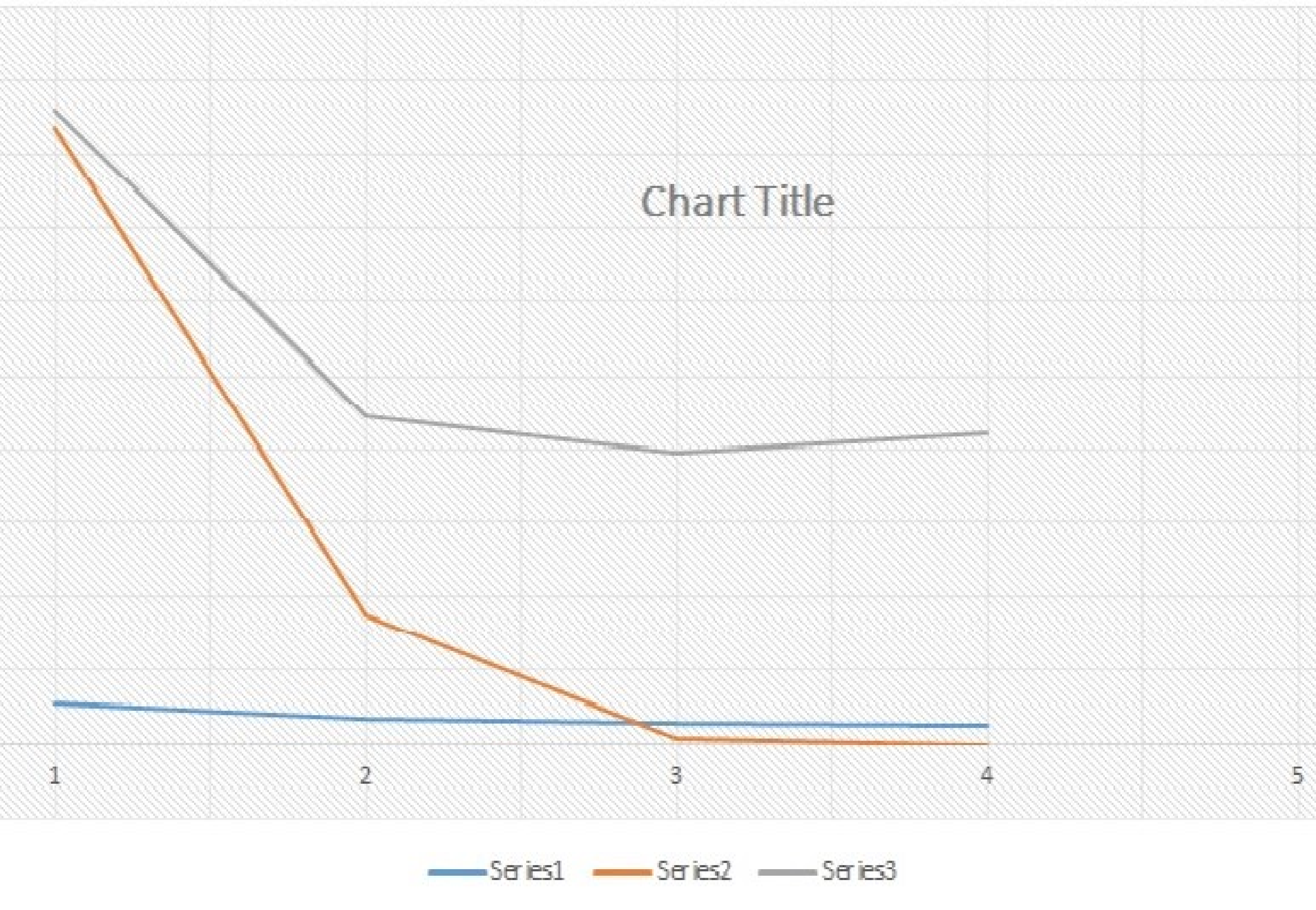
Chart Title
1
2
3
4
5
Series1
Series2
Series3

5 we are intersted in definition of third_order numrical quadrature in interval $(0,1)$

a) Determine the minimum number of integral Point and specify the integration Point weight

b) Is it Possible to obtain a third-order quadrature with the following four integration Point $x_0 = 1/4$, $x_1 = 1/2$, $x_2 = 3/4$ and x_3

$$x = \frac{(b-a)Z+b+a}{2} \qquad \sum w_i f(Z_i)$$

$$f(x) = Ax^3 + Bx^2 + Cx + D \, dx$$

$$w_0 f(1/4) + w_1 f(1/2) + w_2 f(3/4) + w_3 f(1)$$

$$\left[\frac{Ax^4}{4} + \frac{Bx^3}{3} + \frac{Cx^2}{2} + Dx\right]_0^1 \Rightarrow \frac{A}{4} + \frac{B}{3} + \frac{C}{2} + D$$

(7) $\int_0^1 12x\,dx$ $\qquad$ $\int_0^1 (5x^3+2x)\,dx$

i) Trapezoidal rule over 2 uniform intervals

ii) Simpson's rule over 2 uniform intervals

Compute the error of both approximation Are the method

$$\Delta x = \frac{b-a}{n}$$

Trapezoidal rule

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + 2f(x_2) + \dots + 2f(x_{n-1}) + f(x_n)\right]$$

$$\int_0^1 12x\,dx \qquad \Delta x = \frac{b-a}{n} = \frac{1-0}{2} = \frac{1}{2}$$

$x_i = a + i\Delta x$

$x_0 = 0 + 0(1/2) = 0$

$x_1 = 0 + 1(1/2) = 1/2$

$x_2 = 0 + 2(1/2) = 1$

$$\int_a^b f(x)\,dx \approx \frac{\frac{1}{2}}{2}\left[f(0) + 2f(1/2) + f(1)\right]$$

$f(0) = 0$ $\qquad$ $f(1) = 12$

$f(1/2) = 6$

$$\Rightarrow \frac{1}{4}\left[0 + 2(6) + 12\right] = 6$$

Simpson rule

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + \dots + 4f(x_{n-1}) + f(x_n)\right]$$

$$\Rightarrow \frac{\frac{1}{2}}{3}\left[f(0) + 4f(1/2) + f(1)\right] \Rightarrow \frac{1}{6}\left[0 + 4(6) + 12\right] = 6$$

$$\int_0^1 12x\,dx \Rightarrow \left.\frac{12x^2}{2}\right|_0^1 = 6$$

$$\int_0^1 (5x^3+2x)\,dx \qquad \Delta x = \frac{b-a}{n} = \frac{1-0}{2} = \frac{1}{2} \qquad x_i = a + i\Delta x$$

$x_0 = 0 \rightarrow f(0) = 0$

$x_1 = \frac{1}{2} \rightarrow f(1/2) = \frac{13}{8}$

$x_2 = 1 \rightarrow f(1) = 7$

Trapezoidal rule

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}\left[f(x_0) + 2f(x_1) + f(x_2)\right] \Rightarrow \frac{1}{4}\left[f(0) + 2f(1/2) + f(1)\right]$$

$$\frac{1}{4}\left[0 + 2\left(\frac{13}{8}\right) + 7\right] = 2.56$$

Simpson rule

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + \cdots + 4f(x_{n-1}) + f(x_n)\right]$$

$$\Rightarrow \frac{1}{6}\left[f(0) + 4f(\tfrac{1}{2}) + f(1)\right] \Rightarrow \frac{1}{6}\left[0 + \frac{52}{8} + 7\right] = 2.25$$

$$\int_0^1 5x^3 + 2x\,dx \Rightarrow \frac{5x^4}{4} + \frac{2x^2}{2}\Big|_0^1 = \frac{5}{4} + \frac{4}{4} = \frac{9}{4} = 2.25$$

(10) Perform numrical integration of

$$\int_0^1\int_0^1 (9x^3+8x^2)(y^3+y)\,dx\,dy$$

using Simpson's rule in each direction. Is the approximation behaving as expected?

$$\int_0^1\int_0^1 (9x^3+8x^2)(y^3+y)\,dx\,dy \qquad \Delta x = \frac{b-a}{n} = \frac{1-0}{2} = \frac{1}{2} \qquad x_i = a + i\Delta x$$

$$\int_0^1 (9x^3+8x^2)\,dx$$

$x_0 = 0 \to f(0) = 0$

$x_1 = \frac{1}{2} \to f(\frac{1}{2}) = \frac{25}{8}$

$x_2 = 1 \to f(1) = 17$

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left[f(x_0) + 4f(x_1) + \cdots + 4f(x_{n-1}) + f(x_n)\right]$$

$$\frac{1}{6}\left[f(0) + 4f(\tfrac{1}{2}) + f(1)\right] = \frac{1}{6}\left[0 + 4\left(\frac{25}{8}\right) + 17\right] = 4.91$$

$$\int_0^1 (y^3+y)\,dy \qquad \Delta x = \frac{b-a}{n} = \frac{1}{2} \qquad x_i = a + i\Delta x$$

$x_0 = 0 \to f(0) = 0$

$x_1 = \frac{1}{2} \to f(\frac{1}{2}) = \frac{5}{8}$

$x_2 = 1 \to f(1) = 2$

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\left[f(0) + 4f(\tfrac{1}{2}) + f(1)\right] \Rightarrow \frac{1}{6}\left[0 + 4\left(\frac{5}{8}\right) + 2\right] = 0.75$$

$$\Rightarrow 4.91 \times 0.75 = 3.682$$

$$\int_0^1\int_0^1 (9x^3+8x^2)(y^3+y)\,dx\,dy \Rightarrow \int_0^1\int_0^1 (9x^3y^3 + 8x^2y^3 + 9x^3y + 8x^2y)\,dx\,dy$$

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$$\int_0^1 \left[9\frac{x^4}{4} y^3 + 8\frac{x^3}{3} y^3 + 9\frac{x^4}{4} y + \frac{8x^3}{3} y \right]_0^1 dy = \int_0^1 \frac{9}{4} y^3 + \frac{8}{3} y^3 + \frac{9}{4} y + \frac{8}{3} y \, dy$$

$$\frac{9}{16} y^4 + \frac{8}{12} y^4 + \frac{9}{8} y^2 + \frac{8}{6} y^2 \Big|_0^1 = \frac{9}{16} + \frac{8}{12} + \frac{9}{8} + \frac{8}{6} = 3.520$$