PDE Tools
Computational Mechanics Tools

13th December 2018

Homework 2
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1. Introduction

In this homework we are going to solve a PDE using the `MATLAB PDE Toolbox`. Our test case is

$$\begin{cases} \partial_t u(x,y,t) - \Delta u(x,y,t) = f(x,y,t) \qquad (x,y) \in \Omega = [0,1]^2 \\ f(x,y,t) = -3e^{-3t} \\ u(x,y,t=0) = x^2 + xy - y^2 + 1 \\ \partial_n u(x=0,y,t) = -y \\ \partial_n u(x=1,y,t) = 2+y \\ u(x,y=0,t) = x^2 + e^{-3t} \\ \partial_n u(x,y=1,t) = x-2 \end{cases} \tag{1}$$

and have the following analytic solution

$$u(x,y,t) = x^2 + xy - y^2 + e^{-3t}$$

2. Matlab PDE Toolbox

We follow the tutorial given, using the GUI. We set the boundary conditions as described in appendix A.

Initializing the mesh, we have 10 triangles per side, which means the element size is $\frac{1}{10}$. At each refinement, the triangle size (its edge) is divided by two.

The final solution looks the following way

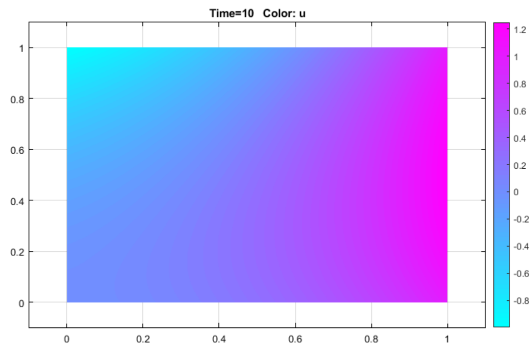

Figure 1: Numerical solution at $t_{end} = 10$

2.1. Convergence

At each refinement, we export the solution at $t = 10$ and compare it to the analytic solution. We plot the result errors over the element size, on a logarithmic scale, and have the following result

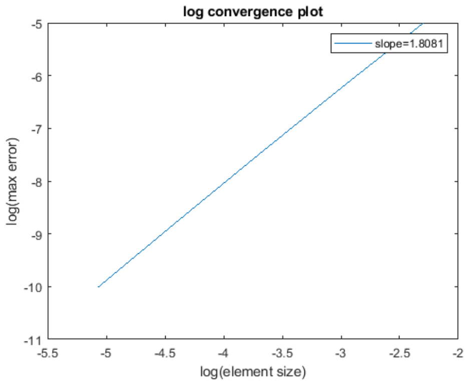

Figure 2: Convergence plot

We can see that the convergence $||e|| \leq Ch^p$ is $p \approx 2$, which is coherent. the linearity over a log scale confirms this previous inequality.

2.2. Time influence

Here we will compare how the time affect the solution. We plot then the solution at time $t = 1, 10, 50$

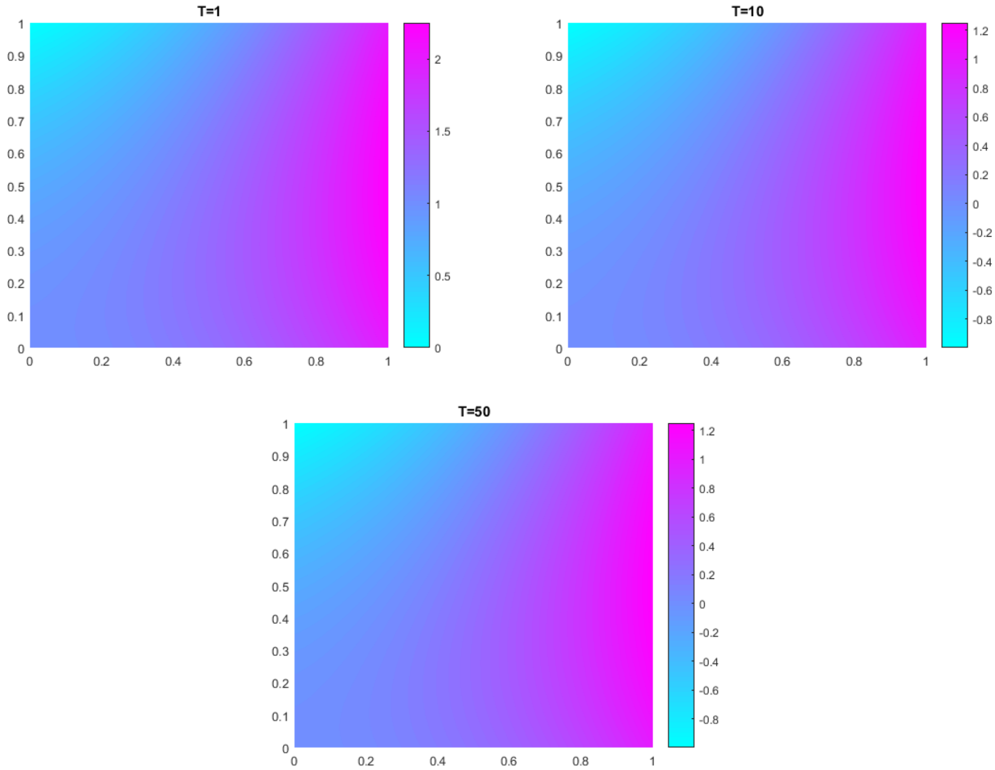

Figure 3: Left: solution at $t = 1$, Right: solution at $t = 10$, Down: solution at $t = 50$

Those graphs are not really relevant on the difference, then we plot the difference between the solution at $t = 1$ and $t = 10$ with the command `pdeplot(p,e,t,'xydata',u(:,1)-u(:,11))`

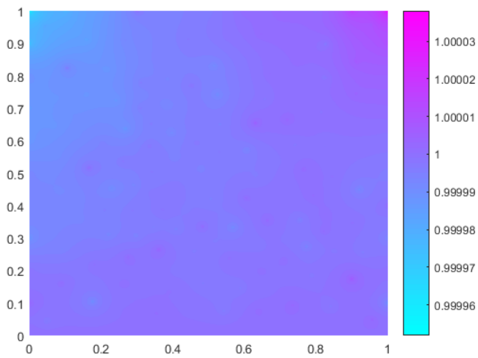

Figure 4: Solution difference between $t = 1$ and $t = 10$

We see that the scale of the difference is considerable. Using the command `max(abs(u(:,1)-u(:,11)))` the maximum difference is 1.0.

Plotting the differences between $t = 10$ and $t = 50$, we can see that the differences are much more smaller. This correspond with the exponential behavior described in the analytical solution.

2.3. Improvement

Here we run our problem until $t = 50$. The simulation takes more than one minute with a $h = \frac{1}{160}$ element size mesh

We notice that the term $e^{-3t} \underset{t=50}{\approx} 0$. Then removing all these terms of our problem, we have a time independent problem, so we redefine it as an elliptic equation.

comparing the numerical parabolic solution at t=50 and the elliptic solution, we have

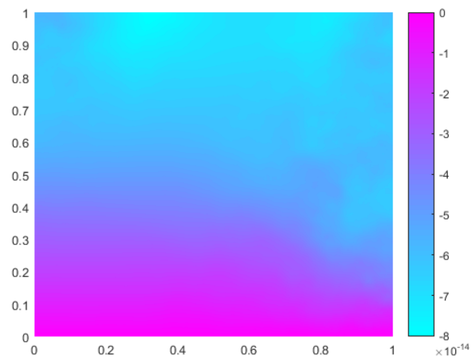

Figure 5: Time dependent parabolic model vs time independant elliptic model difference

The elliptic model is solved in few seconds and the max difference is $8.0158.10^{-14}$ which is close to the numerical zero, so we can consider the two solutions equal.

3. Conclusion

In this homework we solved a problem using the Matlab PDEToolBox. We saw that the convergence rate of the method is around $p = 2$. As any numerical method, more the spaces/time are refined, longer it is to compute.

Some tricks can be useful to solve a problem more efficiently, as its redefinition. We noticed here that our problem could be turned into an elliptic problem, which made a huge time computation improvement.

A. Set boundary conditions

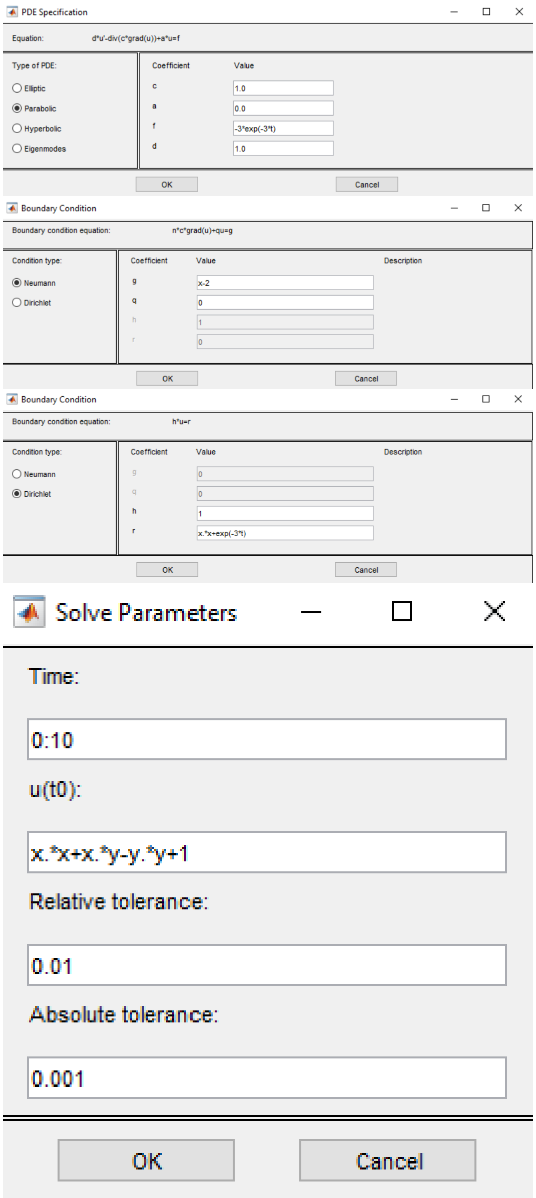

Figure 6: Setting boundary conditions in the PDETools GUI