Assignment 3

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3.1

1. ① $\lambda = \frac{E}{1+\nu} \cdot \frac{\nu}{1-2\nu} = 2\mu \cdot \frac{\nu}{1-2\nu} \Rightarrow \nu = \frac{\lambda}{2(\lambda+\mu)}$

② $\mu = \frac{E}{2\left(1+\frac{\lambda}{2(\lambda+\mu)}\right)} \Rightarrow E = \frac{\mu\cdot(2\mu+3\lambda)}{\lambda+\mu}$

2. ① $\underset{\sim}{E} = \frac{E}{1-\nu^2}\begin{bmatrix} 1 & \nu & 0 \\ \nu & 1 & 0 \\ 0 & 0 & \frac{1-\nu}{2} \end{bmatrix} = \frac{\frac{\mu\cdot(2\mu+3\lambda)}{\lambda+\mu}}{1-\frac{\lambda^2}{4(\lambda+\mu)^2}}\begin{bmatrix} 1 & \frac{\lambda}{2(\lambda+\mu)} & 0 \\ \frac{\lambda}{2(\lambda+\mu)} & 1 & 0 \\ 0 & 0 & \frac{1-\frac{\lambda}{2(\lambda+\mu)}}{2} \end{bmatrix}$

(plane stress)

$$= \frac{4\mu}{\lambda+2\mu}\begin{bmatrix} \lambda+\mu & \frac{\lambda}{2} & 0 \\ \frac{\lambda}{2} & \lambda+\mu & 0 \\ 0 & 0 & \frac{\lambda+2\mu}{4} \end{bmatrix}$$

② $\underset{\sim}{E}^* = \frac{E(1-\nu)}{(1+\nu)(1-2\nu)}\begin{bmatrix} 1 & \frac{\nu}{1-\nu} & 0 \\ \frac{\nu}{1-\nu} & 1 & 0 \\ 0 & 0 & \frac{1-2\nu}{2(1-\nu)} \end{bmatrix}$

$$= \frac{\frac{\mu\cdot(2\mu+3\lambda)}{\lambda+\mu}\cdot\left(1-\frac{\lambda}{2(\lambda+\mu)}\right)}{\left(1+\frac{\lambda}{2(\lambda+\mu)}\right)\left(1-2\cdot\frac{\lambda}{2(\lambda+\mu)}\right)}\begin{bmatrix} 1 & \frac{\frac{\lambda}{2(\lambda+\mu)}}{1-\frac{\lambda}{2(\lambda+\mu)}} & 0 \\ & 1 & 0 \\ \text{symmetry} & & \frac{1-2\cdot\frac{\lambda}{2(\lambda+\mu)}}{2\cdot\left(1-\frac{\lambda}{2(\lambda+\mu)}\right)} \end{bmatrix}$$

$$= \cancel{(\lambda+2\mu)}\begin{bmatrix} \lambda+2\mu & \lambda & 0 \\ \lambda & \lambda+2\mu & 0 \\ 0 & 0 & \mu \end{bmatrix}$$

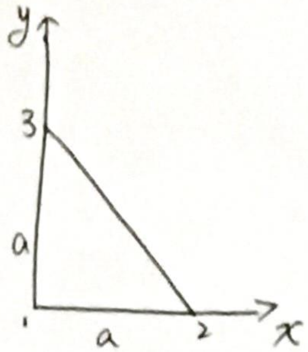
3. $$\underset{\sim}{E}^{*} = \underset{\sim}{E}_{\lambda} + \underset{\sim}{E}_{\mu} = \underbrace{\begin{bmatrix} \lambda & \lambda & 0 \\ \lambda & \lambda & 0 \\ 0 & 0 & 0 \end{bmatrix}}_{\underset{\sim}{E}_{\lambda}} + \underbrace{\begin{bmatrix} 2\mu & 0 & 0 \\ 0 & 2\mu & 0 \\ 0 & 0 & \mu \end{bmatrix}}_{\underset{\sim}{E}_{\mu}}$$

4. $$\underset{\sim}{E}_{\lambda} = \begin{bmatrix} \frac{E}{1+\nu} & 0 & 0 \\ 0 & \frac{E}{1+\nu} & 0 \\ 0 & 0 & \frac{E}{2(1+\nu)} \end{bmatrix};$$

$$\underset{\sim}{E}_{\mu} = \frac{E\nu}{(1+\nu)(1-2\nu)} \begin{bmatrix} 1 & 1 & 0 \\ 1 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

3.2

a) 1. ① plane stress



$$\begin{cases} b_1 = y_2 - y_3 = -1 \\ C_1 = x_3 - x_2 = -1 \end{cases} ; \quad \begin{cases} b_2 = y_3 - y_1 = 1 \\ C_2 = x_1 - x_3 = 0 \end{cases} ; \quad \begin{cases} b_3 = y_1 - y_2 = \\ C_3 = x_2 - x_1 = \end{cases}$$

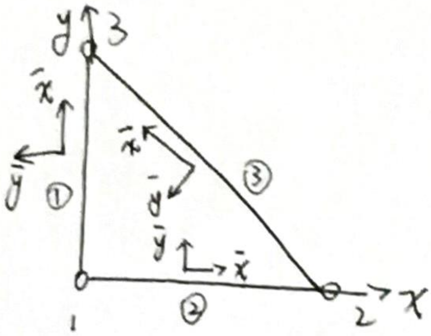
$$\underset{\sim}{B} = \left[\begin{array}{cc:cc:cc} b_1 & 0 & b_2 & 0 & b_3 & 0 \\ 0 & C_1 & 0 & C_2 & 0 & C_3 \\ C_1 & b_1 & C_2 & b_2 & C_3 & b_3 \end{array}\right] = \left[\begin{array}{cc:cc:cc} -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \\ -1 & -1 & 0 & 1 & 1 & 0 \end{array}\right]$$

$$\underset{\sim}{K}_{tri} = \int_{\Omega^e} h \underset{\sim}{B}^T \underset{\sim}{E} \underset{\sim}{B} \, d\Omega^e = A \cdot h \cdot \underset{\sim}{B}^T \underset{\sim}{E} \underset{\sim}{B} ; \quad \underset{\sim}{E} = E \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix}$$

$$= \frac{E}{2} \cdot \underset{\sim}{B}^T \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \frac{1}{2} \end{bmatrix} \underset{\sim}{B}$$

$$= E \cdot \begin{bmatrix} 0.75 & 0.25 & -0.5 & -0.25 & -0.25 & 0 \\ 0.25 & 0.75 & 0 & -0.25 & -0.25 & -0.5 \\ -0.5 & 0 & 0.5 & 0 & 0 & 0 \\ -0.25 & -0.25 & 0 & 0.25 & 0.25 & 0 \\ -0.25 & -0.25 & 0 & 0.25 & 0.25 & 0 \\ 0 & -0.5 & 0 & 0 & 0 & 0.5 \end{bmatrix}$$

② Bar



①: $\begin{cases} C = 0 \\ S = 1 \end{cases} \Rightarrow K^{①} = EA_1 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$

②: $\begin{cases} C = 1 \\ S = 0 \end{cases} \Rightarrow K^{②} = EA_2 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$

③: $\begin{cases} C = -\frac{\sqrt{2}}{2} \\ S = \frac{\sqrt{2}}{2} \end{cases} \Rightarrow K^{③} = \frac{\sqrt{2}}{4} EA_3 \cdot \begin{bmatrix} 1 & -1 & -1 & 1 \\ -1 & 1 & 1 & -1 \\ -1 & 1 & 1 & -1 \\ 1 & -1 & -1 & 1 \end{bmatrix}$

$$\underset{\sim}{K} = EA \begin{bmatrix} 1 & 0 & -1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$+ \frac{\sqrt{2}}{4} EA_3 \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & -1 & -1 & 1 \\ 0 & 0 & -1 & 1 & 1 & -1 \\ 0 & 0 & -1 & 1 & 1 & -1 \\ 0 & 0 & 1 & -1 & -1 & 1 \end{bmatrix}$$

The stiffness matrix are not equivalent. $\underset{\sim}{K}_{bar} \neq \underset{\sim}{K}_{tri}$

3. ① the plane stress Model is different from the Bar Model.

Because Plane Stress Model also consider the shear stress and σ_y while the Bar Model only consider the force along the Bar

4. When $\nu \neq 0$. the stiffness matrix of two Model are still unequivalent.

Conclusions: when we choose the element to represent the object, we need to consider which model should be applied firstly. And then choose the proper element and discrete model for the problem. Because different elements will cause different stiffness matrix and only the right method can get the right solution.