Homework 3

ADVANCED FLUID MECHANICS – 3 DEC 2015

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Problem 1

$$f\left(\frac{\Delta P}{L}, \rho, \bar{V}_0, R, R_1, \mu_1, \mu_2, \sigma\right) = 0$$

8 quantities.

	$\frac{\Delta P}{L}$	ρ	V_0	R	R_1	μ_1	μ_2	σ
L	−2	−3	1	1	1	−1	−1	0
M	1	1	0	0	0	1	1	1
T	−2	0	−1	0	0	−1	−1	−2

Checking the rank of above matrix using Row Transformations

$$\begin{array}{cccccccc} -2 & -3 & 1 & 1 & 1 & -1 & -1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ -2 & 0 & -1 & 0 & 0 & -1 & -1 & -2 \end{array}$$

$$\begin{array}{cccccccc} +1 & 1.5 & -0.5 & -0.5 & -0.5 & 0.5 & 0.5 & 0 \\ 1 & 0 & 0.5 & 0 & 0 & 0.5 & 0.5 & 1 \\ 1 & 1 & 0 & 0 & 0 & 1 & 1 & 1 \end{array}$$

$$\begin{array}{cccccccc} 1 & 1.5 & -0.5 & -0.5 & -0.5 & 0.5 & 0.5 & 0 \\ 0 & 1.5 & -1 & -0.5 & -0.5 & 0 & 0 & -1 \\ 0 & 0.5 & -0.5 & -0.5 & -0.5 & -0.5 & -0.5 & -1 \end{array}$$

$$\begin{array}{cccccccc} 1 & 1.5 & -0.5 & -0.5 & -0.5 & 0.5 & 0.5 & 0 \\ 0 & 1 & -2/3 & -1/3 & -1/3 & 0 & 0 & -2/3 \\ 0 & 0 & 0.5 & 1 & 1 & 1.5 & 1.5 & 0.5 \end{array}$$

$$\begin{array}{cccccccc} 1 & 1.5 & -0.5 & -0.5 & -0.5 & 0.5 & 0.5 & 0 \\ 0 & 1 & -2/3 & -1/3 & -1/3 & 0 & 0 & -2/3 \\ 0 & 0 & 1 & 2 & 2 & 3 & 3 & 2 \end{array}$$

It is in Row-Echelon Form and cannot be further simplified.

rank = no of nonzero rows in matrix = 3

② No. of pi Terms needed = 8 - 3 = $\boxed{5}$

a) $M^0 L^0 T^0 = \left(\frac{\Delta P}{L}\right)(\bar{V}_0)^a (R_1)^b (\rho)^c$

$= \left(\frac{M}{L^2 T^2}\right) \times \left(\frac{L}{T}\right)^a (L)^b \left(\frac{M}{L^3}\right)^c$

$= M^{1+c} \; L^{a+b-2-3c} \; T^{-a-2}$

$c = -1 \quad a = -2 \quad -2+b-2+3 = 0$

$b = 1$

$$\boxed{\Pi_1 = \frac{\Delta P}{L} \times \frac{R_1}{\rho \bar{V}_0^2}}$$

b) $M^0 L^0 T^0 = R\,(\bar{V}_0)^a (R_1)^b (\rho)^c$

$= L \left(\frac{L}{T}\right)^a (L)^b \left(\frac{M}{L^3}\right)^c$

$= M^c \; L^{a+b-3c+1} \; T^{-a}$

$c = 0 \quad a = 0 \quad b = -1$

$$\boxed{\Pi_2 = \frac{R}{R_1}}$$

c) $M^0 L^0 T^0 = \mu_1 (\bar{V}_0)^a (R_1)^b (\rho)^c$

$= \left(\frac{M}{LT}\right)\left(\frac{L}{T}\right)^a (L)^b \left(\frac{M}{L^3}\right)^c$

$= M^{c+1} \; L^{a+b-3c-1} \; T^{-a-1}$

$c = -1 \quad a = -1 \quad -1+b+3-1 = 0$

$b = -1$

$$\boxed{\Pi_3 = \frac{\mu_1}{\bar{V}_0 \rho R_1}}$$

d) $$M^0 L^0 T^0 = \mu_2 (\bar{V_0})^a (R_1)^b (\rho)^c$$

using the similarity with (c)

$$\boxed{\Pi_4 = \frac{\mu_2}{\bar{V_0} \rho R_1}}$$

e) $$M^0 L^0 T^0 = \sigma (\bar{V_0})^a (R_1)^b (\rho)^c$$

$$= \left(\frac{M}{T^2}\right)\left(\frac{L}{T}\right)^a (L)^b \left(\frac{M}{L^3}\right)^c$$

$$= M^{1+c} \; L^{a+b-3c} \; T^{-2-a}$$

$c = -1 \qquad a = -2 \qquad -2+b+3 = 0$

$b = -1$

$$\boxed{\Pi_5 = \frac{\sigma}{\cancel{\rho} \rho \bar{V_0}^2 R_1}}$$

Therefore the equation can be written as,

$$\left(\frac{\Delta P}{L} \times \frac{R_1}{\rho \bar{V_0}^2}\right) = g\left(\frac{\mu_1}{\rho \bar{V_0} R_1}, \frac{\mu_2}{\rho \bar{V_0} R_1}, \frac{R}{R_1}, \frac{\sigma}{\rho \bar{V_0}^2 R_1}\right)$$

Reynolds No. fluid 1 — Reynolds no. fluid 2 — Some ratio — ~~FAV.~~ Weber Number

(Note: since dimensionless, inverses can be taken freely)

Darcy friction factor has similar form.

$$\Delta P = f_D \times \frac{L}{D} \times \frac{\rho V^2}{2}$$

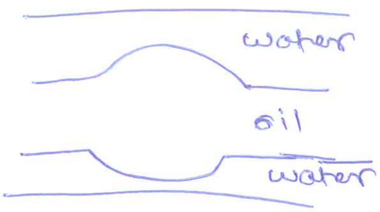
friction factor

$$f_D = \frac{\Delta P}{L} \times \frac{D \times 2}{\rho V^2}$$

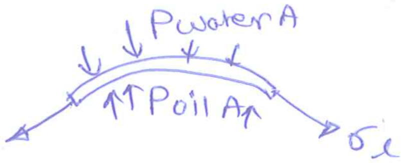
comparing this with out Π term,

$$\left(\frac{\text{Friction factor}}{4}\right) = g\left(Re_1, Re_2, \frac{R}{R_1}, \text{Weber No}\right)$$

(4) formation of waves.



FBD of thin oil curved part

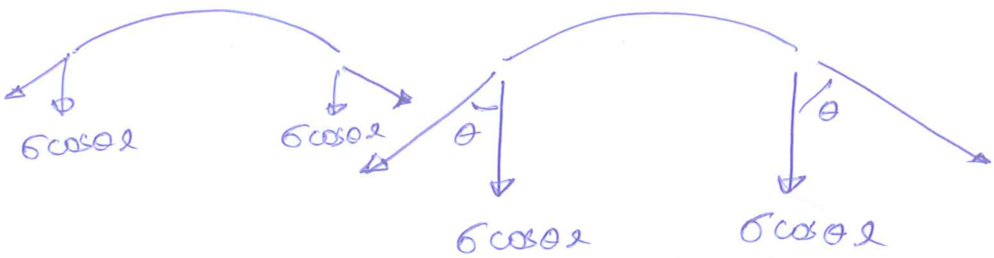


Note
No gravity

The restoring force ~~s~~ is proportional to σ (surface tension)
If there is more surface tension, there will be more restoring force - and it won't allow to form waves.

Less σ

More σ



So at same angle θ, the wave with more surface tension ~~fluid~~ at (fluid interface) will try to come down quickly and hence will resist the wave formation more.

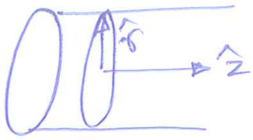
$$\text{So } We = \left(\frac{\sigma}{\rho (V_0)^2 R_1}\right)^{-1} = \frac{\rho V_0^2 R_1}{\sigma}$$

if σ is very high compared to the numerator, we h

$$\boxed{We \ll 1}$$

Gravity will not drive the waves because we have fluids with ~~oth~~ almost same density. So it is like ~~colour~~ adding color to the part of water (with diff. viscosity of course) and observing it. So we are observing immiscible fluids within the pipe. We would want to keep the waves to minimum because if we don't then then the fluids will mix up and it will get difficult to separate them. In general, we can separate oil & water using difference in densities, but here we have adjusted the densities to be equal.

$\frac{\partial p}{\partial z} = -\frac{\Delta P}{L}$ (given)



$v = \begin{bmatrix} 0 \\ 0 \\ v_z(r) \end{bmatrix}$ → seems valid since no radial flow (no waves)

no rotation (symmetry)

and v_z will vary as r changes

steady state given

Navier stokes equation

In r, and θ direction we get $0 = 0$

in z direction,

$$\rho\left(\frac{\partial v_z}{\partial t} + v_z \frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{\partial^2 v_z}{\partial z^2}\right]$$

since only one velocity lets call it $v = v(r)$ instead of v_z

$$\rho\left(\frac{\partial v}{\partial t} + v\frac{\partial v}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\frac{\partial^2 v}{\partial z^2}$$

$\cancel{\rho(\partial}$

steady state $\partial = 0$; $v_r = 0$; $v_\theta = 0$; $\frac{\partial v_z}{\partial z} = 0$

$$\rho\left(\frac{\partial v_z}{\partial t} + v_r\frac{\partial v_z}{\partial r} + \frac{v_\phi}{r}\frac{\partial v_z}{\partial \theta} + v_z\frac{\partial v_z}{\partial z}\right) = -\frac{\partial p}{\partial z} + \mu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 v_z}{\partial \theta^2} + \frac{\partial^2 v_z}{\partial z^2}\right] + \rho g_z$$

$\frac{\partial v_z}{\partial \theta} = 0$; $\frac{\partial v_z}{\partial z} = 0$; $g_z = 0$

$$-\frac{\partial p}{\partial z} + \frac{\mu}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) = 0$$

for two fluids -

$$-\frac{\partial p}{\partial z} + \frac{\mu_1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) = 0 \qquad 0 \le r \le R_1$$

$$-\frac{\partial p}{\partial z} + \frac{\mu_2}{r}\frac{\partial}{\partial r}\left(r\frac{\partial v_z}{\partial r}\right) = 0 \qquad R_1 \le r \le R$$

⑥ * BCs,

a) at $r=R$ No-slip condition

$$\boxed{V_2=0}$$

d) $\left.\frac{\partial V_1}{\partial r}\right|_{r=0}=0$ symmetry and for equilibrium

b) at interface velocity field should be continous.
Left hand limit of velocity = Right hand limit of velocity.
(at $r=R_1$)

c) Also at interface shear stresses should be equal.

Solving the diff equation,

$$-\frac{\partial P}{\partial z}+\frac{\mu_1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial V_z}{\partial \cancel{z}}\right)=0$$

$$-\frac{\Delta P}{L}=\frac{\mu_1}{r}\frac{\partial}{\partial r}\left(r\frac{dV}{\partial r}\right)$$

$$-\frac{\Delta P\, r^2}{2\mu_1 L}+C_1=r\frac{dv}{dr}$$

$$-\frac{\Delta P\, r}{2\mu_1 L}+\frac{C_1}{r}=\frac{dv}{dr}$$

$0<r<R_1$

$\mu_1=\mu_{oil}$

$$\boxed{V_1=-\frac{\Delta P}{2\mu_1 L}\frac{r^2}{2}+C_1\ln r+C_2}$$

$R_1<r<R$

$\mu_2=\mu_{water}$

$$\boxed{V_2=\frac{-\Delta P}{2\mu_2 L}\frac{r^2}{2}+C_3\ln r+C_4}$$

$$\left.V_2\right|_{r=R}=0$$

$$0=\frac{-\Delta P}{4\mu_2 L}R^2+C_3\ln R+C_4 \quad —①$$

$$\mu_2\left.\frac{\partial V_2}{\partial r}\right|_{r=R_1}=\mu_1\left.\frac{\partial V_1}{\partial r}\right|_{r=R_1}$$

$$-\mu_1\frac{\Delta P R_1}{2\mu_1 L}+\frac{C_1}{R_1}=-\mu_2\frac{\Delta P R_1}{2\mu_2 L}+\frac{C_3}{R_1}$$

$$C_1-C_3=-\frac{\Delta P R_1^2}{2L}\left(\frac{\mu_2}{\mu_2}-\frac{\mu_1}{\mu_1}\right) \quad —②$$

$C_1=C_3$

$$\frac{-\Delta P}{2\mu_1 L}\frac{R_1^2}{2} + c_1 \ln R_1 + c_2 = \frac{-\Delta P}{2\mu_2 L}\frac{R_1^2}{2} + c_3 \ln R_1 + c_4$$

$$c_1 = c_3$$

$$c_2 - c_4 = \frac{\Delta P R_1^2}{4L}\left(\frac{1}{\mu_1} - \frac{1}{\mu_2}\right) \quad \text{— (3)}$$

$$\left.\frac{\partial V}{\partial r}\right|_{r=0} = 0$$

$$\frac{-\Delta P r^2}{2\mu_1 L} + c_1 = 0$$

$$\boxed{c_1 = 0} \quad \text{— (4)}$$

from (2),

$c_3 = 0$

from (1),

$$\boxed{c_4 = \frac{\Delta P}{4\mu_2 L} R^2}$$

$$\boxed{V_2 = \frac{\Delta P}{4\mu_2 L}\left(R^2 - r^2\right)} \quad R_1 \le r \le R$$

from (3)

$$c_2 = c_4 + \frac{\Delta P R_1^2}{4L}\left(\frac{1}{\mu_1} - \frac{1}{\mu_2}\right)$$

$$= \frac{\Delta P}{4\mu_2 L}\left(R^2 - r^2\right) + \frac{\Delta P R_1^2}{4L}\left(\frac{1}{\mu_1} - \frac{1}{\mu_2}\right)$$

$$\cancel{\frac{\Delta P}{4\mu_1 L}}\left(\right.$$

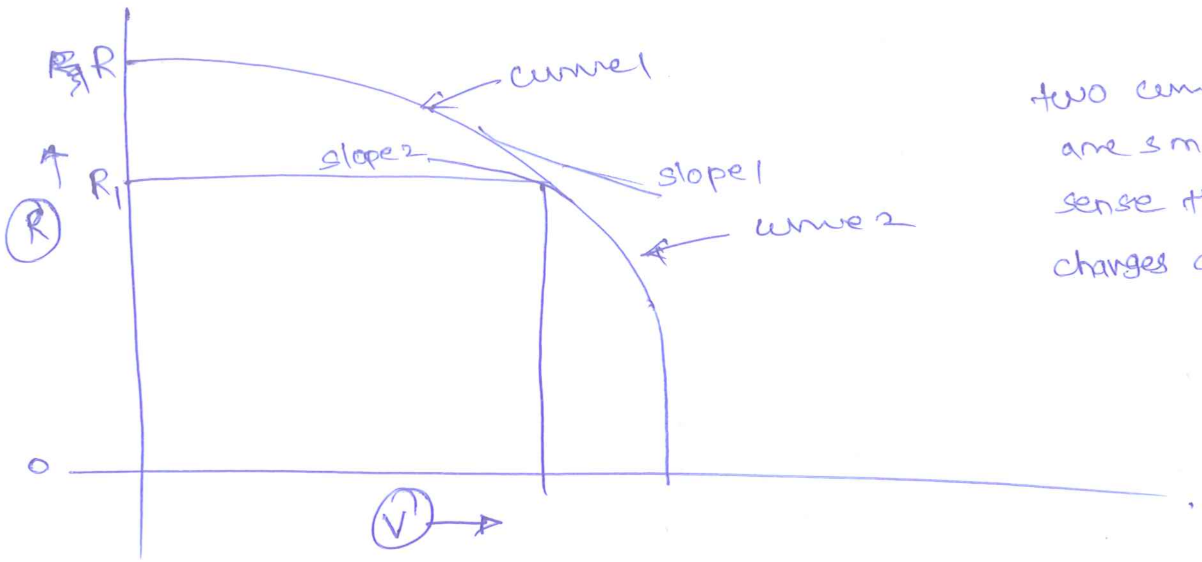
$$V_1 = \frac{-\Delta P r^2}{4\mu_1 L} + \frac{\Delta P}{4\mu_2 L}\left(R^2 - r^2\right) + \frac{\Delta P R_1^2}{4L}\left(\frac{1}{\mu_1} - \frac{1}{\mu_2}\right)$$

$$\boxed{V_1 = \frac{\Delta P}{4\mu_1 L}\left(R_1^2 - r^2\right) + \frac{\Delta P}{4\mu_2 L}\left(R^2 - R_1^2\right)}$$

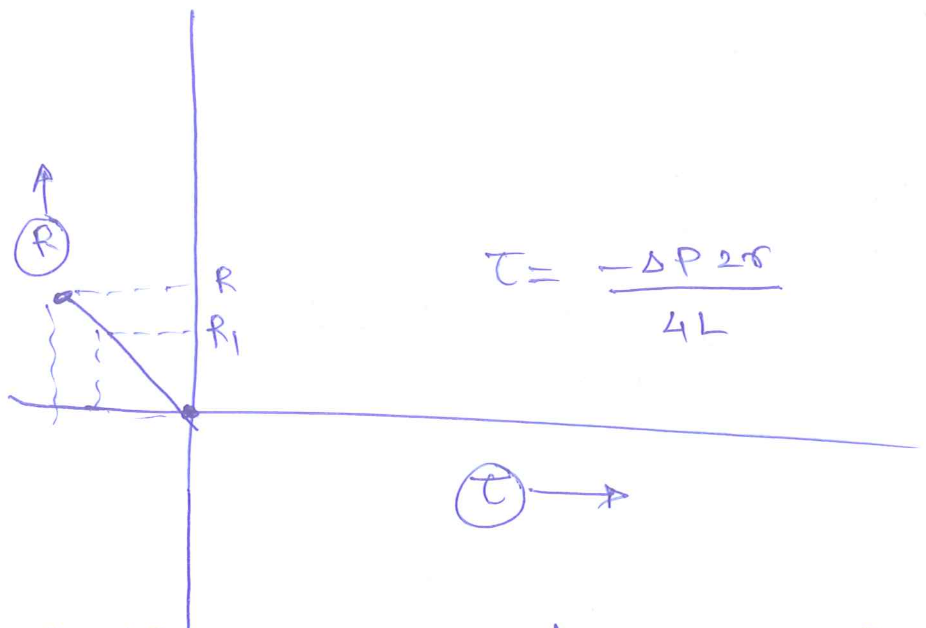
⑧ Interface velocity,

$V_1\big|_{r=R_1}$ or $V_2\big|_{r=R_1}$ both same,

$$\boxed{V_{interface} = \frac{\Delta P}{4\mu_2 L}\left(R^2 - R_1^2\right)}$$



two curves but are smooth in the sense that derivative changes at boundary.



$$\tau = \frac{-\Delta P\, 2r}{4L}$$

$0 \le r \le R_1$ $\quad \tau = \mu_1 \frac{dV_1}{dr} = \frac{-\Delta P\, \cancel{2R_1}\, 2r}{4L}$

$R_1 \le r \le R$ $\quad \tau = \mu_2 \frac{dV_2}{dr} = \frac{-\Delta P\, 2r}{4L}$

Same slope
Straight line.

Volume flow rate Q_o oil $\quad Q_w =$ water

$$Q_{oil} = \int_0^{R_1} 2\pi r\,dr\,V_1$$

$$= \frac{\Delta P\,2\pi}{4L\mu_1}\int_0^{R_1}\left(R_1^2 r - r^3\right)dr + \frac{\Delta P\,2\pi}{4L\mu_2}\int_0^{R_1}\left(R^2 r - R_1^2 r\right)dr$$

$$= \frac{\Delta P\,2\pi}{4L\mu_1}\left[\frac{R_1^2 R_1^2}{2} - \frac{R_1^4}{4}\right] + \frac{\Delta P\,2\pi}{4L\mu_2}\left[\frac{R^2 R_1^2}{2} - \frac{R_1^{\cancel{4}}}{\cancel{2}}\right]$$

$$= \boxed{\frac{\Delta P\,\pi}{8L\mu_1}R_1^4 + \frac{\Delta P R_1^2}{4L\mu_2}\left(R^2 - R_1^2\right)}$$

$$Q_{water} = \int_{R_1}^{R} 2\pi r\,dr\,V_2$$

$$= \int \frac{\Delta P\,2\pi}{4\mu_2 L}\int_{R_1}^{R} \cancel{2\pi}\left(R^2 r - r^3\right)dr$$

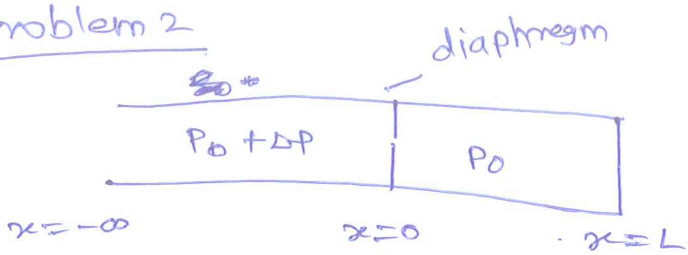
$$= \frac{\Delta P\,2\pi}{4\mu_2 L}\left[\frac{R^2\left(R^2 - R_1^2\right)}{2} - \frac{R^4 - R_1^4}{4}\right]$$

$$= \frac{\Delta P}{2\mu_2 L}\left[\frac{R^4}{2} - \frac{R^2 R_1^2}{2} - \frac{R^4}{4} + \frac{R_1^4}{4}\right]$$

$$= \frac{\Delta P}{2\mu_2 L}\left[\frac{R^4}{4} + \frac{R_1^4}{4} - \frac{R_1^2 R^2}{2}\right]$$

$$= \boxed{\frac{\Delta P}{8\mu_2 L}\left(R^2 - R_1^2\right)^2}$$

(10) Problem 2



$\rho = \rho_0 + \Delta\rho$

$\Delta P \ll P_0$

$V \approx 0$

Assumming compressible Inviscid fluid (air) and 1D velocities

$$\frac{\partial \rho}{\partial t} + \nabla\cdot(\rho \bar{V}) = 0$$

$$\frac{\partial}{\partial t}(\rho U) + \nabla\cdot(\rho \bar{V}\otimes\bar{V} + pI) = \rho \bar{b}$$

$$\frac{\partial}{\partial t}(\rho E) + \nabla\cdot((\rho E + p)\bar{V}) = \bar{V}\cdot\rho\bar{b}$$

~~Assuming Ideal gas~~

$\rho = \rho_0 + \Delta\rho$

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho_0 u) + \cancel{\frac{\partial}{\partial x}(\Delta\rho u)} \rightarrow 0$$ as $\Delta\rho$ is very small.

$$\boxed{\frac{\partial \rho}{\partial t} + \rho_0 \frac{\partial u}{\partial x} = 0} \quad \text{— (1)}$$

$P = P_0 + \Delta P$ ΔP is very small.

Neglecting $V\otimes V$ (higher order term) in Euler Eqs.

since u is very small.

and $\bar{b} = 0$ (no body forces)

$$\boxed{\rho_0 \frac{\partial u}{\partial t} + \frac{\partial p}{\partial x} = 0} \quad \text{— (2)}$$

Defining new ~~density~~, variable $\rho_n = \dfrac{\rho - \rho_0}{\rho_0}$
to simplify the PDE.

$$\Downarrow$$

$$\boxed{\rho = (1+\rho_n)\rho_0}$$

$$\frac{\partial \rho}{\partial t} = \frac{\partial \rho_n}{\partial t}\rho_0$$

eqn. 1 becomes,

$$\boxed{\frac{\partial \rho_n}{\partial t} + \frac{\partial u}{\partial x} = 0} \quad \text{— (3)}$$

Assuming Ideal gas obeying $\dfrac{P}{\rho^\gamma} = \dfrac{P_0}{\rho_0^\gamma}$ $\quad \gamma = \dfrac{c_p}{c_v}$

(Isentropic after the disturbance is created)

$$\frac{P}{(1+\rho_n)^\gamma \cancel{\rho_0^\gamma}} = \frac{P_0}{\cancel{\rho_0^\gamma}}$$

$$P = P_0(1+\rho_n)^\gamma$$

$$P \approx P_0(1+\gamma\rho_n)$$

since ρ_n is very small because ~~since~~ $\Delta\rho$ is small

$(1+\Delta x)^n = 1 + n\Delta x$

$$\frac{\partial P}{\partial x} = \gamma P_0 \frac{\partial \rho_n}{\partial x}$$

eqn. 2 becomes

$$\rho_0 \frac{\partial u}{\partial t} + \gamma P_0 \frac{\partial \rho_n}{\partial x} = 0$$

$$\boxed{\frac{\partial u}{\partial t} + m\frac{\partial \rho_n}{\partial x} = 0} \quad \text{where } m = \frac{\gamma P_0}{\rho_0} \quad \text{— (4)}$$

$$\frac{\gamma P_0}{\rho_0} = c^2$$

c = speed of sound

3 & 4 is a system of PDEs,

use with ~~have to~~

4 becomes, after using the fact $\dfrac{\gamma P_0}{\rho_0} = c^2$

$$\frac{\partial u}{\partial t} + c^2\frac{\partial \rho_n}{\partial x} = 0$$

(12) Sys of Equations to solve,

$$\boxed{\begin{aligned} &\frac{\partial \rho}{\partial t} + \frac{\partial u}{\partial x} = 0 \quad \text{— (5)} \\ &\frac{\partial u}{\partial t} + c^2 \frac{\partial \rho}{\partial x} = 0 \quad \text{— (6)} \end{aligned}}$$

~~dropp~~ writing ρ_n as ρ (to simplify writing)

$$\rho_n = \frac{\rho - \rho_0}{\rho_0}$$

Now onwards ρ_n is written as ρ

(5)+(6)

$$\cancel{\frac{\partial \rho}{\partial t} + \frac{\partial u}{\partial x} + \frac{\partial u}{\partial t} + c^2 \frac{\partial \rho}{\partial x} = 0}$$

~~Dividing by c~~

$$\cancel{\frac{\partial}{\partial t}(u+\rho) + \frac{\partial}{\partial x}(u + c^2\rho) = 0}$$

$$\cancel{\frac{1}{c}\frac{\partial \rho}{\partial t} + \frac{1}{c}\frac{\partial u}{\partial x} + \frac{1}{c}\frac{\partial u}{\partial t} + c\frac{\partial \rho}{\partial x} = 0}$$

$$\cancel{\frac{\partial}{\partial t}\left(\frac{u}{c} + \frac{\rho}{c}\right) + c\frac{\partial}{\partial x}\left(\rho + \frac{u}{c^2}\right) = 0}$$

(5) + (6) → divide (6) by c before adding

$$\frac{\partial \rho}{\partial t} + \frac{\partial u}{\partial x} + \frac{1}{c}\frac{\partial u}{\partial t} + c\frac{\partial \rho}{\partial x} = 0$$

$$\frac{\partial}{\partial t}\left(\frac{u}{c} + \rho\right) + c\frac{\partial}{\partial x}\left(\frac{u}{c} + \rho\right) = 0$$

$$\frac{u}{c} + \rho = J$$

$$\boxed{\frac{\partial J}{\partial t} + c\frac{\partial J}{\partial x} = 0}$$ where $J = \frac{u}{c} + \rho$

(5) − (6) → divide (6) by c before subtracting (5) from (6)

$$-\frac{\partial \rho}{\partial t} + \frac{\partial u}{\partial x} + \frac{1}{c}\frac{\partial u}{\partial t} + c\frac{\partial \rho}{\partial x} = 0$$

$$\frac{\partial}{\partial t}\left(\frac{u}{c} - \rho\right) + c\frac{\partial}{\partial x}\left(\frac{u}{c} - \rho\right) = 0$$

$$\boxed{\frac{\partial k}{\partial t} - c\frac{\partial k}{\partial x} = 0}$$ where $k = \frac{u}{c} - \rho$

so the new system to be solved is

$$\boxed{\frac{\partial J}{\partial t} + c\frac{\partial J}{\partial x} = 0}$$

$$J = \frac{u}{c} + \rho_n$$

$$\rho_n = \frac{\rho - \rho_0}{\rho_0}$$

$$\boxed{\frac{\partial K}{\partial t} - c\frac{\partial K}{\partial x} = 0}$$

$$K = \frac{u}{c} - \rho_n$$

$$\rho_n = \frac{\rho - \rho_0}{\rho_0}$$

Using characteristic line method. for eqn. 1

$$\frac{\partial J}{\partial t} + c\frac{\partial J}{\partial x} = 0$$

J → x → s, J → t → s

$$\frac{dJ}{ds} = \frac{\partial J}{\partial x}\frac{dx}{ds} + \frac{\partial J}{\partial t}\frac{dt}{ds}$$

if $\frac{dx}{ds} = c$ &

$\frac{dt}{ds} = 1$ we get

$$\frac{dx}{ds} = c \qquad \frac{dt}{ds} = 1$$

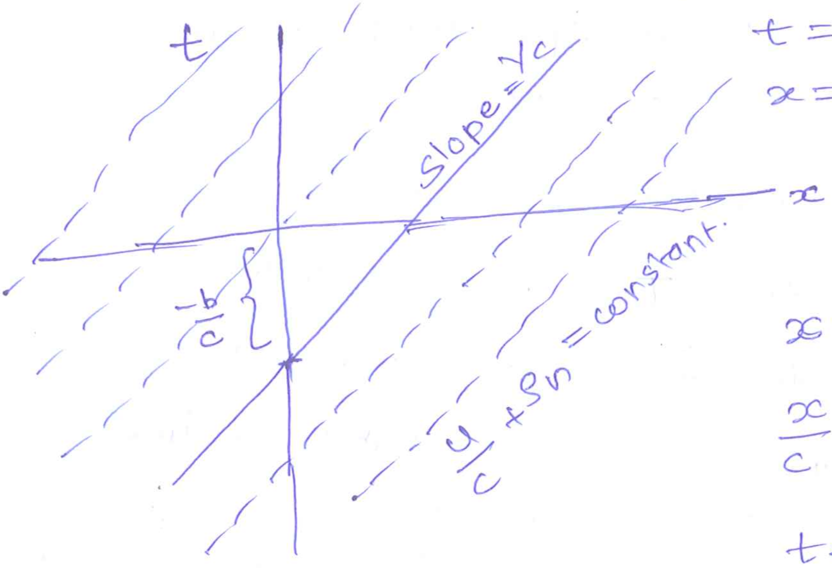
Assume
$t(0) = 0$

$t = s$

$x = c\cancel{s} + b$

$$\frac{dJ}{ds} = \frac{\partial J}{\partial t} + c\frac{\partial J}{\partial x} = 0$$

J = constant along s



$$x = ct + b$$

$$\frac{x}{c} - \frac{b}{c} = t$$

$$t = \frac{x}{c} - \frac{b}{c}$$

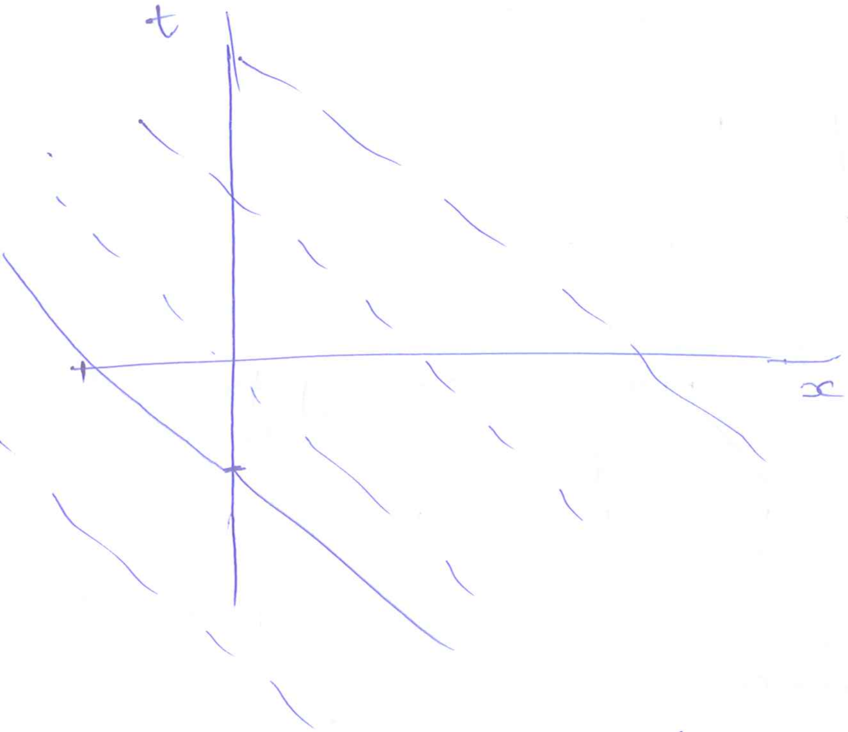
slope $= \frac{1}{c}$ (+ve)

now depending on value of b (Initial condition)
value of x at t=0

we will have family of lines shown by dotted lines.

(1h) Using characteristic line method on eqn (2)

$$\frac{\partial k}{\partial t} - c\frac{\partial k}{\partial x} = 0$$



$\frac{dx}{ds} = -c \qquad \frac{dt}{ds} = 1$

$t = s$
assume
$t(0) = 0$

~~$x = ct$~~

$x = -ct + d$

$-\frac{x}{c} - \frac{d}{c} = t$

slope $= \frac{-1}{c}$ (−ve)

We get multiple dotted lines based on IC.

Our problem is a combination of these two graphs.

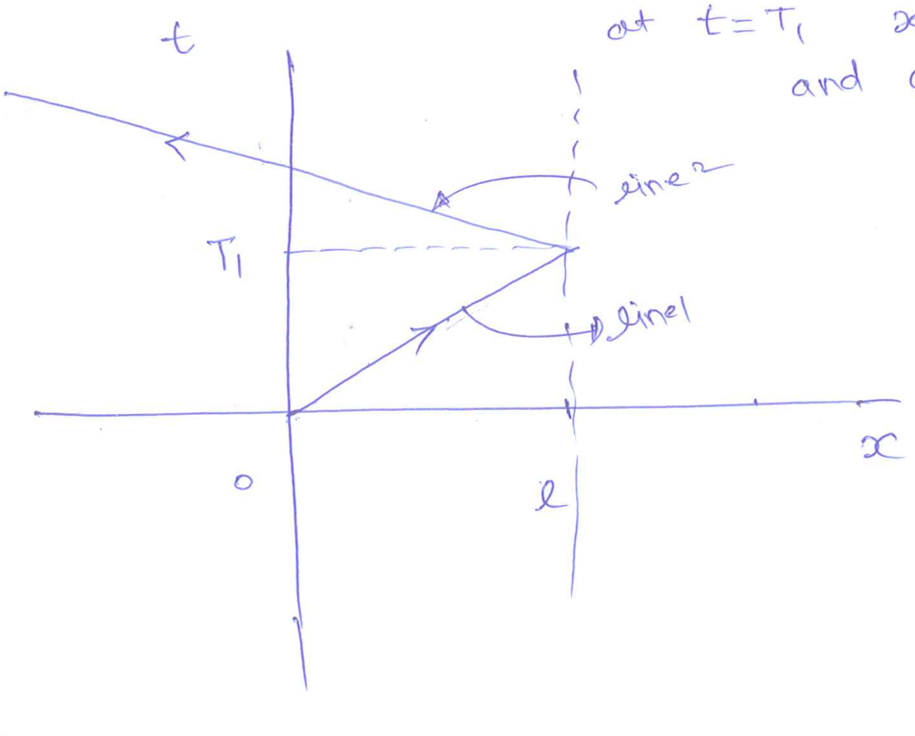
for time $= 0$ to T_1 forward wave

for time $= T_1$ to ∞ backward wave

Also at $t=0$ $x=0$ (initial condition)

at $t=T_1$ $x=x_1$ (wall)

and continuity of characteristic line.



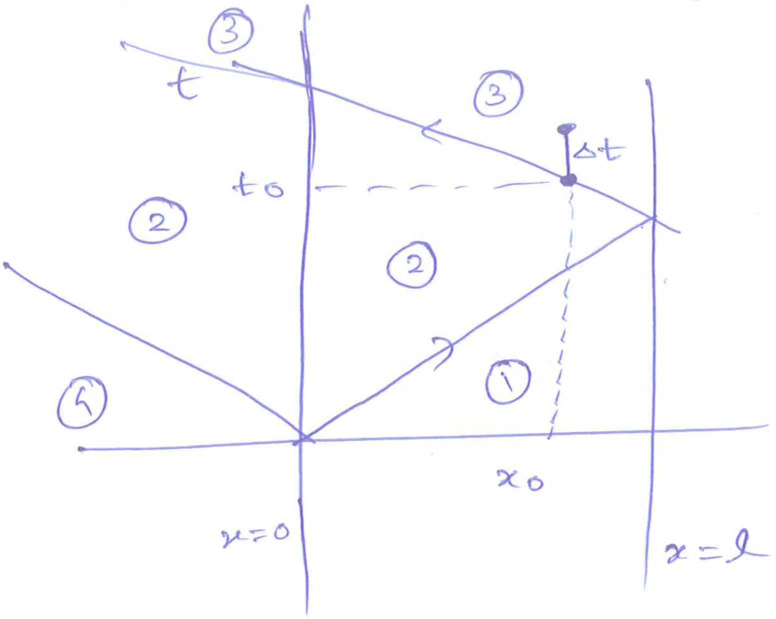
line 1 eqn.

$t = \frac{x}{c}$

line 2 eqn

$x = -c(t - T_1) + l$

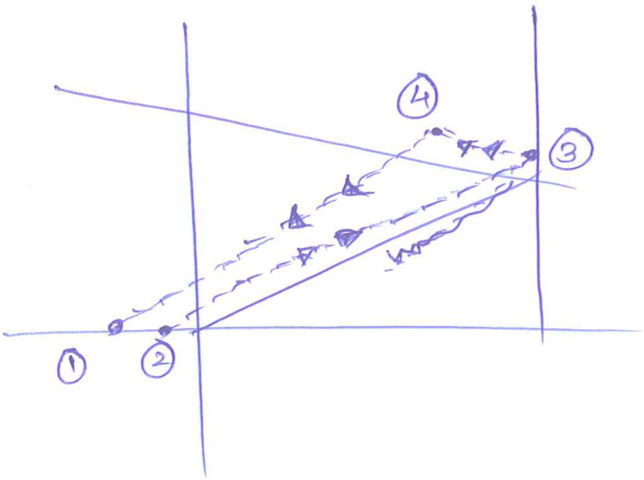
Lets draw the regions of interest.



we want to know solution ~~to~~ after the wave has passed.

So we start at a point and measure u, ρ after the wave has passed.

Say ~~at~~ we are $x = x_0$ then we wait for time t_0 so that wave passes and then wait for time Δt (very small) so that we are behind wave and then measure u & ρ



① - ④ $\frac{u}{c} + \rho_n$ = constant

③ - ④ $\frac{u}{c} + \rho_n$ = constant

② - ③ $\frac{u}{c} + \rho_n$ = constant.

at ① $\rho = \rho_1$ $(\rho + \Delta\rho \neq \rho_1)$

$u = 0$

at ② $\rho = \rho_1$ (same region)

$u = 0$

$$\frac{u_1}{c} + \rho_1 = \frac{u_4}{c} + \rho_4$$

$$\boxed{\rho_1 = \frac{u_4}{c} + \rho_4} \quad (a)$$

$$\frac{u_2}{c} + \rho_2 = \frac{u_3}{c} + \rho_3$$

$u_3 = 0$ at boundary

$$\rho_2 = \frac{u_3}{c} + \rho_3$$

$$\boxed{\rho_3 = \rho_2} = \rho_1$$

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between 3 & 4

$$\frac{u_3}{c} - \rho_3 = \frac{u_4}{c} - \rho_4$$

$$\boxed{-\rho_1 = \frac{u_4}{c} - \rho_4} \quad \text{— b}$$

a & b are two ~~si~~ eqn. with 2 unknowns.

$$\rho_1 = \frac{u_4}{c} + \rho_4$$

$$-\rho_1 = \frac{u_4}{c} - \rho_4$$

Adding we get

$$0 = u_4 + 0$$

$$\boxed{u_4 = 0}$$

subtracting we get

$$2\rho_1 = 2\rho_4$$

$$\boxed{\rho_4 = \rho_1}$$

This is actually ρ_n

$$\frac{\rho_4 - \rho_0}{\rho_0} = \frac{\rho_1 - \rho_0}{\rho_0}$$

$$\rho_4 = \rho_1$$

To find pressure,

~~so~~

$$\frac{P_4}{(\rho_4)^\gamma} = \frac{P_1}{(\rho_1)^\gamma}$$

$$\boxed{P_4 = P_1}$$

$$\boxed{\text{so just behind wave, } \quad u = 0 \quad P = P_1}$$

END