Assignment 2 - Transfinite PDE Tool-Box

Daniel Díez Rodríguez

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Solve the following problem with the MATLAB PDE ToolBox:

$$u_t - \triangle u = f \tag{1}$$

$$\text{in } \Omega = [0,1]^2$$

$$f(x,y,t) = -3e^{-3t} \tag{2}$$

We consider an initial condition at t=0:

$$u(x,y,t=0) = x^2 + xy - y^2 + 1 \tag{3}$$

And the following boundary conditions:

$$\begin{gathered} u_n(x=0,y,t) = -y \\ u_n(x=1,y,t) = 2+y \\ u(x,y=0,t) = x^2 + e^{-3t} \\ u_n(x,y=1,t) = x-2 \end{gathered}$$

The analytical solution of this problem is given by:

$$u(x,y,t) = x^2 + xy - y^2 + 3e^{-3t} \tag{4}$$

1. Solve the problem and refine the initial mesh up to 4 times. Verify that the theoretical convergence order holds.
2. How is the solution affected when we modify the final time?
3. We are interested in obtaining the solution at time $t_{end} = 50$. Find a more efficient manner to solve this problem. You do not need to prove the equivalence mathematically, but you need to provide numerical evidence of the new method.

1 Solve the problem and refine the initial mesh up to 4 times. Verify that the theoretical convergence order holds.

h=element size

h	$error$
0.1	$7.62825e-3$
0.05	$2.16183e-3$
0.025	$6.05323e-4$
0.0125	$1.67808e-4$
0.00625	$4.62822e-5$

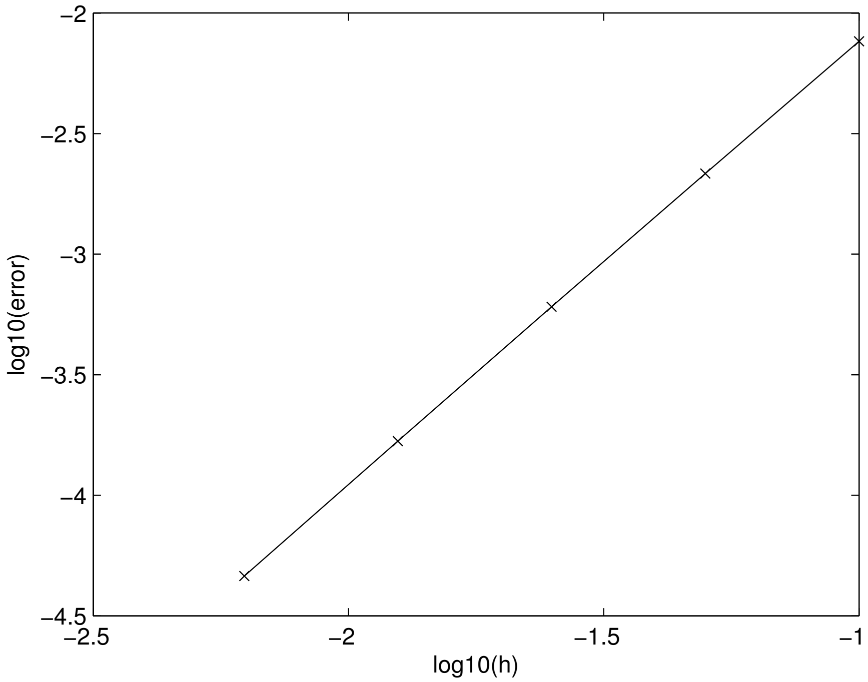

Figure 1: Convergence function

2 How is the solution affected when we modify the final time?

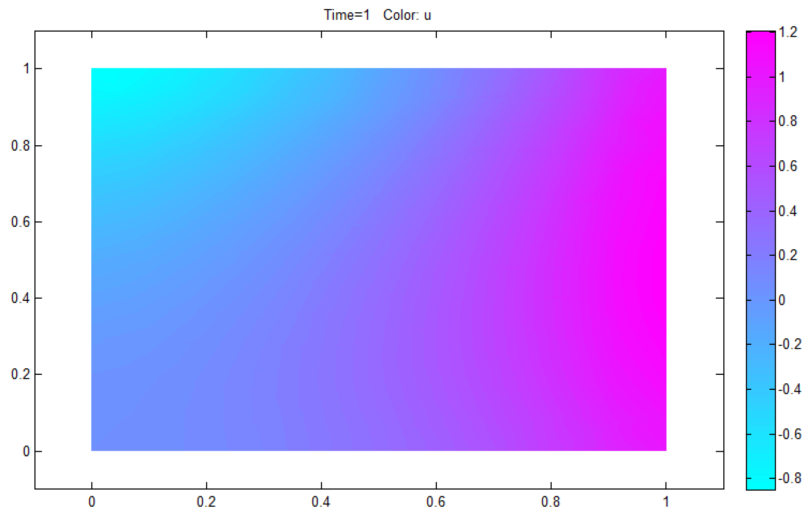

Figure 2: Solution for t=1

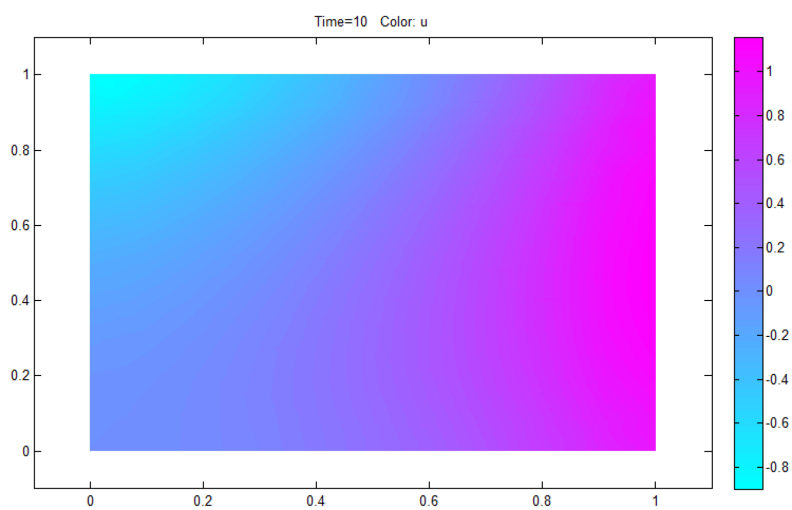

Figure 3: Solution for t=10

The solution varies very little with time.

3 We are interested in obtaining the solution at time $t_{end} = 50$. Find a more efficient manner to solve this problem. You do not need to prove the equivalence mathematically, but you need to provide numerical evidence of the new method.

For $t = 50$, the time dependent terms tend to 0 ($e^{-50} \approx 0$). The problem can be considered stationary. The problem that we solve now is:

$$u_t - \triangle u = 0 \quad (5)$$

$$\text{in } \Omega = [0,1]^2$$

And the following boundary conditions:

$$u_n(x = 0, y) = -y$$
$$u_n(x = 1, y) = 2 + y$$
$$u(x, y = 0) = x^2$$
$$u_n(x, y = 1) = x - 2$$

Computing the greatest error between the u obtained in this new problem and the value of u that we obtain solving the previous problem with t=50 we obtain:

$error = 2.9252 * 10^{-12}$

So the equivalence is proved.