Assignment-1
Computational Solid Mechanics
Master of Science in Computational Mechanics 2016

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April 8, 2016

Part 1: Rate Independent Models

Total 6 different models were simulated to verify the correctness of algorithm. Parameters such as, `Loading Data, Hardness Type, Hardness Modulus, Model Type` were varied. For all models of this section, Young's Modulus (E), Poisson's Ratio (ν) and Yield Stress (σ_y) were kept constant, 20000, 0.3 and 100. All constituent models are simulating a plane-strain condition and are inviscid. Total duration of simulation is taken as 10.

In general, all curves are labeled using numbers. They serve the purpose of indentifying curve directions and are also used descritption in text. Note that same numerbered locations between two distinct figures are in general not same.

Model 1

This simulates 1D loading/unloading for a non-symmetric model (n=2) with linear $H = 0.5$. Undamaged stresses are as follows: Step 1. $\sigma_1 = 400$, σ_2=0. Step 2. σ_1=-2000,σ_2=0. Step 3. $\sigma_1 = 1100$, σ_2=0. 10 sub-steps were used for each load-step. (For simplicity of depiction, sub-steps for plotting damage surfaces were reduced).

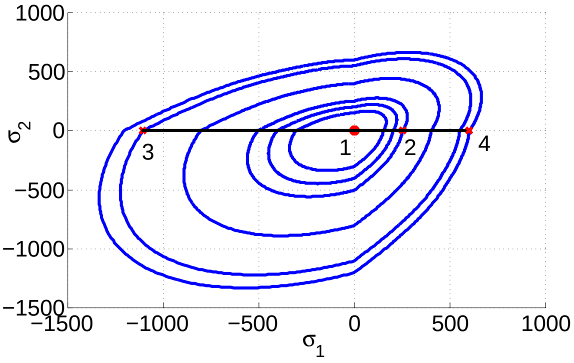

Figure 1: Model 1, Damage Surface Evolution

The evolution of q (Figure 2) indicates three increments in q. It is justified, since there the damage surface is forced outwards three times. First damage happens when load reaches yield stress between 1-2. At point 2, unloading begins and then as it further decreases and reaches the compressive damage surface (Contour of point 2) and damage increases till point 3. The third occurence of damage begins, since the loading increases in tension side crossing the new damage surface (Contour of point 3) to reach point 4.

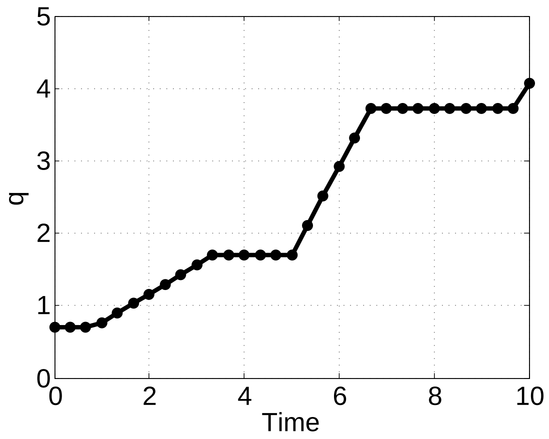

Figure 2: Model 1, Damage Parameter Evolution

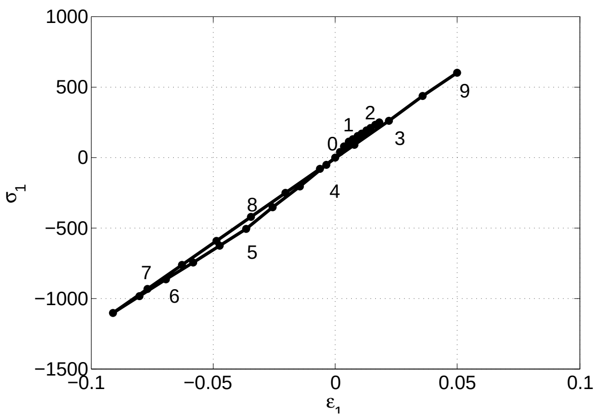

Figure 3: Model 1, Stress vs Strain

Figure 3, shows that the loading and unloading curves are not overlapping exactly. This is due to the hardening modulus. Also, since there is no plasticity, the curves must pass through origin when loading is reversed. The assumption of constant H was done for this model, which can be verified from the plot of q vs r as shown in Figure 4, which has a constant slope.

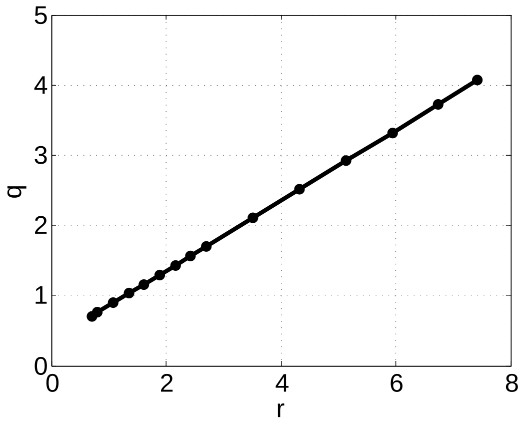

Figure 4: Model 1, Constant H

Model 2

This simulates 1D loading/unloading for a Tension-Only model with linear $H = 0.5$. Undamaged stresses are same as that of Model 1.10 sub-steps were used for each load-step.

The evolution of damage surface (Figure 5) is asymptotic to -Y and -X axes which is justified since, the material cannot reach damage surface in compression. From 1-2, when material reached yield stress, damage surface is pushed outwards (since H¿0), damage increases.(First increment in 'd' shown Figure 6). From 2-3 (unloading and compression phase), damage would remain constant (Tension Only). When reloading occurs during 3-4, damage surface is pushed again from damage contour of point 2 to point 4.

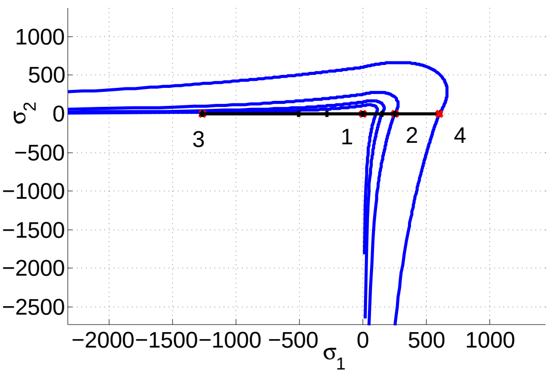

Figure 5: Model 2, Damage Parameter Evolution

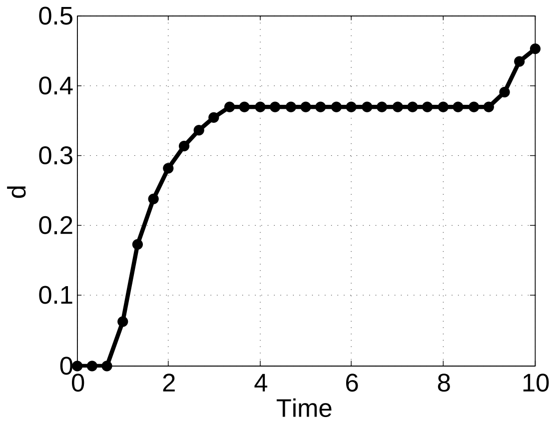

Figure 6: Model 2, Damage vs Time

The stresses in compression zone are retraced back as shown in Figure 7 (since no damage in compression). But it is not the case in tension, and some deviation in the loading and unloading curves can be observed. Figure 8 shows the evolution of σ_1 with time.

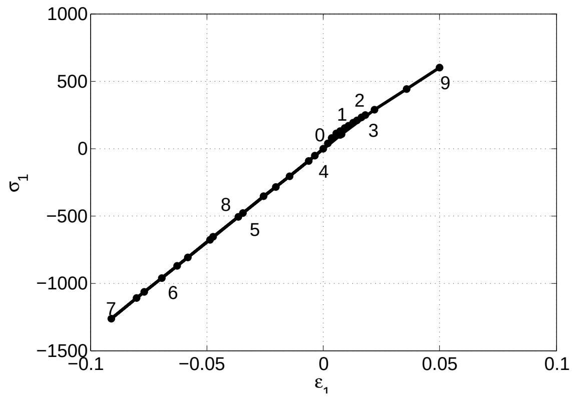

Figure 7: Model 2, Stress vs Strain

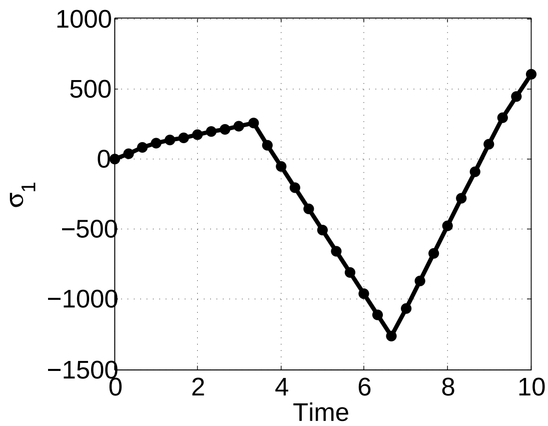

Figure 8: Model 2, Damage vs Time

Model 3

This simulates 1D loading/unloading for a tension only model with exponential type such that $H = 0.3$ at $r = r_0$. q_∞ is taken as $1.1r_0$. Undamaged stresses are as follows: Step 1. $\sigma_1 = 200$, σ_2=0. Step 2. σ_1=100,σ_2=0. Step 3. $\sigma_1 = 500$, σ_2=0. 2s0 sub-steps were used for each load-step.

Two damage increments occur. First between 1-2 when load exceeds the yield stress. The second increment is between 3-4 when loading pushes the new damage surface (contour of point 2), similar to Model 2. (9)

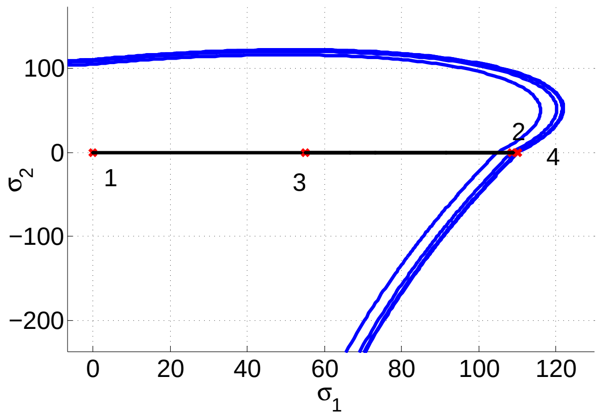

Figure 9: Model 3, Damage Surface Evolution

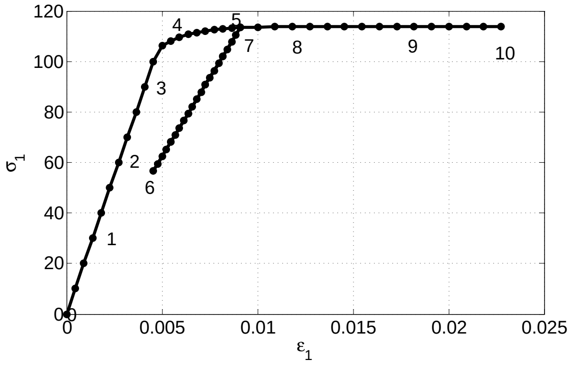

Figure 10: Model 3, Stress vs Strain

Another observation is that, in inelastic loading phase, the subsequent damage surfaces are nearer to each other and damage surface are approaching a limiting surface. At this point, H would be zero. Thus, the stress-space would be bounded by this surface even though there would be no such limit on strain-space.

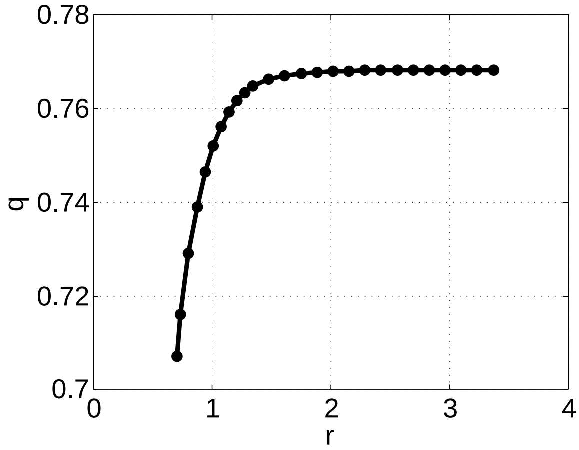

Figure 11: Model 3, Exponential Hardening

Stress-Strain curve (Figure 10) indicates that 1-2-3 is linear since load is less than yield stress. From 3 onwards, nonlinearity kicks in, to follow path 3-4-5 at which unloading starts. After 6, reloading occurs via same curve till 5. 5-10 is almost flat, since the modulus of hardening has reduced to almost H. The hardening modulus decreases with respect to r therefore, the curve becomes more and more flat. The exponential hardening can be observed in Figure 11 since $q'(r) \to 0$.

Model 4

This simulates 1D loading/unloading for a nonsymmetric model (n=2) with exponential type such that $H = -0.3$ at $r = r_0$. q_∞ is taken as $0.9r_0$. Undamaged stresses are same as that of Model 3. Since, the range of loading is in first quadrant of principal stress, Non-symmetric model and tension-only model would behave identically. 20 sub-steps were used for each load-step.

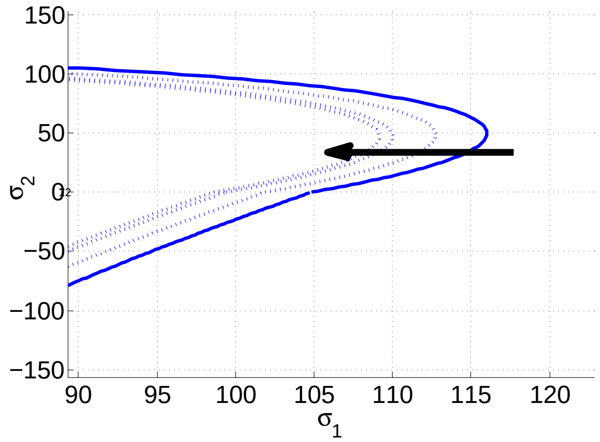

Figure 12: Model 4, Damage Surface Evolution

Since, this is a softening model, the damage surface moves inwards as the inelastic loading begins.This is evident from Figure 12. Similar to exponential hardening, the damage surfaces are approaching a limit damage surface which is inside the original damage surface.

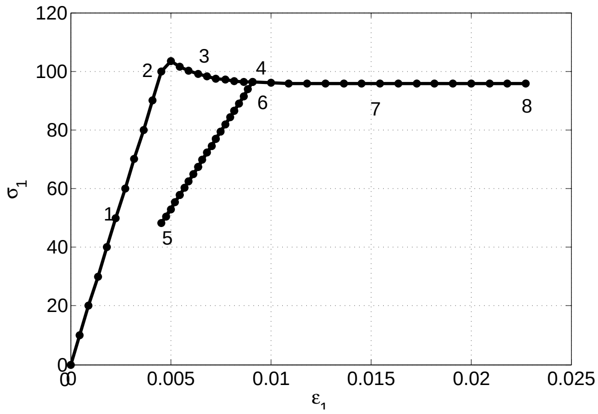

Figure 13: Model 4, Stress vs Strain

Comparison of Figure 13 with Figure 10, gives valuable insight into hardening and softening. The stress-behavior is identical till yield stress. But in case of hardening, the stress would be asymptotic to a value greater than the yield. On the other hand, in case of softening, stress would be asymptotic to a value lesser than the yield.

From evolution of q over time, it is observed that, q got reduced in two processes and remained constant otherwise. First decline is during the loading, (as soon as load reaches yield, q would start decreasing). Then for the entire time of unloading and loading (until current state), q would be constant since there is no inelastic loading. The second decline occurs when the stress reaches point 6 in Figure 13.

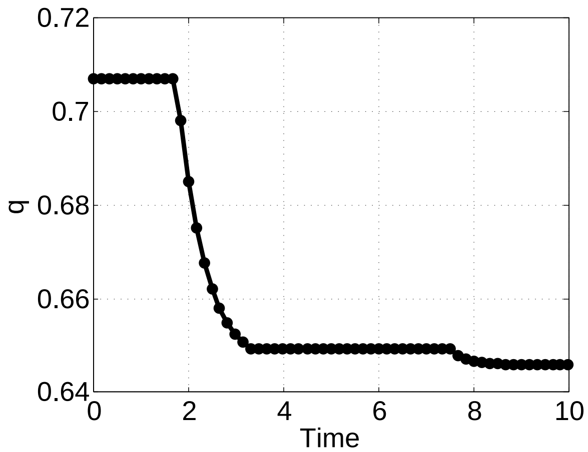

Figure 14: Model 4, q vs Time

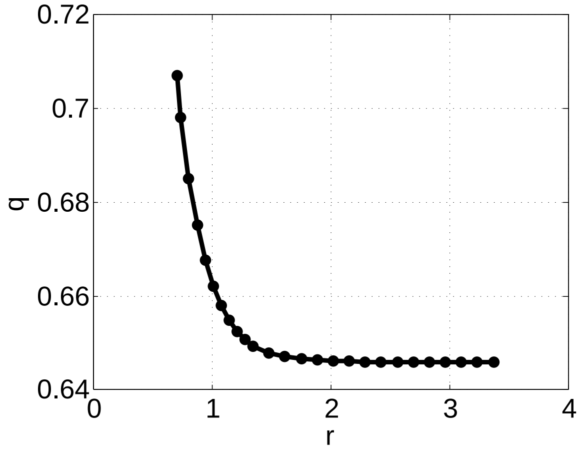

Figure 15: Model 4, Exponential Softening

The hardening modulus increases (approaches zero from negative side) with respect to r therefore, the curve becomes more flat. The exponential hardening can be observed in Figure 15 since $q'(r) \to 0$.

Model 5

This simulates 2D loading/unloading for a non-symmetric model (n=2) with linear $H = 0.3$. Undamaged stresses are as follows: Step 1. $\sigma_1 = 200$, σ_2=0. Step 2. σ_1=100,σ_2=-100. Step 3. $\sigma_1 = 500$, σ_2=300. 10 sub-steps were used for each load-step.

The red lines in Figure 16 depict the undamaged model. During loading along path 1-2, the damage surface is pushed outwards once the elastic loading reaches damage surface. 2-3 is elastic unloading (since it is within the new damage surface), thus no change in damage surface. 3-2 is elastic loading and after 2-4, it is inelastic loading, thus creating damage.

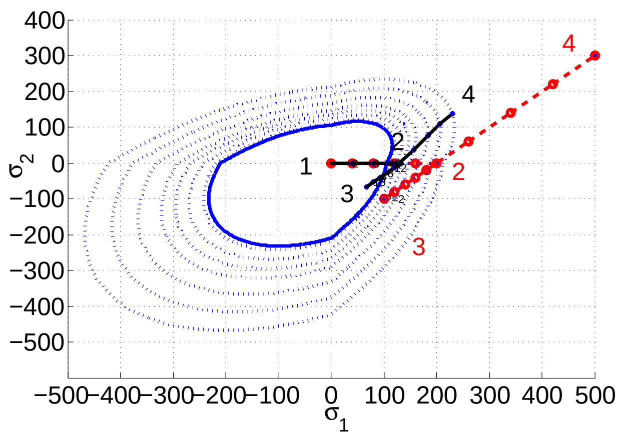

Figure 16: Model 5, Damage Surface Evolution

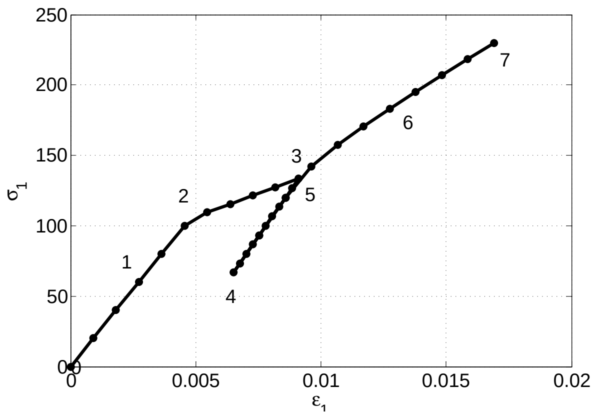

Figure 17: Model 5, Stress vs Strain, First Principal

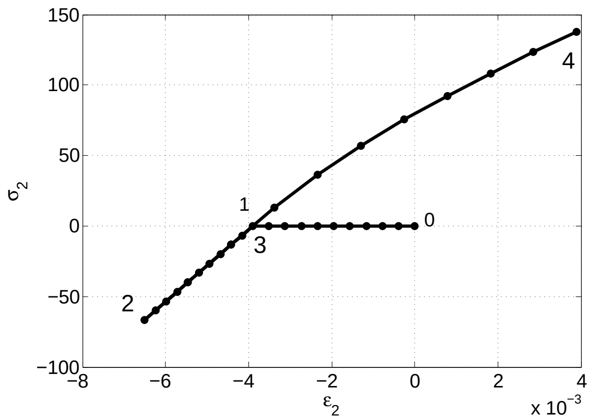

Figure 18: Model 5, Stress vs Strain, Second Principal

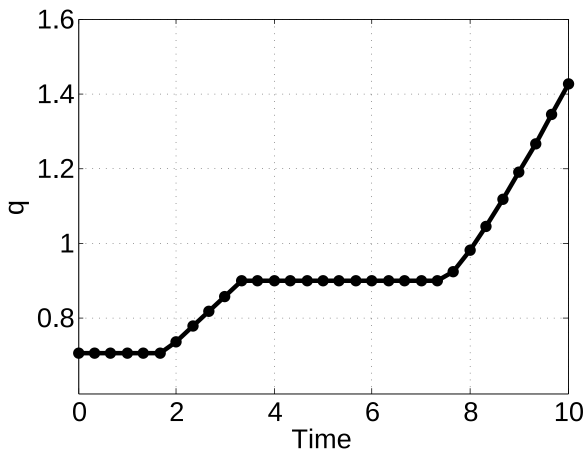

Figure 19: Model 5, q vs Time

With refernce to Figure 17, till point 2, σ_2 is 0. Thus, it is an uniaxial model effectively. Therefore, when σ_1 reaches 100 (σ_y), hardening begins (linear hardening) (2-3). 3-4-5 is elastic unloading-loading. From 5-6-7 it is again linear hardening. With reference to Figure 18, from 0-1, strain decreases, due to poisson's ratio (dependence on σ_1). from 1-2-3 is elastic unloading-loading. From 3-4 inelastic-loading.

Since, H is constant, in inelastic-loading, q would be linear which is evident from Figure 19. The phase with constant q is the elastic unloading-loading phase.

Model 6

This simulates 2D loading/unloading for a non-symmetric model (n=2) with linear $H = 0.3$. Undamaged stresses are as follows: Step 1. $\sigma_1 = 200$, σ_2=200. Step 2. σ_1=100,σ_2=100. Step 3. $\sigma_1 = 500$, σ_2=500. 10 sub-steps were used for each load-step.

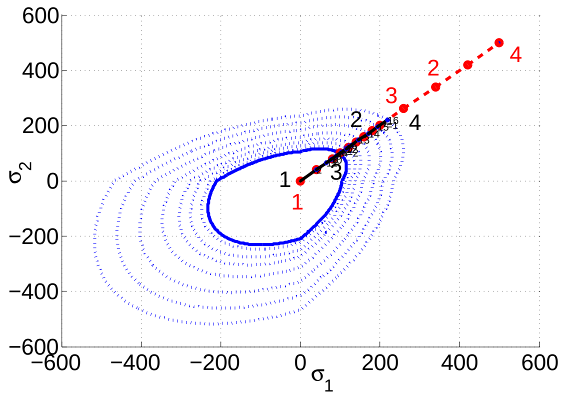

Figure 20: Model 6, q vs Times

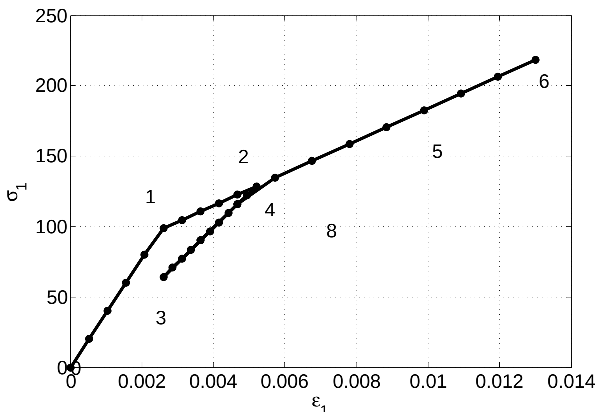

Figure 21: Model 6, q vs Times

Since, this is a symmetric loading in terms of σ_1 and σ_2, both stresses should behave exactly same. (Isotropic nature of constitutive model). This fact is evident from Figure 21 to Figure 24.

The red line indicates the undamaged stress states. Since, loading and unloading of both stress is same in quantity, the lines of loading and unloading overlap in principal stress space. 2-3-2 path is indicative of this elastic unloading-loading. Both the stresses behave identically as expected.

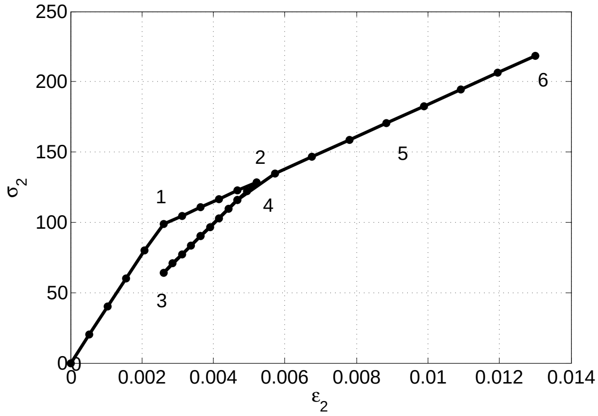

Figure 22: Model 6, q vs Times

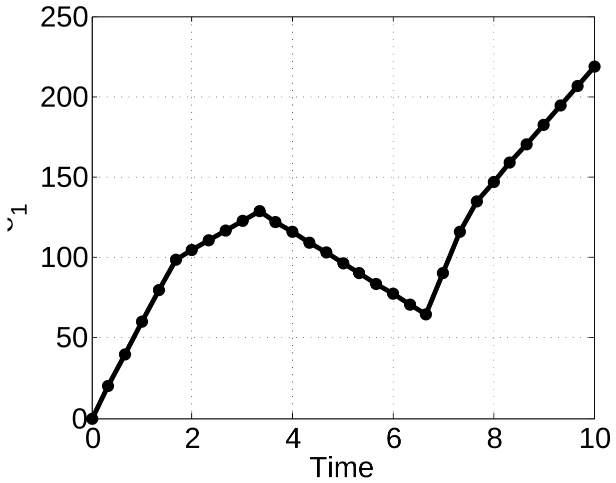

Figure 23: Model 6, q vs Times

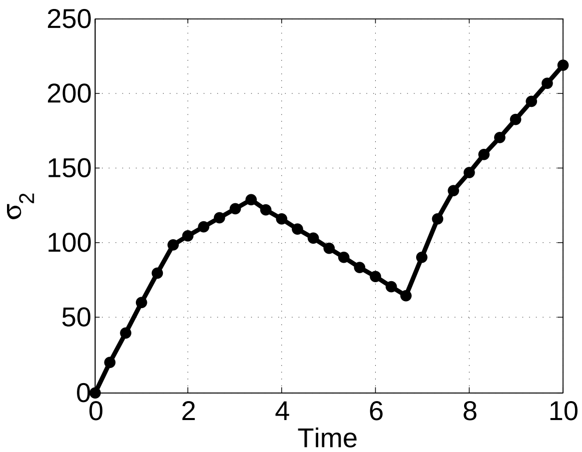

Figure 24: Model 6, q vs Times

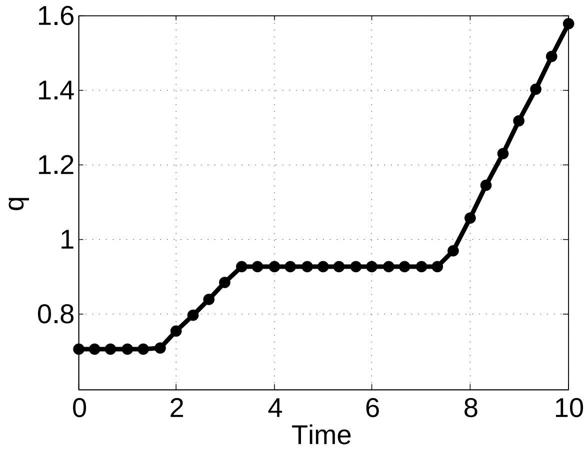

Figure 25: Model 6, q vs Times

In general, it important to note the following observations:
1. For all 6 models, the stress could never go outside damage surface. (Which is based on KKT conditions for inviscid damage models)
2. Values of d, r, $\dot{d}$ and $\dot{r}$ remained non-negative. This is based on the definition of internal variable and 2nd law of thermodynamics. A check was implemented in the program to verify the above conditions.
3. Some values, need to satisfy certain conditions which were checked in the program. $H \leq \frac{q}{r}$ based on 2nd law of thermodynamics. Another condition is $A > 0$ so that as $t \to \infty$, $q \to q_\infty$

Part 2: Rate Dependent Models

Dependence on η

A model with following parameters was simulated. $E = 20000$, $\nu = 0.3$, $H = 0.2$ at $r = r_0$, $\sigma_y = 100$, Plane Strain, Symmetric Model, Exponential Hardening, $\alpha = 1$, total time = 10, sub-steps = 10, Undamaged stresses = [200,0],[400,0],[600,0].

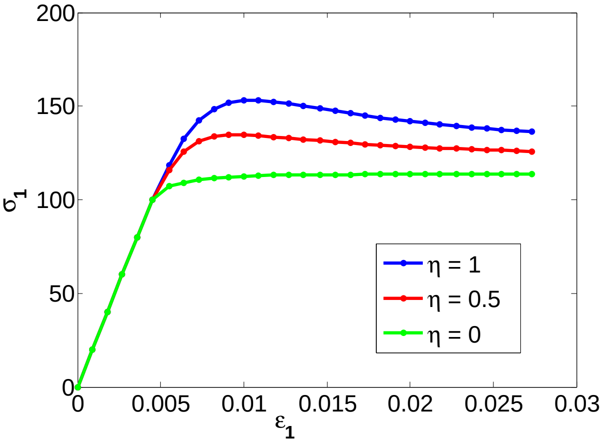

Figure 26: Stress vs Strain, η Variation

The behavior is as shown in Figure 26. As η approaches 0, the model starts to loose its viscous behavior. Also, η is a measure of how far a damage surface can move outwards, with $\eta = 0$ means, surface can move very large distance, if the r_{n+1} demands it. So, η allows the state of stress to be outside the damage surface, but would catch upto it eventually. So, in summary, more the value of η, more distance, the state of stress can be outside the damage surface. This is evident from the Figure 26, since $\eta = 1$ has the maximum allowance (how high it is from σ_y)

Dependence on $\dot{\epsilon}$

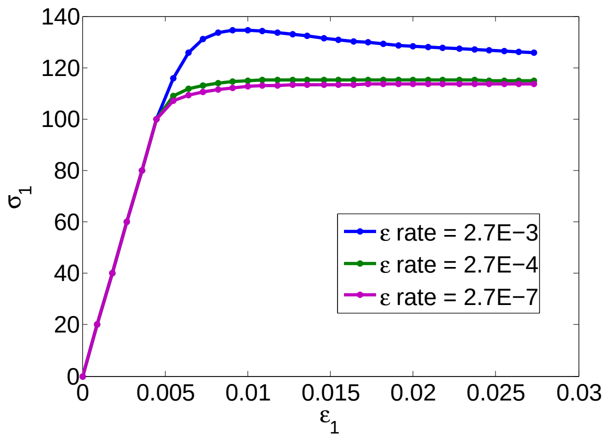

Figure 27: Stress vs Strain, $\dot{\epsilon}$ Variation

A model with following parameters was simulated. $E = 20000$, $\nu = 0.3$, $H = 0.2$ at $r = r_0$, $\sigma_y = 100$, Plane Strain, Symmetric Model, Exponential Hardening, $\alpha = 1$, $\eta = 0.5$, sub-steps = 10, Undamaged stresses = [200,0],[400,0],[600,0]. Different strain rates were obtained by changing the total time.
The behavior is as shown in Figure 27. As $\dot{\epsilon}$ approaches 0, the model starts to loose its viscous behavior. It becomes more like a static loading scenario. This model thus simulates the known fact that materials resist loads with higher strain rate more easily. To summarize, as $\dot{\epsilon}$ decreases, the solution approaches towards the inviscid solution which is evident from the Figure.

Dependence on α

The integration methods for $\alpha < 0.5$ are not stable from theory. This is also seen in the practice. The model with following parameters showed the above statement is true. The key to get oscillations, according to the author, is to get τ_n in elastic doamin, but τ_{n+1} outside elastic surface. So when they are averaged (using weighting factor α), one might get a value which is still inside the elastic region. So a situation would arise, that the loading is outside elastic domain, but the method has failed to identify it since, its weighted average is still inside elastic regime.

Following parameters were used to simulate this model. $E = 20000$, $\nu = 0.3$, $H = 0.2$ at $r = r_0$, $\sigma_y = 100$, Plane Strain, Symmetric Model, Exponential Hardening, , total time = 100, $\eta = 0.5$, sub-steps = 10, Undamaged stresses = [190,0],[400,0],[600,0].

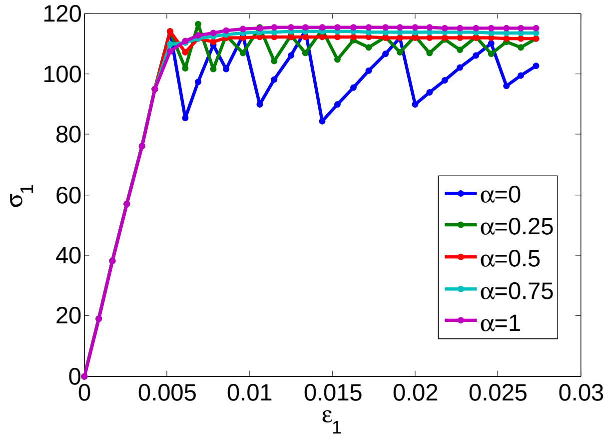

Figure 28: Stress vs Strain, α Variation

Dependence of C_{11} on α

The algorithm to calculate Algorithmic and Tangent constitutive operators was written as a post process since all the parameters are known and are stored

in a custom-written database. As shown in Figure 29, as α reduces to 0, difference between two tangent operators decreases and ultimately becomes 0 when $\alpha = 0$ which is as sexpected from the theory.
Following parameters were used to simulate this model. $E = 20000$, $\nu = 0.3$, $H = 0.2$ at $r = r_0$, $\sigma_y = 100$, Plane Strain, Symmetric Model, Exponential Hardening, , total time = 10, $\eta = 0.5$, sub-steps = 10, Undamaged stresses = [190,0],[400,0],[600,0].

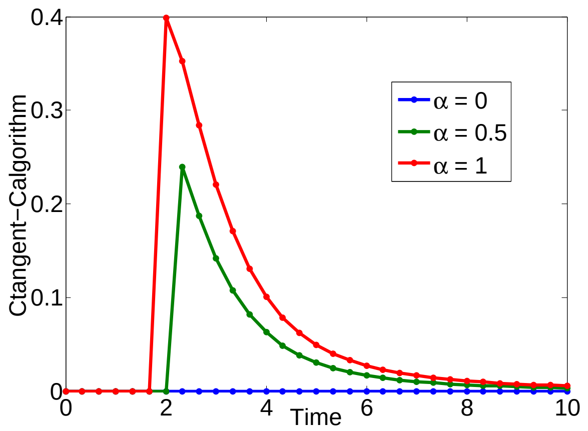

Figure 29: Difference of C_{11} vs Time, α Variation

Modified Code

Please refer to the attached .rar folder for the code. I have written my own post processing routines for easy plotting of multiple variables at the same time.