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Assignment 3: Non-linear Elastic Block

a) Identify in the code (file, lines) the following items:

(a) The definition of the example (loading, geometry)

The choice of the example is done in line 8 of file “main_buckling.m”.

The definition of the loading and geometry of the example is done in file “preprocessing.m”. From line 8 to 59, the basic parameters used to define the geometry (the coordinates of two vertices are defined), mesh (the number of divisions in both x and y directions) and loading (which is the magnitude of the applied force) for the chosen example are defined, as well as the selection of applied forces or applied displacements. In these lines, the vector lambda, which defines the increments of the load, is also defined. From line 61 to 75 the matrices X and T of the mesh are created (using, amongst other, the function “CreaMalla”). The setting of the loading and boundary conditions are finished in lines 108 to 136.

(b) The choice of the solution method (Newton’s method with or without line-search)

The choice of the solution method is done in lines 18 and 20 of file “main_buckling.m”. First, the solver: vanilla Newton-Rapshon, Newton-Rapshon (NR), Modified NR, L-BFGS or Conjugate Gradient. However, in the function “Equilibrate” only the Newton-Rapshon and its vanilla and modified versions are implemented, so both the L-BFGS of Conjugate Gradient are actually not available. In line 20, the line-search method can be activated or deactivated.

Other options can be selected from line 14 to 31 of the file “main_buckling.m”, such as the maximum number of iterations, the tolerances used or the type of line-search.

(c) The implementation of the solution method

The different solution methods are implemented between lines 9 and 94 of “Equilibrate.m” file. If the line-search option is activated, the “LineSearch” function is called. Two different versions of the line-search method are implemented in file “LineSearch.m”: A “simple backtracking version” or a version which uses Matlab functions.

(d) The implementation of the incremental-iterative strategy, with smart initial guesses for imposed displacements

The incremental-iterative strategy is implemented in lines 66 to 97 of “main_buckling.m” as a for loop on the load increments. The initial guesses for the displacements are implemented in lines 70 to 74 (only used in examples 1 and 2, the other ones do not have imposed displacements).

(e) The introduction of random perturbations in the initial guesses of the solution method

The introduction of random perturbations in the initial guesses of the solution is implemented in line 76 of file “main_buckling.m”. This line is written as a commentary, but random perturbations can be implemented just removing the symbol “%”.

b) Run the different examples and comment the features of the nonlinear model.

The upsetting of a block is represented in examples 0 and 1. Since it is not a slender geometry, it makes no sense to use the linearized buckling analysis, so nonlinear analysis will be compared only with linear elasticity results.

Example 0 uses an imposed force to upset the block. Two different forces have been studied: f=3 (Fig. 1) and f=-3 (Fig. 2) using the Newton-Rhapson method without line-search. As is depicted in the figures, the linear analysis (Black) is not able to represent the non-linear deformation (red), but can be used to have an idea of the shape of the final geometry. Another conclusions that can be extrapolated from example 0 is the fact that both the elongation and transversal deformation obtained with the linear elasticity model is symmetric with respect to the sign of the load, while for the non-linear case there is a non-symmetric deformation with respect to the sign of the load (under a traction load there is a large elongation, but the magnitude of the displacements is lower under a compression force).

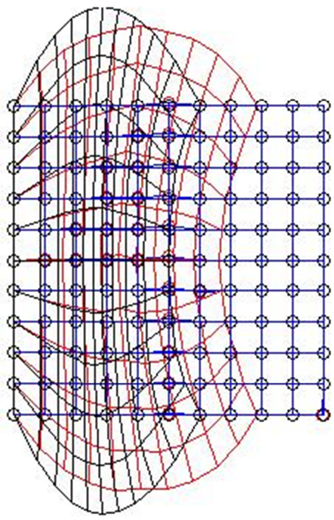

Fig. 1-Example 0 with f=3. Blue for the initial mesh, red for non-linear analysis and black for linear analysis.

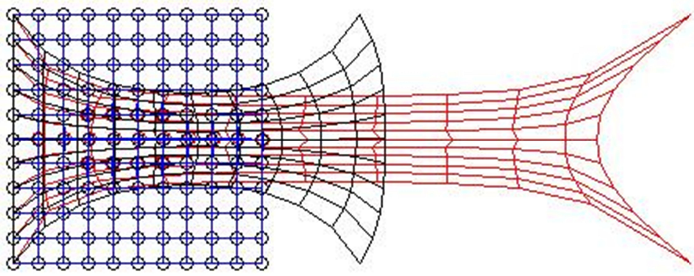

Fig. 2-Example 0 with f=3. Blue for the initial mesh, red for non-linear analysis and black for linear analysis.

Example 1 represents the deformation of the block due to imposed displacements. The results obtained when imposing different positive stretches are depicted in Fig. 3. The nonlinear analysis has been done using Newton-Rhapson's method without line-search. The results obtained show that the linear analysis can provide a good approximation to the transversal deformation obtained for small deformations (Fig. 3a). However, the results obtained for larger deformations do not match non-linear analysis, especially for very large deformations where very unrealistic results are obtained (Fig. 3 c and d).

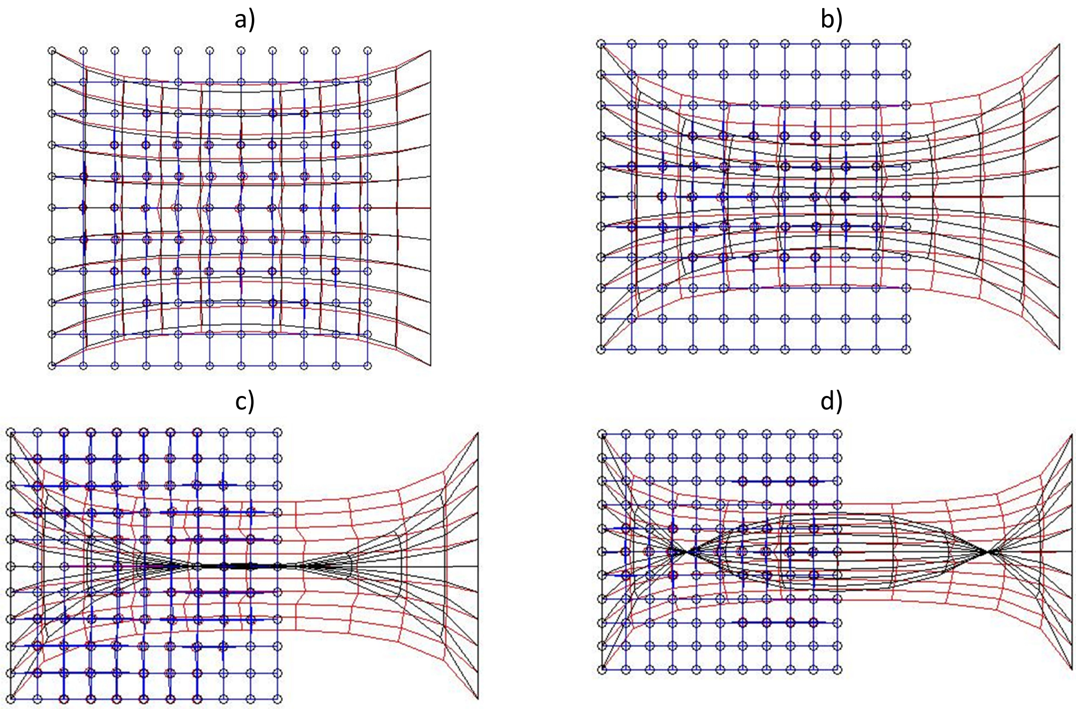

Fig. 3-Example one with positive displacement. Blue for the initial mesh, red for non-linear analysis and black for linear analysis. a) $\boldsymbol{\lambda} = [\mathbf{1:0.025:1.2}]$ b) $\boldsymbol{\lambda} = [\mathbf{1:0.025:1.5}]$ c) $\boldsymbol{\lambda} = [\mathbf{1:0.025:1.75}]$ d) $\boldsymbol{\lambda} = [\mathbf{1:0.025:2}]$

The results obtained when imposing different negative stretches in Example 1 are depicted in Fig. 4. Since large compressions can produce failure in the nonlinear analysis, Newton-Rhapson's method with line-search has been used. The results obtained are similar to the ones obtained for positive stretches: for low displacements (Fig. 4 a) the linear elasticity analysis can provide acceptable results while for larger deformations (Fig. 4 b) the nonlinear behavior cannot be explained using linear analysis.

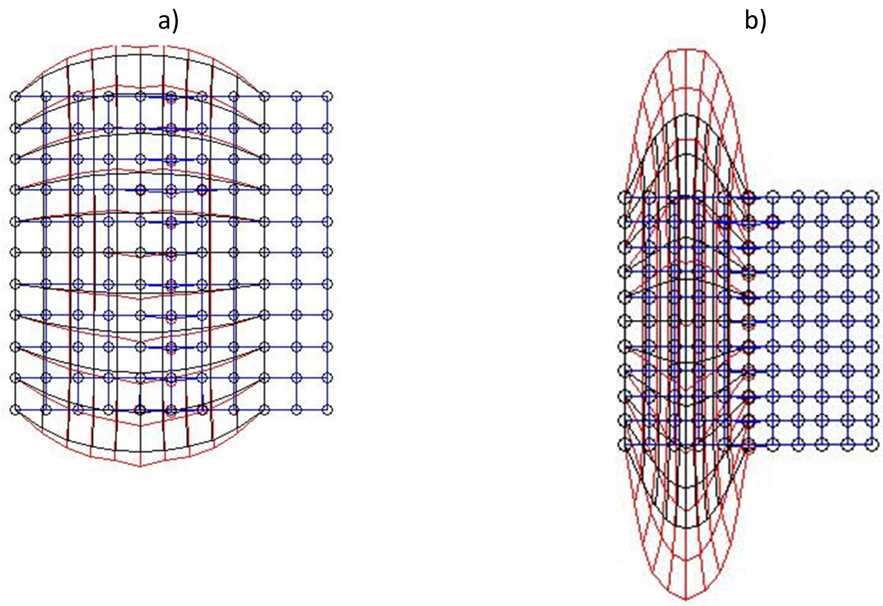

Fig. 4-Example one with negative displacement. Blue for the initial mesh, red for non-linear analysis and black for linear analysis. a) $\boldsymbol{\lambda} = [\mathbf{1}: -\mathbf{0.025}: \mathbf{0.8}]$ b) $\boldsymbol{\lambda} = [\mathbf{1}: -\mathbf{0.01}: \mathbf{0.5}]$

Examples 2 and 3 represent the compression of a slender beam. In example 2, the compression is induced due to imposed displacements. The result obtained without using line-search or random perturbations is depicted in Fig. 5. As can be seen, the result obtained in the nonlinear analysis is very similar to the one obtained using linear analysis. However, as can be inferred from the linear buckling analysis, the presence of buckling is expected in this problem. The non-linear analysis can be improved using random perturbations (Fig. 6). As can be seen, the results obtained adding random perturbations is completely different, so it can be concluded that the solution of the non-linear problem is not unique. This result has been obtained using the line-search method because the computation fails when it is not used.

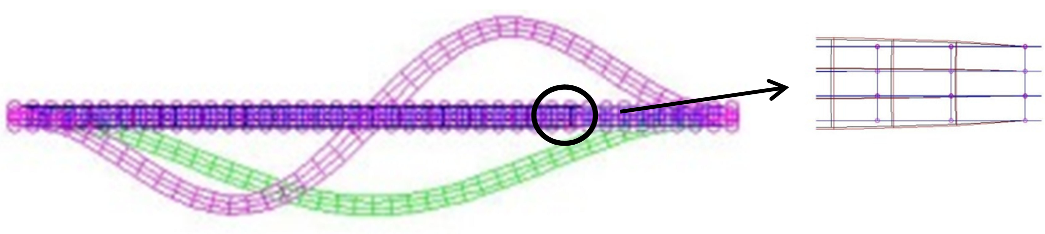

Fig. 5- Example 2 without random perturbations. Blue for the initial mesh, red for non-linear analysis, green and violet for first and second buckling modes and black for linear analysis.

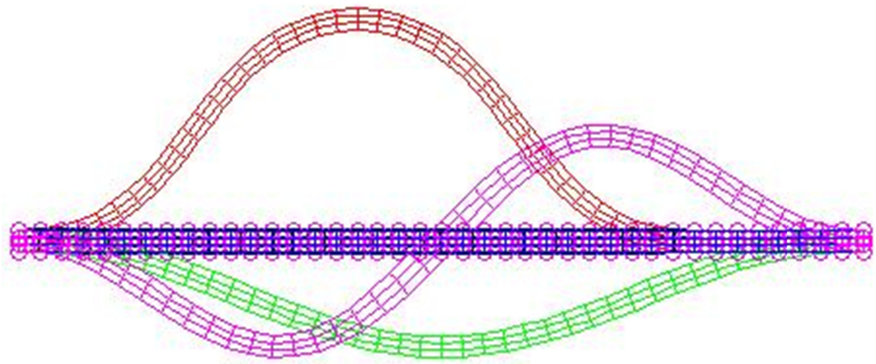

Fig. 6- Example 2 witht random perturbations. Blue for the initial mesh, red for non-linear analysis, green and violet for first and second buckling modes and black for linear analysis.

In example 3 the compression is induced by a force. This problem is solved using Newton-Rhapson method with line-search because convergence cannot be achieved without line-search. The result obtained is depicted in Fig. 7. If random perturbations were used, the same result would be obtained. As is depicted there's buckling, so the linear elastic analysis cannot be used to analyze this problem. The first buckling mode approximates the nonlinear solution, but not the second buckling mode.

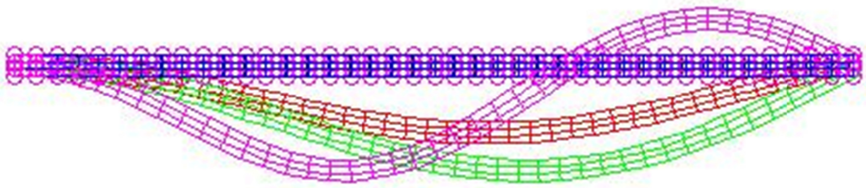

Fig. 7- Example 3. Blue for the initial mesh, red for non-linear analysis, green and violet for first and second buckling modes and black for linear analysis.

Examples 4 and 5 represent an arch with different dead loads. In example 4, the dead load is placed at the center of the arch, while in example 5 the dead load is placed near the supports. This problem is not stable, so line-search method is needed to achieve convergence, but the utilization of random perturbation does not affect the result. The results obtained for a force equal to one are depicted in Fig. 8 and Fig. 9. As can be seen, neither the linear elastic analysis or linear buckling analysis can capture the final deformation obtained using the non-linear analysis.

The solution of the nonlinear problem is unstable. For example, solving example 4 with forces equal to 0.55 and 0.6 leads to very different results. This is depicted in Fig. 10 and Fig. 11.

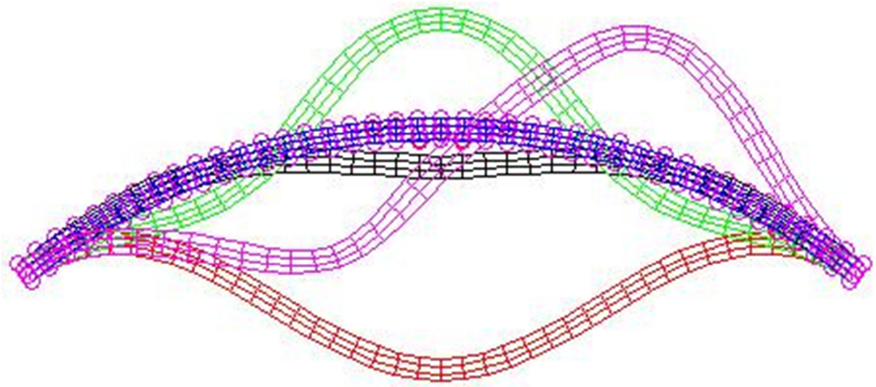

Fig. 8- Example 4 without random perturbations and using Newton's method with line-search. Force equal to 1.

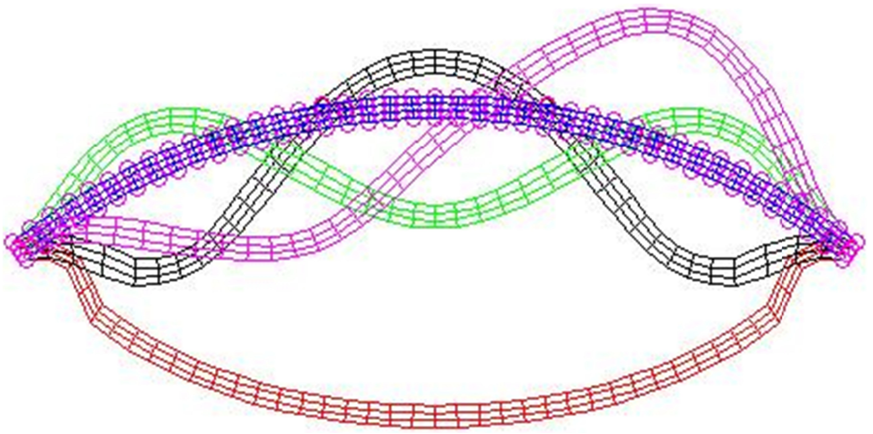

Fig. 9- Example 5 without random perturbations and using Newton's method with line-search. Force equal to 1.

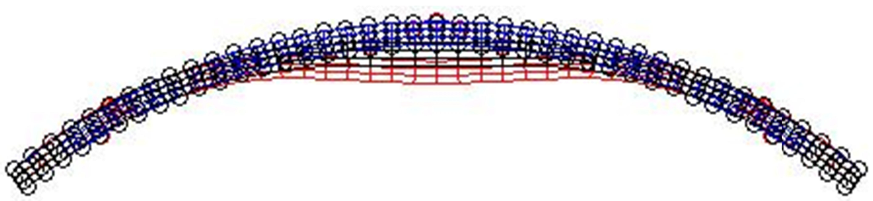

Fig. 10- Example 4. Force equal to 0.55.

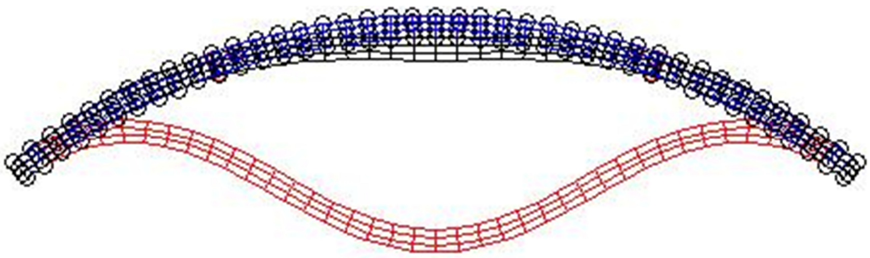

Fig. 11- Example 4. Force equal to 0.6.

Several conclusions arise from this work. First, it can be concluded that the linear analysis can only be used for very small deformations. For large deformations, the buckling modes can be used to know if buckling appears in the problem, but the solution obtained is very poor. It can also be concluded that the non-linear problem is non-stable, has no unique solution, there is no symmetry with respect to the sign of the loads and there is no proportionality with respect to the loads. This makes the non-linear problem much harder to solve and produces the need of the utilization of techniques such as the line-search method, the utilization of smart initial guesses and an incremental-iterative strategy and the introduction of small random perturbation. However, not all these methods are necessary or useful for every case, so the solution of different cases may need the utilization of different techniques.