Extended Abstract

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Effective Thermal Conductivity of Trapped Fluids

1. Introduction

Study of trapped fluids has some important applications in Oil&Gas industry. In the subsea Oil domain, the thermal analysis of Wellhead and Manifolds is performed to study the temperature behavior after the main oil valve is closed/ opened. The main purpose is to study hydrate formations which start forming once the temperature reaches below a certain value. The Wellhead and Manifold have a lot of annular spaces which contain trapped fluids, mainly sea-water. Therefore, to study the thermal response of the system, it is important to take into consideration these convective currents in the trapped fluids.

One way to analyze such thermal response is to perform a CFD within the trapped region and solve the CFD problem along with the solid conduction problem for the surrounding solids. But this method is computationally very costly. Especially, in the preliminary design stages where the design keeps changing from time to time, it is not an efficient method. Also, mainly the region of interest is the Wellhead and not the trapped fluids. So, an alternative method has been proposed in the literature and books.

2. Convection in Trapped Fluids

A trapped fluid is any fluid, which is enclosed by solids on all sides. A general, simplified case has been shown below in Figure 1. In a rectangular void, the fluid has been filled. One of the vertical walls is at a higher temperature than the other wall.

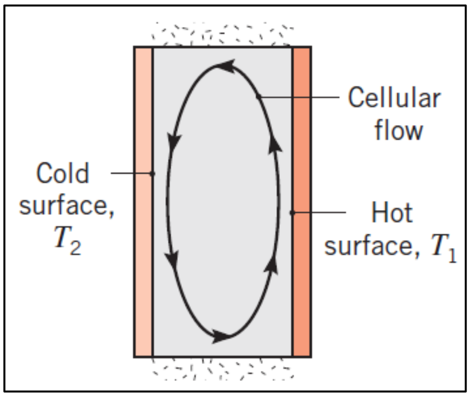

Figure 1: A Trapped Fluid

Rayleigh Number is defined to quantify the convective versus conductive effects for the trapped fluid. It is defined as follows.

$$Ra = Gr\ Pr$$

Where,

$$\text{Prandtl Number (Pr)} = \frac{C_p\ \mu}{k}$$

$$\text{Grashof's Number (Gr)} = \frac{g\beta l^3}{\nu^2} \times (T_1 - T_2)$$

The Grashof's number is defined as the Ratio of Buoyancy to Viscous Forces. This number (and Rayleigh's number since it is dependent on Grashof's number) plays a key role to decide the scale of natural convection. If Grashof's number is big, more natural convection will occur.

The phenomenon of natural convection is trapped fluids occurs as follows. There is always two quantities which are counteracting each other. One is the temperature difference in the walls. If this value is more, then it will try to drive the flow due to density difference created because of temperature variation in the domain. But this driving force will be opposed by the viscous forces in the fluid. So if the temperature difference is small (T1-T2), then the viscous forces dominate and there is no flow. This implies that the heat is only transferred via conduction. But if the temperature difference (T1-T2) exceeds a certain value, the flow driving forces dominate over the viscous forces and there is a natural convection within the fluid. Hence, the heat is transferred using conduction plus advection. (Together they are called as convection)

Figure 2 shows this phenomenon. Figure 2(a) shows the flow when the Rayleigh number is 2E6. Figure 2(b) shows the flow when the Rayleigh number is 4E7 and Figure 2(c) shows the flow when the Rayleigh number is 1E10.

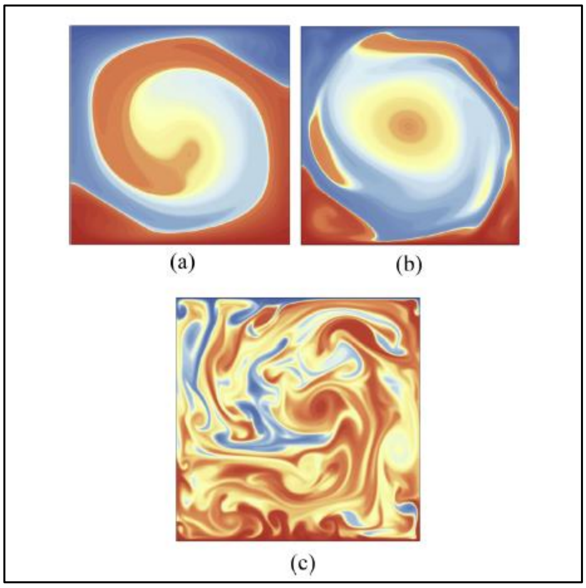

Figure 2: Natural Convection in Trapped Fluids

This connective phenomenon in fluids is called as Benard cells, which assist the transfer of heat. Figure 3 shows the Benard Cells for a hexahedral cavity.

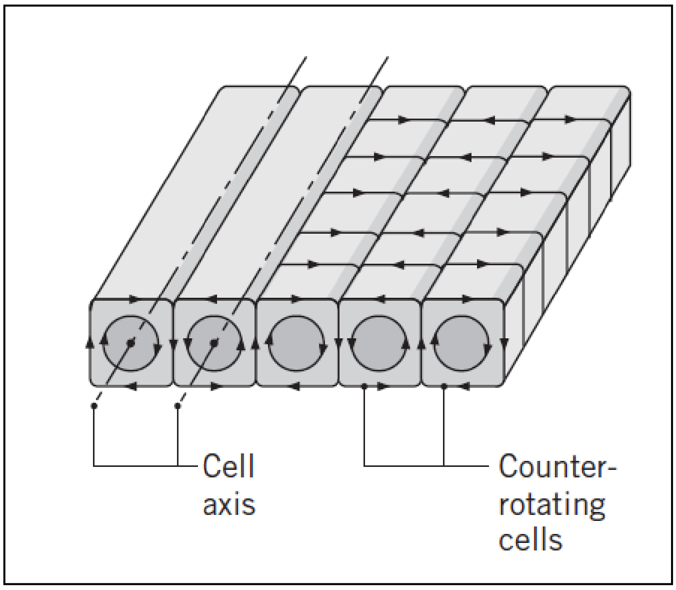

Figure 3: Benard Cells

3. The Concept of $K_{effective}$

In general, when there is no convection, the fluid will behave as a solid with the fluid's conductivity. But with convection, there is additional heat transfer. In order to account for this additional convective heat transfer, the fluid is treated like a solid but is assumed to conduct heat with a higher conductivity called as 'Effective Thermal Conductivity'. This parameter depends on various parameters such as Prandtl Number, Grashof Number, Geometrical Dimensions, and Orientation to name a few. The relations of relating these parameters with the effective thermal conductivity are mainly empirical in nature. A general form of $K_{effective}$ can be written as follows,

$$K_{effective} = K \times C \times (Gr\ Pr)^n \times \left(\frac{L}{l}\right)^m$$

Where,

C,m,n = constants obtained from empirical relations
L,l = Geometry Parameters
Pr = Prandtl Number
Gr = Grashof Number

Experiments have been conducted to obtain the empirical relations and parameters C,m and n are obtained from these experiments.

4. The limitation of the $K_{effective}$

Consider a simplified version of a typical problem in Oil&Gas domain as shown in Figure 4. One of the vertical walls of the solid is at T1 and the other is at T2. Now to obtain the $K_{effective}$ of the trapped fluid, the temperatures at the boundary of the fluid are needed. But unfortunately it is never known. In order to perform the analysis one needs to know the $K_{effective}$ of the trapped fluid, but to obtain the $K_{effective}$ one needs to know the temperatures at the boundary of the fluid. These temperatures can be obtained only after performing the analysis. So there is a dependency and this puts the limitation on the use of this method.

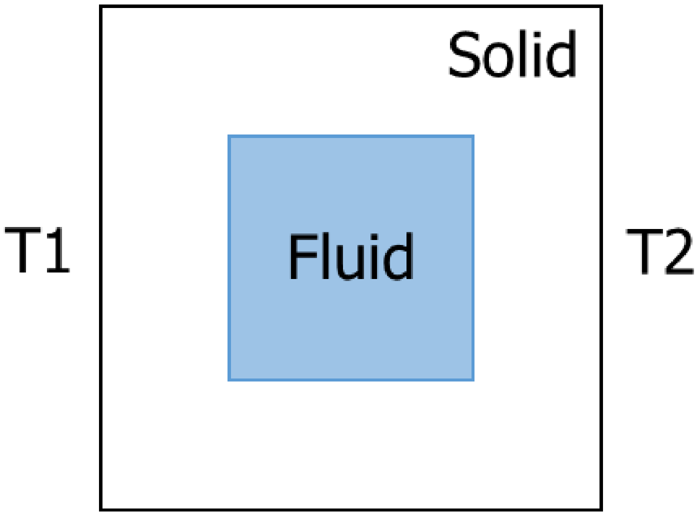

Figure 4: A simplied General Problem

5. Solution: Fixed Point Iteration Method

The solution to tackle this limitation is to perform a fixed point iteration method to solve this implicit behavior. The proposed algorithm is as follows.

1. Assume Some Temperature T1, T2
2. Calculate $K_{effective}$
3. 3. Use $K_{effective}$ as input in FEA
4. 4. Use $K_{effective}$ as input in FEA
5. Recalculate $K_{effective}$ but with new T1 and T2
6. Compare new $K_{effective}$ with old $K_{effective}$
7. If the difference in K's is acceptable then stop, else go to step 3 with the new $K_{effective}$

6. Limitations of Iterative Approach

The major limitation is that the convergence is not guaranteed. Another major limitation is that the empirical formulae to calculate the $K_{effective}$ change as the range changes. This pose problems in the convergence.

7. Conclusion

Despite the limitations, the method works most of the time based on author's professional experience. The time reduced by the method (by avoiding the CFD) is significant. The errors in the method are less, since the formulae for $K_{effective}$ are empirical in nature. Also, automation can be performed for the iterations and calculation of $K_{effective}$.

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