

Assignment 2

Computational Solid Mechanics

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MSc Computational Mechanics

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J2 Computational plasticity

The material data with which the numerical simulation has been carried out are considered hypothetically to be:

- 1) Elastic Modulus = 25 (units)
- 2) Yield Stress = 10 (units)
- 3) Poisson's ratio = 0.3

And the following uniaxial cyclic loading is considered to be the loadcase:

$$\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 20 & 0 & 0 & 0 & 0 & 0 \\ -30 & 0 & 0 & 0 & 0 & 0 \\ 20 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

Perfect Plasticity Model

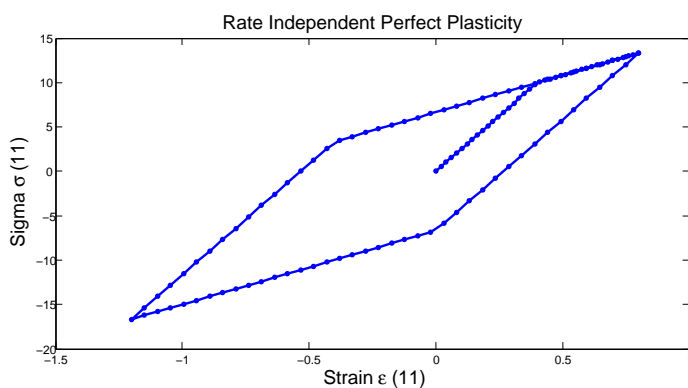


Figure 1: Rate Independent Plasticity (Stress11 vs. Strain11)

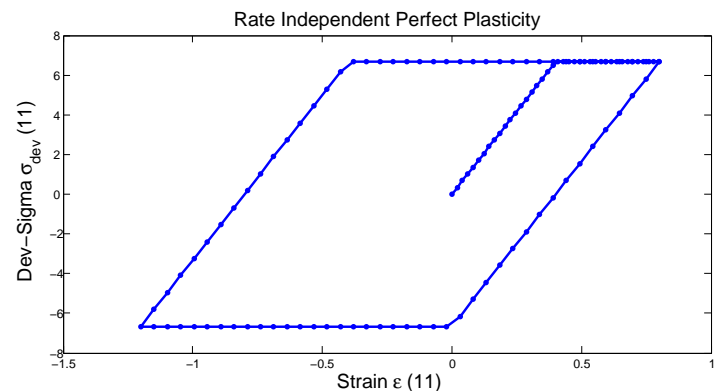


Figure 2: Rate Independent Plasticity (Dev-Stress11 vs. Strain11)

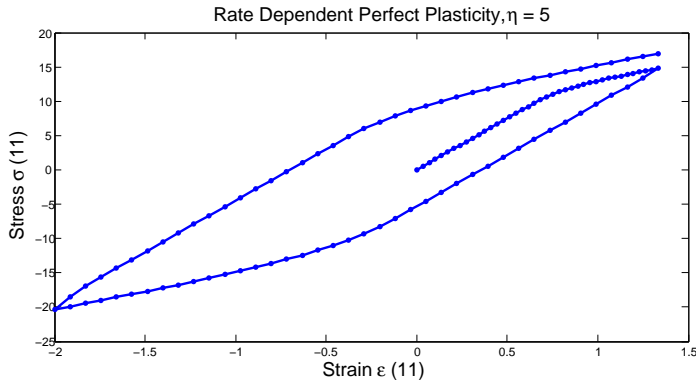


Figure 3: Rate Dependent Plasticity(Stress11 vs. Strain11)

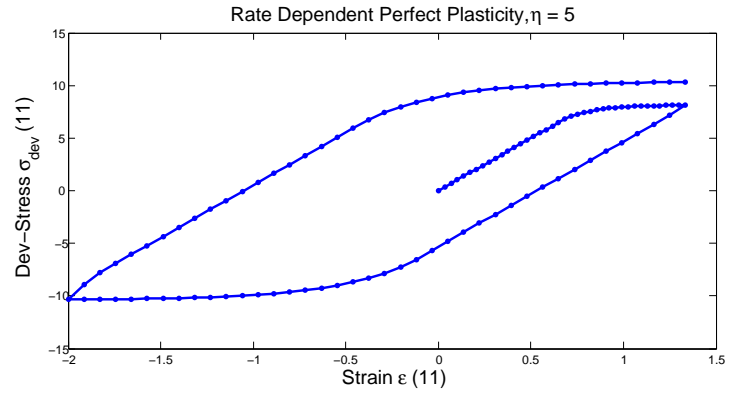


Figure 4: Rate Dependent Plasticity(Dev-Stress11 vs. Strain11)

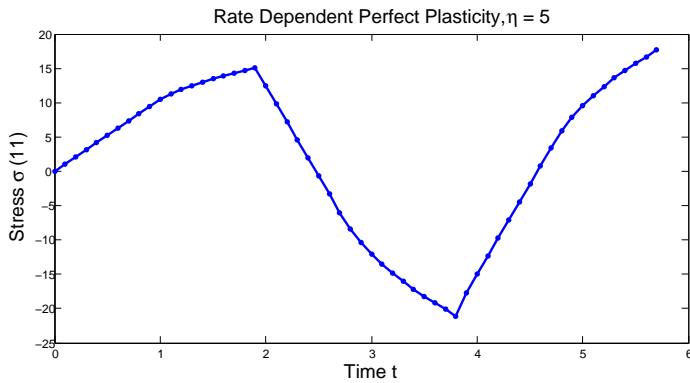


Figure 5: Rate Dependent Plasticity(Stress11 vs. Time)

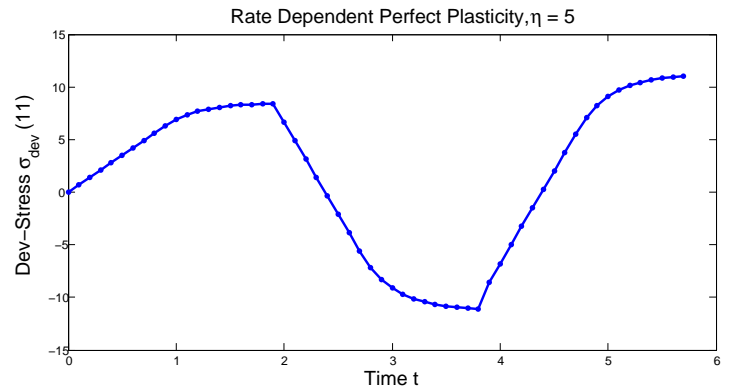


Figure 6: Rate Dependent Plasticity(Dev-Stress11 vs. Time)

For the rate independent case, when cyclic loading in x-direction $[0,20,-30,20]$ (units) is applied, the deviatoric stress(11) exhibits perfectly plastic response and it does not cross the $\frac{2}{3}$ rd of yield surface (6.67 units) due to the absence of any hardening parameter as seen in Figure 2. Once Stress11 reaches the intersection of Stress11 and Yield surface, then Stress22 and Stress33 may start increasing so that the stress point may still lie on the yield surface in 3-D space, therefore Stress11 is allowed to grow beyond Yield point as seen in Figure 1.

However, in case of Rate dependent perfect plasticity, due to the presence of viscosity, when tensile loading of Stress11=20 (units) is applied, Deviatoric Stress(11) crosses the $\frac{2}{3}$ rd of yield surface. Further when 30 (units) compressive loading is applied, the Dev-stress go further beyond the $\frac{2}{3}$ rd of yield surface as seen in Figure. 4.

Linear Isotropic Model

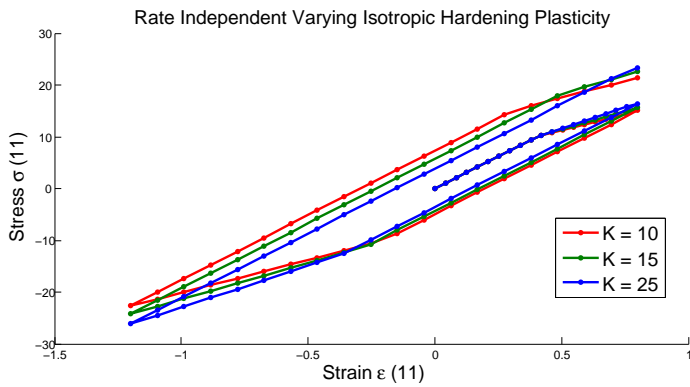


Figure 7: Rate Independent Varying Linear Isotropic Hardening(Stress11 vs. Strain11)

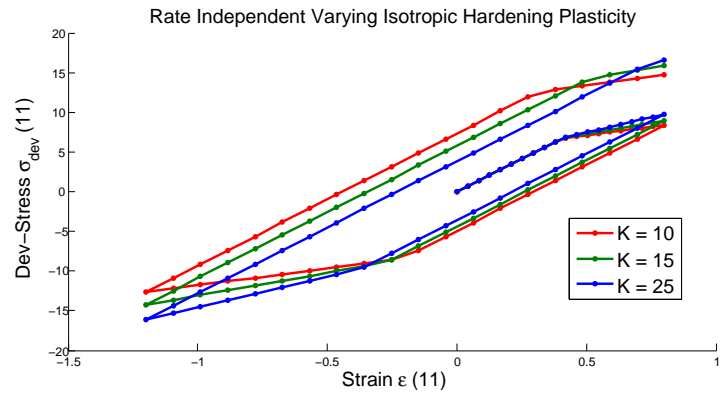


Figure 8: Rate Independent Varying Linear Isotropic Hardening(Dev-Stress11 vs. Strain11)

Here a cyclic loading in x-direction $[0,20,-30,20]$ (units) is applied. The yield stress during initial tension loading increases due to hardening and then the compression yield stress grows the same amount for varying Isotropic Hardening coefficient K . As we increase the value of K , the slope of the deformation changes and approaches the value of Young's Modulus (25 units) as seen in Figure. 8. The material starts behaving elastically when isotropic hardening coefficient equals young modulus .

Linear Kinematic Model

Here a cyclic loading in x-direction $[0,30,-40,30]$ (units) is applied. The Yield stress is 10 (units), Young's modulus = 25 (units). We consider a rate independent plasticity model and we vary the Kinematic coefficient (K) to understand its influence.

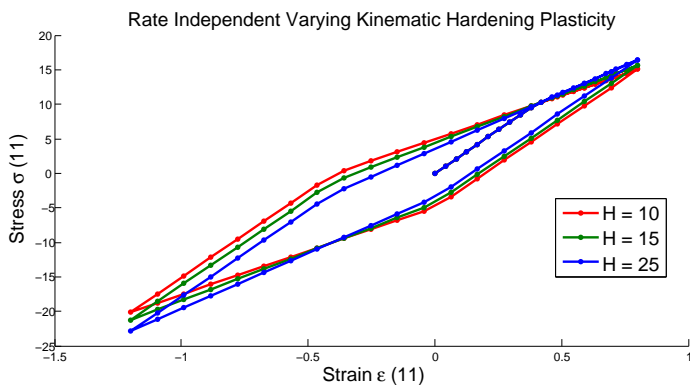


Figure 9: Rate Independent Varying Linear Kinematic Hardening(Stress11 vs. Strain11)

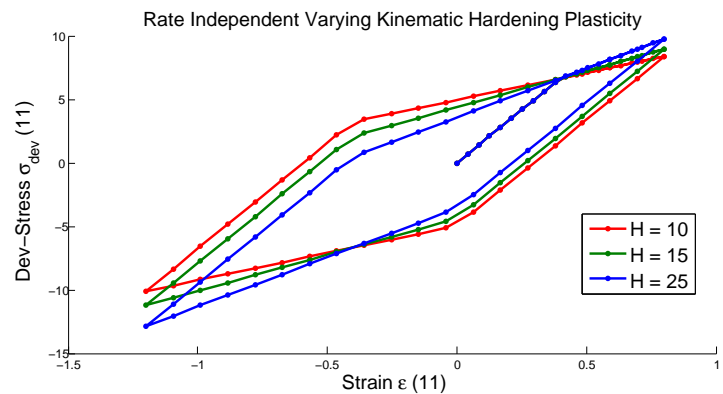


Figure 10: Rate Independent Varying Linear Kinematic Hardening(Dev-Stress11 vs. Strain11)

Kinematic hardening correctly accounts for Bauschinger effect. When kinematic hardening is applied, the material softens in compression and thus can correctly model cyclic behaviour. As the Kinematic Hardening coefficient increases, the axisymmetric nature of tension-compression is gradually lost as the material softens more in compression as seen in 10.

Rate Dependent Plasticity

Effect of Varying Viscosity

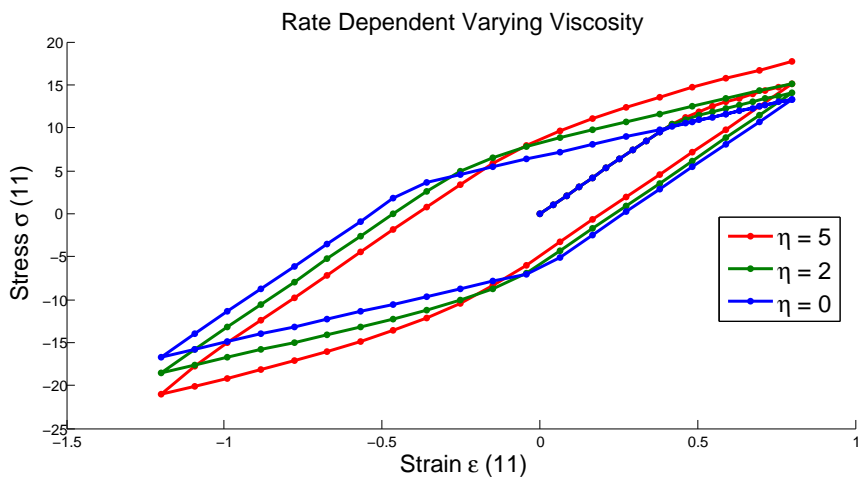


Figure 11: Varying Viscosity with constant Loading Rate(Stress11 vs. Strain11)

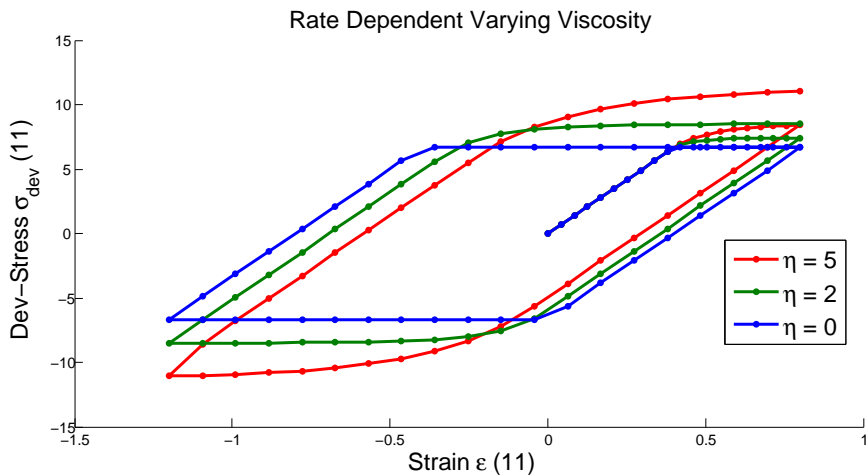


Figure 12: Varying Viscosity with constant Loading Rate(Dev-Stress11 vs.Strain11)

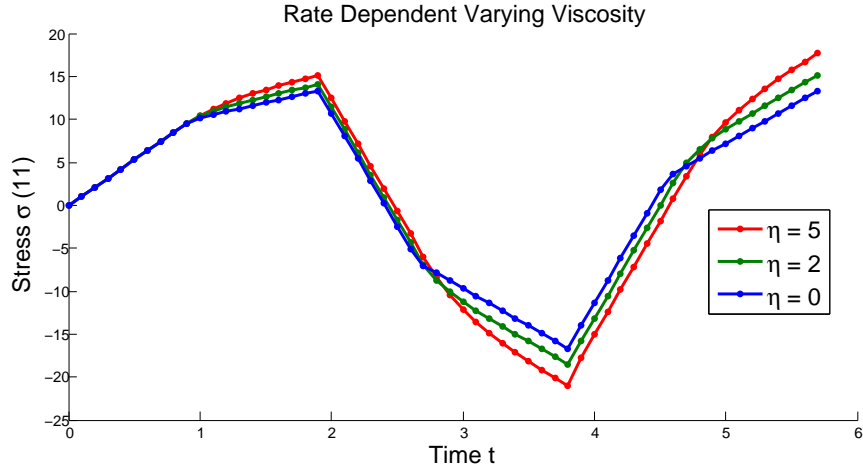


Figure 13: Varying Viscosity with constant Loading Rate(Stress11 vs. Time)

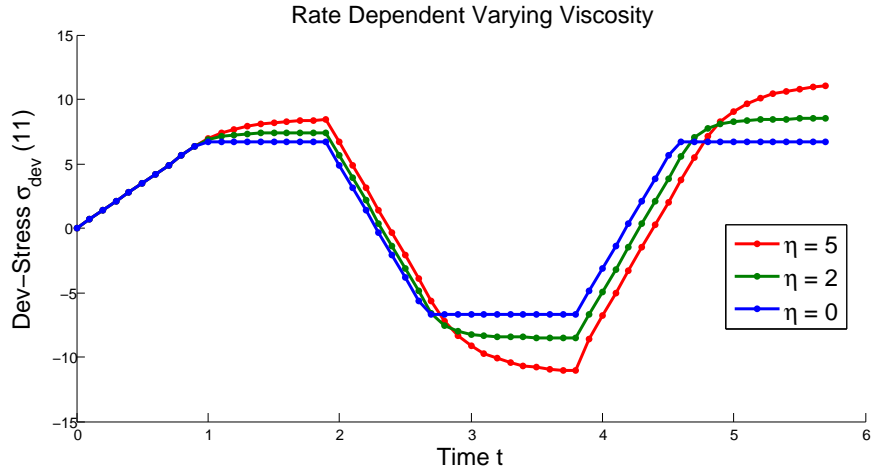


Figure 14: Varying Viscosity with constant Loading Rate(Dev-Stress11 vs.Time)

Here a cyclic loading in x-direction $[0,20,-30,20]$ (units) is applied.. The Yield stress is 10 (units), Young's modulus = 25 (units). We consider a rate dependent plasticity model with constant strain rate and we vary the viscosity coefficients to understand the influence of viscosity parameter (η). The isotropic and kinematic hardening coefficients are taken to be zero for better understanding the effects of viscosity.

When the viscosity is zero, the material behaves like perfect plasticity condition with deviatoric stresses(11) not going beyond $\frac{2}{3}$ rd of yield stress.As the viscosity is increased, the material gets deformed and the Dev-Stress(11) go beyond the $\frac{2}{3}$ rd of yield stress. Upon further increase of viscosity, the streses go much further than the $\frac{2}{3}$ rd of yield surface as seen in Figure. 12. If the coefficient of viscosity is taken to of be very high value, the material will start behaving elastically. The plastic strain decreases with the increase of viscosity as seen in Figure 15 and 16 below.

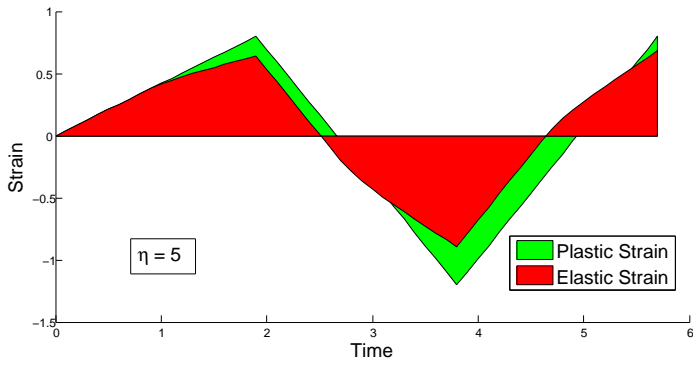


Figure 15: Strain11 vs. Time at $\eta = 5$

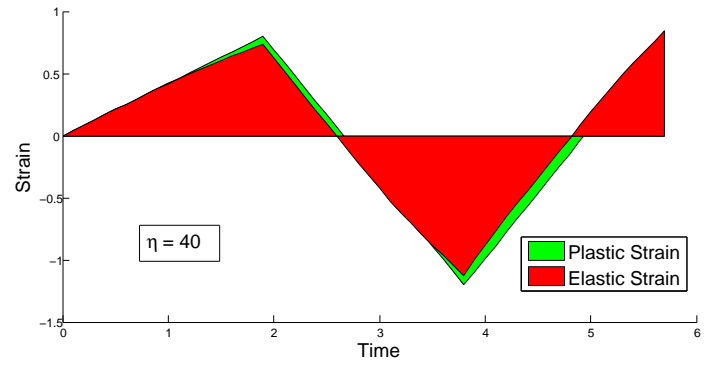


Figure 16: Strain11 vs. Time at $\eta = 40$

Effect of Varying Loading Rate

Here a cyclic loading in x-direction $[0,20,-30,20]$ (units) is applied. The Yield stress is 10 (units), Young's modulus = 25 (units). We consider a rate dependent plasticity model with constant viscosity and we vary the loading rate to understand the influence of loading rate (dt). The isotropic and kinematic hardening coefficients are taken to be zero for better understanding of the effects.

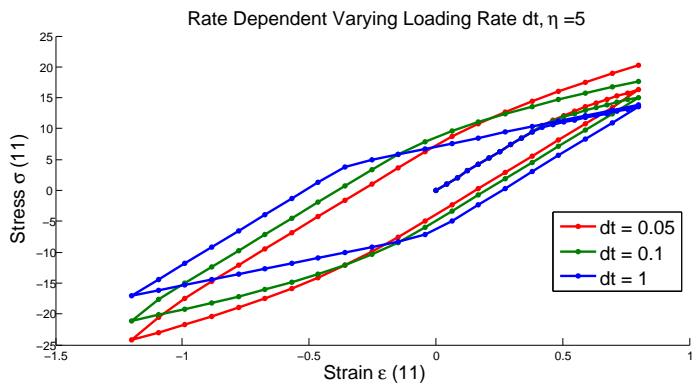


Figure 17: Varying Loading Rate with constant Viscosity (Stress vs. Strain (Stress11 vs. Strain11))

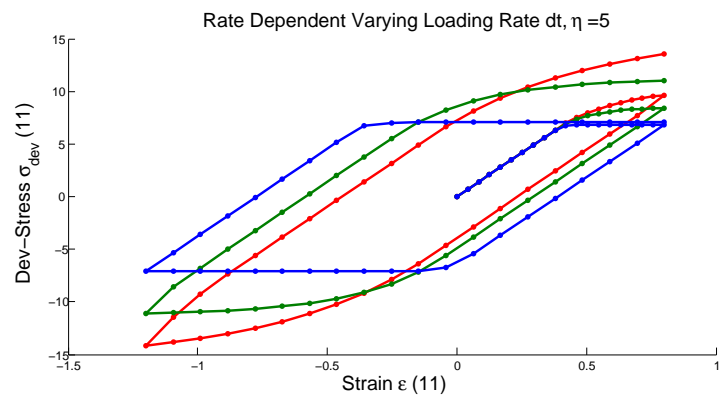


Figure 18: Varying Loading Rate with constant Viscosity (Dev-Stress11 vs. Strain11)

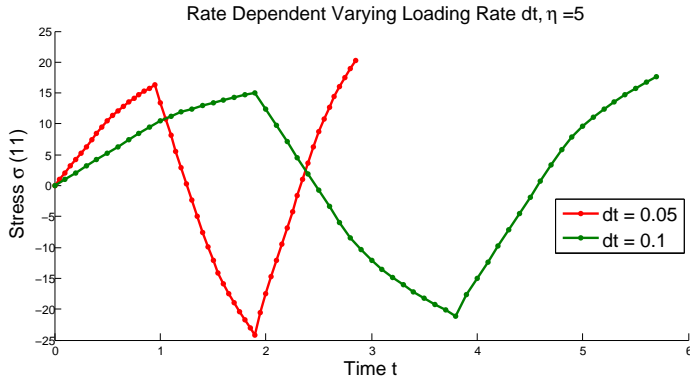


Figure 19: Varying Loading Rate with constant Viscosity(Stress vs. Strain(Stress11 vs. Time))

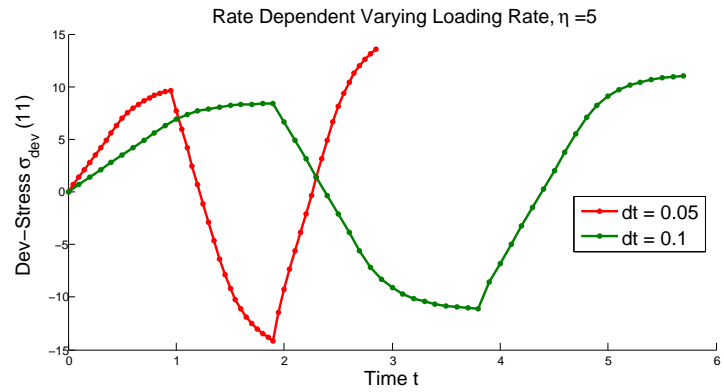


Figure 20: Varying Loading Rate with constant Viscosity(Dev-Stress11 vs. Time)

When dt is low, i.e. the strain rate is high, the effects of viscosity is clearly visible. The Dev-stress11 exceeds the $\frac{2}{3}$ -rd of yield stress as seen in Figure 18. As the strain rate is decreased, i.e. dt is increased the rate of increase of stress becomes gradually lower as seen in Figure 20. Upon further increase of dt , i.e. when strain rate is made much lower, the material approaches quasi-static state. If the loading rate is very low, the material will start behaving perfectly plastic with little or no effect of viscosity.

Recovery of Rate Independent response from Rate Dependent Result

Here a cyclic loading in x-direction $[0,20,-30,20]$ (units) is applied. The Yield stress is 10 (units), Young's modulus = 25 (units). We consider a rate dependent plasticity model with constant loading rate and we vary the viscosity to recover the rate independent response. The isotropic and kinematic hardening coefficients are taken to be zero for better understanding.

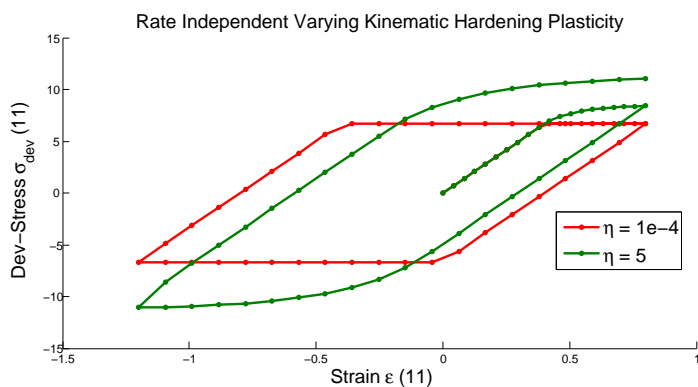


Figure 21: Varying Viscosity with constant Loading Rate(Dev-Stress11 vs. Strain11)

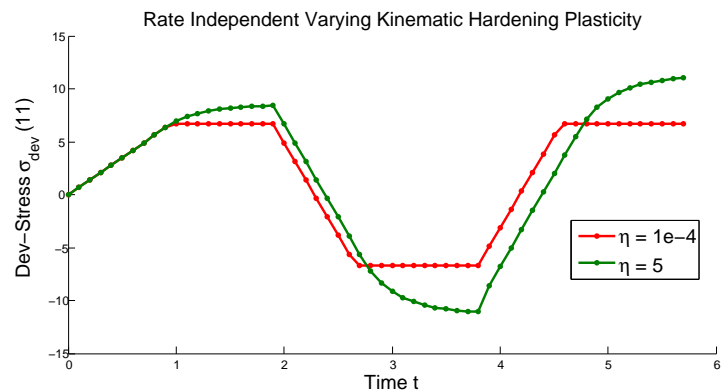


Figure 22: Varying Viscosity Rate with constant Loading Rate(Dev-Stress11 vs. Time)

From the Figure. 21 & 22 above, we observe that, when very small value of viscosity coefficient is considered, we recover the Rate Independent response from Rate Dependent result.

Non-Linear Isotropic Hardening

Here a cyclic loading in x-direction [0,20,-30,20] (units) is applied. The Yield stress is 10 (units), Young's modulus = 25, $\sigma_\infty = 15$ (units). We consider a rate independent non linear isotropic plasticity model for simplicity and better understanding. While calculation of non-linear isotropic hardening, the linear hardening effect is not added to the exponential saturation law, since we are considering **only the saturation law**.

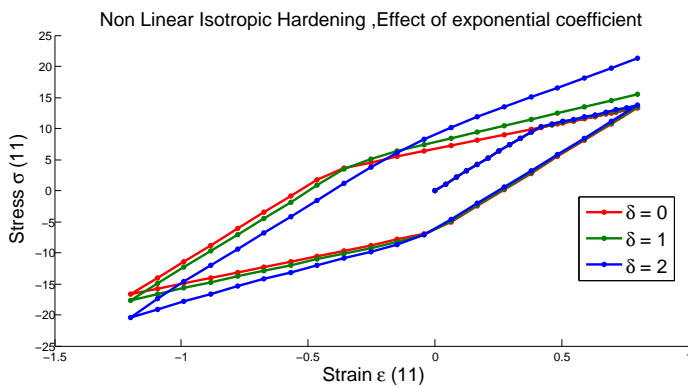


Figure 23: Rate Independent Non-linear Isotropic Hardening Plasticity with varying delta(Stress11 vs.Strain11)

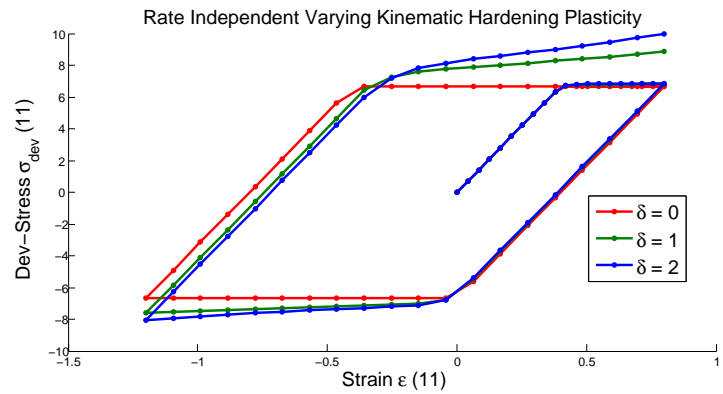


Figure 24: Rate Independent Non-linear Isotropic Hardeing Plasticity with varying delta(Dev-Stress11 vs. Strain11)

When the exponential coefficient δ is zero, the material gives perfect plastic response. As the value of δ is increased we start to get a non-linear exponential hardening plot which approaches the value of $\frac{2}{3}$ rd of σ_∞ . The dev-stress11 will never go beyond the $\frac{2}{3}$ rd of saturation point. With increasing value of δ , the tangential slope of hardening, approaching $\frac{2}{3}$ rd of σ_∞ increases, as seen in Figure 24.

APPENDIX

```
1 clear all
2 clc
3
4 %%----- Material Properties -----
5 young      = 25;      %Young's Modulus
6 nu         = 0.3;    %Poisson's Ratio
7 yield      = 10;     %Yield Strength
8 mu         = young / ( 2 * (1 + nu) );
9 sigma_infy = 15;
10 eta       = 5;      %Viscosity Coefficient
11 K         = 10;     %Isotropic Hardening
12 H         = 10;     %Kinematic Hardening
13 %%-----
14
15 delta     = 1;
16
17 dt        = .1 ;
18
19 disp('[1] Linear case')
20 disp('[2] Nonlinear case')
21 method=input('Choose method:');
22
23 %%----- Loading -----
24 nSteps    = 20 ;
25 sigmaP    = [ 0 0 0 0 0 0;
26              20 0 0 0 0 0;
27              -30 0 0 0 0 0;
28              20 0 0 0 0 0];
29
30 load_Steps = size(sigmaP,1)-1 ;
31 timeSteps  = nSteps*ones(1,load_Steps);
32 %%-----
33
34 strain =StrainHistory(young,nu,sigmaP,timeSteps);
35
36
37 [ce] = elastic_tensor(young,nu);
38
39 if method==1      %Linear Case
40
41 sigma_trial = zeros(6 , size(strain,2)) ;
42 f_trial     = zeros(1, size(strain,2)) ;
43 zeta_trial  = zeros(6 , size(strain,2)) ;
44 stress      = zeros(6 , size(strain,2)) ;
45 qbar        = zeros(6 , size(strain,2)) ;
46 q           = zeros(1, size(strain,2)) ;
47 strain_plas = zeros(6 , size(strain,2)) ;
48
49 for n=1 : size(strain,2)-1
50
51     sigma_Temp      = ce * (strain(:,n+1) - strain_plas(:,n)) ;
52     sigma_trial(:,n+1) = sigma_Temp ;
53     devsigma_Temp   = dev_sigma(sigma_trial(:,n+1)) ;
54     zeta_trial(:,n+1) = devsigma_Temp - qbar(:,n);
55     f_trial(n+1)    = norm(zeta_trial(:,n+1)) - sqrt(2/3)*( yield - q(n) ) ;
56
57     if f_trial(:,n+1) <= 0      %% Within elastic limit
58
59         strain_plas(:,n+1) = strain_plas(:,n);
60         qbar(:,n+1)        = qbar(:,n);
```

```

61     q(n+1)                = q(n) ;
62     stress(:,n+1)        = sigma_trail(:,n+1);
63
64     else                    %% Plasicity starts
65         %Updation of internal variables takes place
66
67         delta_gamma      = f_trial(:,n+1) / (2*mu + 2/3*K + 2/3*H + eta/dt) ;
68
69         stress(:,n+1) = sigma_trail(:,n+1) - delta_gamma * 2*mu ...
70                                 * unit_vector(zeta_trial(:,n+1));
71
72         strain_plas(:,n+1) = strain_plas(:,n) + delta_gamma...
73                                 *unit_vector(zeta_trial(:,n+1));
74
75         qbar(:,n+1) = qbar(:,n) + delta_gamma * 2/3*H ...
76                                 * unit_vector(zeta_trial(:,n+1)) ;
77
78         q(n+1) = q(n) - delta_gamma * sqrt(2/3)*K ;
79     end
80 end
81
82 elseif method==2          %Non-Linear case
83
84
85 sigma_trail = zeros(6 , size(strain,2)) ;
86 zeta_trial  = zeros(6 , size(strain,2)) ;
87 stress      = zeros(6 , size(strain,2)) ;
88 f_trial     = zeros(1, size(strain,2)) ;
89 qbar        = zeros(6 , size(strain,2)) ;
90 q           = zeros(1, size(strain,2)) ;
91 alpha       = zeros(1, size(strain,2)) ;
92 strain_plas = zeros(6 , size(strain,2)) ;
93 gamma       = zeros(1, size(strain,2)) ;
94
95 for n=1 : size(strain,2)-1
96
97     sigma_Temp = ce * (strain(:,n+1) - strain_plas(:,n)) ;
98     sigma_trail(:,n+1) = sigma_Temp ;
99     devsigma_Temp = dev_sigma(sigma_trail(:,n+1)) ;
100    zeta_trial(:,n+1) = devsigma_Temp - qbar(:,n);
101    f_trial(n+1) = norm(zeta_trial(:,n+1)) - sqrt(2/3)*( yield - q(n) ) ;
102
103    if f_trial(:,n+1) <= 0        %% Within elastic limit
104
105        strain_plas(:,n+1) = strain_plas(:,n);
106        qbar(:,n+1) = qbar(:,n);
107        q(n+1) = q(n) ;
108        stress(:,n+1) = sigma_trail(:,n+1);
109        alpha(n+1) = alpha(n) ;
110
111    else                    %plastic state
112        %Updation of internal variables takes place
113
114        gamma(n+1)= Newton(f_trial(n+1),dt,mu,H,eta,alpha(n),delta...
115                            ,sigma_infy,yield) ;
116
117        stress(:,n+1) = sigma_trail(:,n+1) - gamma(n+1)*dt * 2*mu ...
118                                * unit_vector(zeta_trial(:,n+1));
119
120        strain_plas(:,n+1) = strain_plas(:,n) + gamma(n+1)*dt ...

```

```

121                                     * unit_vector(zeta_trial(:,n+1)) ;
122
123     qbar(:,n+1) = qbar(:,n) + gamma(n+1)*dt * 2/3*H ...
124                                     * unit_vector(zeta_trial(:,n+1)) ;
125
126     q(n+1) = q(n) - gamma(n+1)*dt * sqrt(2/3)*K ;
127     alpha(n+1) = alpha(n) - gamma(n+1)*dt * sqrt(2/3) ;
128
129     end
130 end
131
132 end
133
134 for I=1:size(strain,2)
135     devStress(:,I) = dev_sigma(stress(:,I)) ;
137 end
138
139 %----- Post-Processing-----

```

```

1 function [value] = unit_vector(vector)
2
3 value = vector / norm(vector) ;
4
5 return

```

```

1 function gamma2=Newton(trial_f,dt,mu,H,eta,alpha,delta,sigma_infy,yield)
2 gamma1=0;
3 relaError=1;
4
5 while (relaError>1e-13)
6
7     gamma2=gamma1-residue(gamma1,trial_f,dt,mu,H,eta,alpha,delta,sigma_infy,↵
yield)...
8     /D_residue(gamma1,dt,mu,H,eta,alpha,delta,sigma_infy,yield);
9     relaError=abs(gamma2-gamma1);
10    gamma1=gamma2;
11
12 end
13
14 end

```

```
1 function value = residue(gamma,trial_f,dt,mu,H,eta,alpha,delta,sigma_infy,yield)
2
3 value = trial_f - gamma * dt * (2*mu + 2/3*H + eta/dt) -...
4     sqrt(2/3)*(phi(alpha + gamma * dt * sqrt(2/3),delta,sigma_infy,yield) -
...
5     phi(alpha,delta,sigma_infy,yield));
6 end
```