Assignment 3

Martí Burcet Rodríguez

January 8th, 2016

1 Introduction:

The aim of this assignment is to look at a code to solve 2D nonlinear elastic problems and compare the results with the linear elasticity ones. The solver implemented to assess the nonlinear regime is base on Newton-Raphson method. In addition the code has the option to combine a Line-Search algorithm in order to distinguish between local and global stable points. The solver can also introduce random perturbations to overcome the possible unstable solutions and find the good one.

2 Approach to the main code:

The main code that controls the process of the main problem is called *main_buckling_iterative.m* . This file is where the user can choose his preferences to solve the problem. After this the same script will call other subroutines to compute the solution.

Some of the most useful aspects to know about this code are:

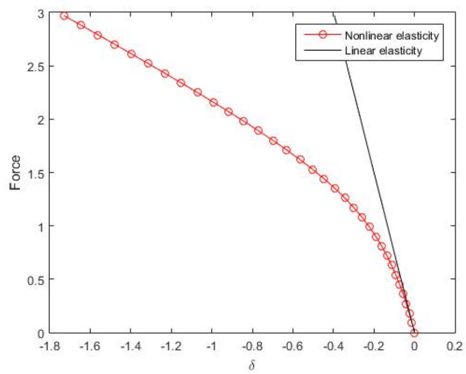
1. The definition of the example: **line 11**.
2. The choice of solution method: **line 20**. To choose between: "*0: plain Newton-Raphson, 1: Newton-Raphson with line search*".
3. The implementation of the solution method: **line 53**.
4. The implementation of the incremental-iterative strategy, with smart initial guesses for imposed displacements: **line 42**.
5. The introduction of random perturbations in the initial guesses of the solution method: **line 50**.

3 Comparison between methods:

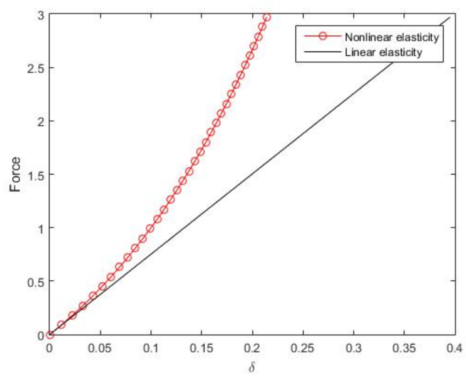
To show the effect of the different methods to the solution of problems of nonlinear elasticity some examples have been run using combinations of the methods explained in the introduction (Newton-Raphson with and without line search and with and without perturbations). In addition to clearly state the effect of the nonlinear behaviour of the model the linear solution has always been computed for comparison purposes.

3.1 Example 0

The Example 0 represents the upsetting of a block under a dead load and what we want to see is the influence of the sign of the force to



(a) Traction; f=-3e0



(b) Compression; f=3e0

Figure 1: Response of the block under dead load using Newton-Raphson without line search and with perturbations.

the response of the theory. The solving algorithm chosen is the Newton-Raphson without line search but with random perturbations. In Figure 1 we can see how the sign of the force applied (traction in (a) and compression in (b)) makes the response completely behave different from one model to the other. While in linear elasticity the response under traction or compression is identically proportional, with the only difference of the sign of the slope, the nonlinear elastic curve show a much more rigid response in the compressive case than in the traction one. The response of the nonlinear case under traction can represent the influence of successive damages to the block produced by the large deformation. This damage of course will effect the stiffness of the material so the larger the deformation, the smaller the resistance and therefore the faster the elongation.

3.2 Example 1

Similarly to Example 0, Example 1 represents the upsetting of a block but instead of under a dead load under imposed displacements of opposite sign. Again the Newton-Raphson method without line search and with perturbations has been selected.

In this case what is interesting to see from the results is how the linear elastic model gives a deformed mesh which doesn't have physical sense. In Figure 2 the deformed shape of the block given by the linear elasticity is compatible with the equilibrium equations, otherwise the code hasn't reached a solution, but is not the real one. This shows that within the theory of linear elasticity there isn't uniqueness of solutions and that the solver used in this exampled cannot distinguish between compatible and incompatible solutions.

3.3 Examples 2 and 3

The next set of examples run were 2 and 3 which represent the deformation of a slender beam under imposed displacements and dead load respectively. These models were run with 4 different solving schemes each one: Newton-Raphson with and without line search and with and without random perturbations. In Figure 3 it is shown how

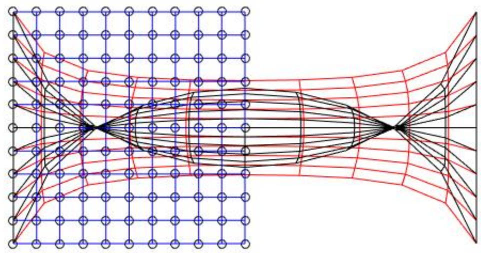
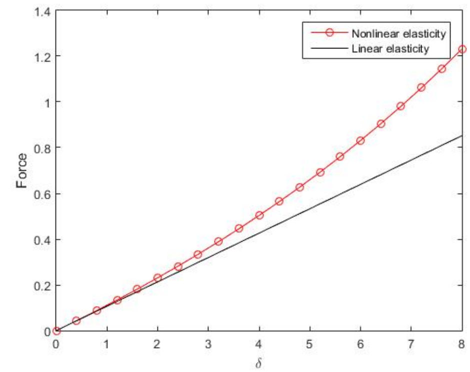
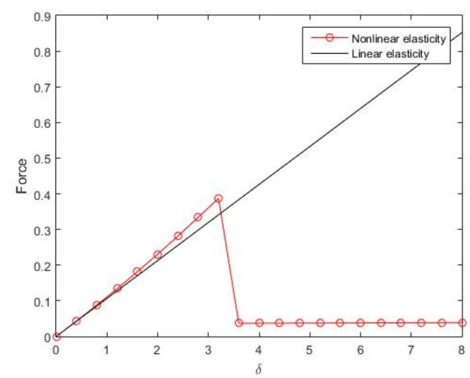


Figure 2: Deformed mesh under an imposed displacement of 0.5 applied in 50 increments of 0.01.

the introduction of perturbations make reach different solutions, therefore there isn't a unique solution although only one is the global minimum potential. Looking at Figure 3b what is seen is how the nonlinear elasticity theory find a solution where the beam breaks (plasticised) and doesn't hold anymore the load and deform indefinitely. It is the combination of the line search and the random perturbations that make the code find the solution of breaking of the beam, because all the other cases tested weren't able to predict the plastification of the beam.



(a) Newton-Raphson without line search without perturbations.

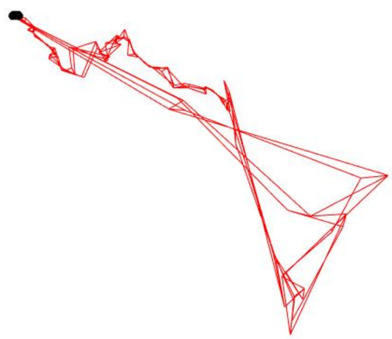


(b) Newton-Raphson with line search with perturbations.

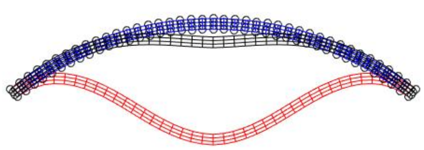
Figure 3: Force-displacement graphs of Example 2.

3.4 Examples 4 and 5

The last two examples run with this code model the deformation of the deformation of an arch upset with dead loads at the centre and near the supports. For this purpose Newton-Raphson with and without line search and without random perturbations have been studied. We won't enter to study the different response of the arch depending on where the load is applied but to see the influence of each method to the stability of the solution. In Figure 4a we can easily see that the solution that reaches the nonlinear elasticity without line search and without perturbations is completely unstable. This is due to the fact that the Hessian used to advance in the iterative process have negative eigenvalues and this makes the algorithm unstable and that equilibrium isn't reached after some load has been applied, so when large deformation begin to occur and when the arch starts to deviate from the linear elas-



(a) Newton-Raphson without line search without perturbations.



(b) Newton-Raphson with line search without perturbations.

Figure 4: Deformed shapes resulting from Example 4.

ticity field. On the other hand, Figure 4b that was computed using Newton-Raphson combined with the line search scheme gives for the nonlinear elasticity case a stable and plausible solution. It is not presented in this report, but the big difference between the linear and nonlinear solutions arises from the fact that at some load the arch looses stiffness and deforms quickly for small increments of load. Of course these discontinuities cannot be captured by the linear elasticity that always present the same response don't matter how big is the load or the strain applied. Just a final remark about Example 4, if one looks at Figure 4b it is possible to see that a second mode of deformation is becoming to be formed in the nonlinear case. This behaviour is well captured in the force vs. displacement curve presented in 5. It is seen how near force equal 0.3 there is a fast deformation with a small increment of load. This correspond to the first mode of deformation of the structure. After this episode, the arch starts resisting again, which correspond to the beginning of formation of the second mode of deformation.

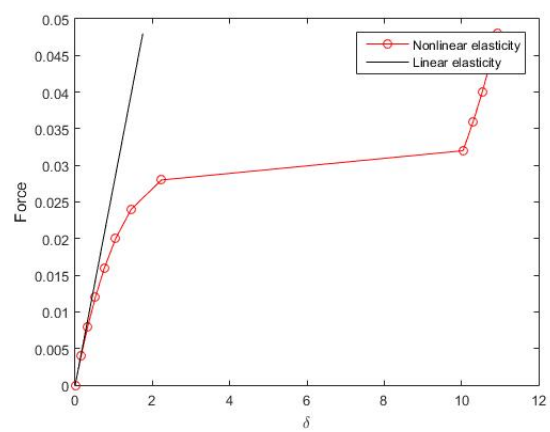


Figure 5: Force vs. displacement curve for the Example 4 with line search and without perturbations.

4 Conclusions:

In this report the influence of the different solving methods have been exposed. A first conclusion to state is that the linear elastic regime is only valid on the field of small deformations. In this zone the proportionality between stresses and strains are almost linear, so reversible, and also symmetric with respect to the sign of the force. In this zone the nonlinearities are so small than can be neglected without big loose of precision. However once important deformations are occurring, the damage of the micro-structure, changes in the geometry and other inhomogeneities start playing an important role and the response diverge from the linear one.

Regarding the solving methods, it has been checked that Newton-Raphson without line search can lead to unstable situations where equilibrium isn't achieved. In addition the introduction of random perturbations to the iterative process helps to overcome intermediate unstable solutions that don't correspond to global minimum potentials. Therefor after all these trials the best option to perform calculations under the assumptin of nonlinear elasticity is a solving iterative method both with line search and with random perturbations.