①

| | $V_f$ | $a$ | $\rho_s$ | $\rho_l$ | $\mu_l$ | $g$ |
|---|---|---|---|---|---|---|
| M | 0 | 0 | 1 | 1 | 1 | 0 |
| L | 1 | 1 | -3 | -3 | -1 | 1 |
| T | -1 | 0 | 0 | 0 | -1 | -2 |

$n=6$
$r=3$
$n-r=6-3=3$

$$\Pi_1 = V_f\, a^a \rho_l^b \mu_l^c$$

$$M^0L^0T^0 = LT^{-1}\, L^a\, M^b L^{-3b}\, M^c L^{-c} T^{-c}$$
$$= M^{b+c}\, L^{1+a-3b-c}\, T^{-1-c}$$

$$\left.\begin{array}{l} b+c=0 \\ 1+a-3b-c=0 \\ -1-c=0 \end{array}\right\} \begin{array}{l} a=-1+3-1=1 \\ b=1 \\ c=-1 \end{array}$$

$$\boxed{\Pi_1 = \frac{a\rho_l V_f}{\mu_l}}$$

$$\Pi_2 = V_f\, a^a \rho_s^b g^c$$

$$M^0L^0T^0 = LT^{-1}\, L^a M^b L^{-3b}\, L^c T^{-2c}$$
$$= M^b\, L^{1+a+3b+c}\, T^{-1-2c}$$

$$\left.\begin{array}{l} b=0 \\ 1+a+3b+c=0 \\ -1-2c=0 \end{array}\right\} \begin{array}{l} a=-1/2 \\ b=0 \\ c=-1/2 \end{array}$$

$$\boxed{\Pi_2 = \frac{V_f}{\sqrt{a \cdot g}}}$$

$$\boxed{\Pi_3 = \frac{\rho_s}{\rho_l}}$$

$$\boxed{\frac{a\rho_l V_f}{\mu_l} = F\left(\frac{V_f}{\sqrt{ag}}, \frac{\rho_s}{\rho_l}\right)}$$

$$R_e = F\left(F_r, \frac{\rho_s}{\rho_l}\right)$$

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(2) $\frac{D}{Dt}\int_{V_t} \rho s \, dV \geq -\int_{S_t} \frac{q \cdot n}{T} \, dS$ Fluido Newtoniano $K \geq 0$, $\mu > 0$

Ley Fourier $q = -K\nabla T$, $K>0$

a) Reynolds $\int_{V_t} \frac{\partial(\rho s)}{\partial t} dV + \int_{S_t} \rho s \underline{v} \cdot n \, dS + \int_{S_t} \frac{q \cdot n}{T} dS \geq 0$

$$\int_{V_t} \frac{\partial(\rho s)}{\partial t} dV + \int_{V_t} \rho s \nabla \underline{v} \, dV + \int_{V_t} \frac{\nabla q}{T} dV \geq 0$$

$$\int_{V_t} \left(\frac{\partial(\rho s)}{\partial t} + \rho s \nabla \underline{v}\right) dV + \int_{V_t} \frac{\nabla q}{T} dV \geq 0$$

$$\frac{\partial(\rho s)}{\partial t} + \rho s \nabla \underline{v} + \frac{\nabla q}{T} \geq 0$$

$$\rho \frac{Ds}{Dt} + \frac{\nabla q}{T} \geq 0$$

b) $T \, ds = de + p \, d\left(\frac{1}{\rho}\right)$

$$\underbrace{\rho \frac{De}{Dt}}_{\text{conservación energía}} + \rho p \frac{D}{Dt}\left(\frac{1}{\rho}\right) + \nabla \frac{q}{T} \geq 0$$

$$\sigma : \nabla v - \nabla q + \rho \cdot p \frac{\partial(1/\rho)}{\partial t} + \cancel{\rho} \cdot p \cdot \frac{1}{\cancel{\rho}} \nabla v + \nabla \frac{q}{T} \geq 0$$

$$\sigma : \nabla v + p \nabla \cdot v - \frac{q \cdot \nabla T}{T} \geq 0$$

c) $q = -K\nabla T$

$$\sigma : \nabla v + p \nabla v + \frac{K (\nabla T)^2}{T} \geq 0 \quad \text{Si } K > 0 \Rightarrow \text{Cumple}$$

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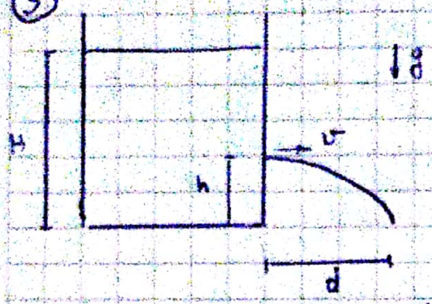


Bernoulli

$$\frac{1}{2}\cancel{\frac{v_2^2}{g}}^{0} + \cancel{\frac{P_2}{\rho g}} + H = \frac{1}{2}\frac{v_1^2}{g} + \cancel{\frac{P_1}{\rho g}} + h \qquad P_1 = P_2 = P_{atm} \Rightarrow v_2 \approx 0$$

$$v_1^2 = 2g(H-h)$$

$$v_1 = \sqrt{2g(H-h)}$$

a) $$\boxed{v \begin{cases} u = \sqrt{2g(H-h)} \\ w = -gt \end{cases}}$$

$$\begin{cases} \dfrac{dx}{dt} = \sqrt{2g(H-h)} \Rightarrow x\Big|_0^d = \sqrt{2g(H-h)}\, t\Big|_0^t \Rightarrow d = \sqrt{2g(H-h)}\, t \\ \dfrac{dy}{dt} = -gt \Rightarrow y\Big|_0^{-h} = -\dfrac{1}{2}gt^2\Big|_0^t \Rightarrow -h = -\dfrac{1}{2}gt^2 \end{cases}$$

$$h = \frac{1}{2}gt^2$$

$$t(h) = \sqrt{\frac{2h}{g}}$$

$$d(h) = \sqrt{2g(H-h)}\sqrt{\frac{2h}{g}}$$

b) $$\boxed{d(h) = 2\sqrt{h(H-h)}}$$

Si $H = 10$

$$d(h) = 2\sqrt{h(10-h)}$$

$$\frac{d}{dh}(d(h)) = 2\left(\frac{1}{2}h^{-1/2}(10-h)^{1/2} + \frac{h^{1/2}}{2}(10-h)^{-1/2}(-1)\right) = 0$$

$$= \frac{\sqrt{10-h}}{\sqrt{h}} - \frac{\sqrt{h}}{\sqrt{10-h}} = \frac{10-h-h}{\sqrt{h}\sqrt{10-h}} = \frac{2(5-h)}{\sqrt{h(10-h)}} = 0$$

$$2(5-h) = 0$$

c) $$\boxed{h = 5\,m}$$

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