Assignment 6

CSMD

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1. Problem a

The computation of stiffness matrix is given by following equation:

$$K^e = \int_{l^e} B^T GAB dx = \frac{1}{2} \cdot w \cdot B^T(\xi) GAB(\xi) \cdot l^e$$

$$B(\xi) = [-\frac{1}{l^e}, \frac{(\xi - 1)}{2}, \frac{1}{l^e}, -\frac{(\xi + 1)}{2}]$$

For one Gauss point:

$$\xi = 0$$
$$w = 2$$

So, after computing, we get:

$$K^e = GA \cdot \begin{bmatrix} \frac{1}{l^e} & \frac{1}{2} & -\frac{1}{l^e} & \frac{1}{2} \\ \frac{1}{2} & \frac{l^e}{4} & -\frac{1}{2} & \frac{l^e}{4} \\ -\frac{1}{l^e} & -\frac{1}{2} & -\frac{1}{l^e} & -\frac{1}{2} \\ \frac{1}{2} & \frac{l^e}{4} & -\frac{1}{2} & \frac{l^e}{4} \end{bmatrix}$$

Based on this stiffness matrix, I have changed the Timoshenko code to Timoshenko Reduced one:

```
K_s = [   1   ,   len/2 ,   -1   ,   len/2 ;
        len/2 , len^2/4 , -len/2 , len^2/4 ;
         -1   ,  -len/2 ,    1   ,  -len/2 ;
        len/2 , len^2/4 , -len/2 , len^2/4 ];
```

Figure 1. The change of the Timoshenko code

2. Problem b

After implementing the code for three situations:

i. Euler Bernulli element
ii. Timoshenko full integrate element
iii. Timoshenko reduce integration element

I have gotten the relationships between a/L and above three different models.

I. The maximum displacement of three models with respect to a/L

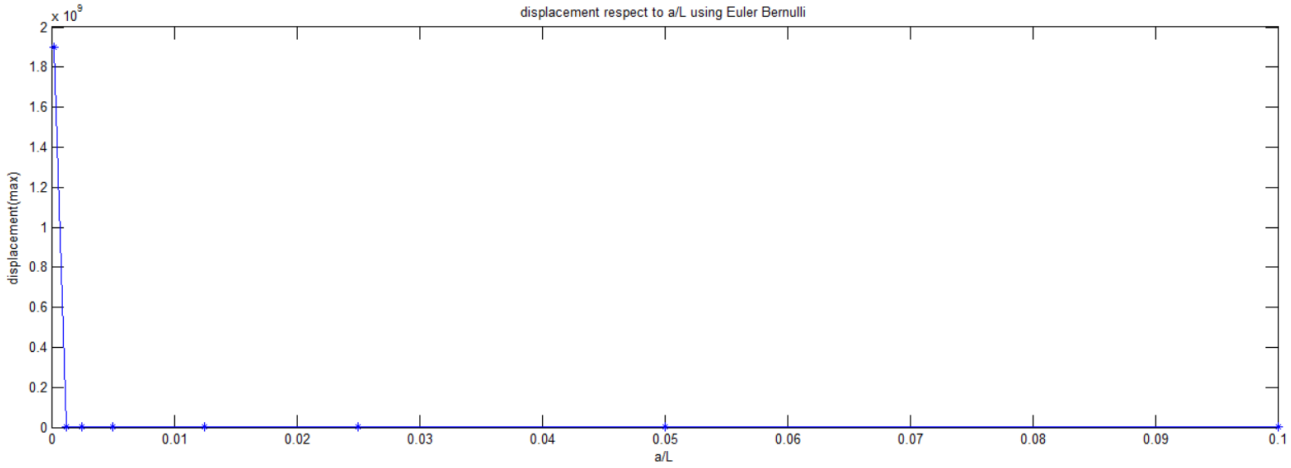

Figure 2. Maximum displacement respect a/L using Euler Bernulli

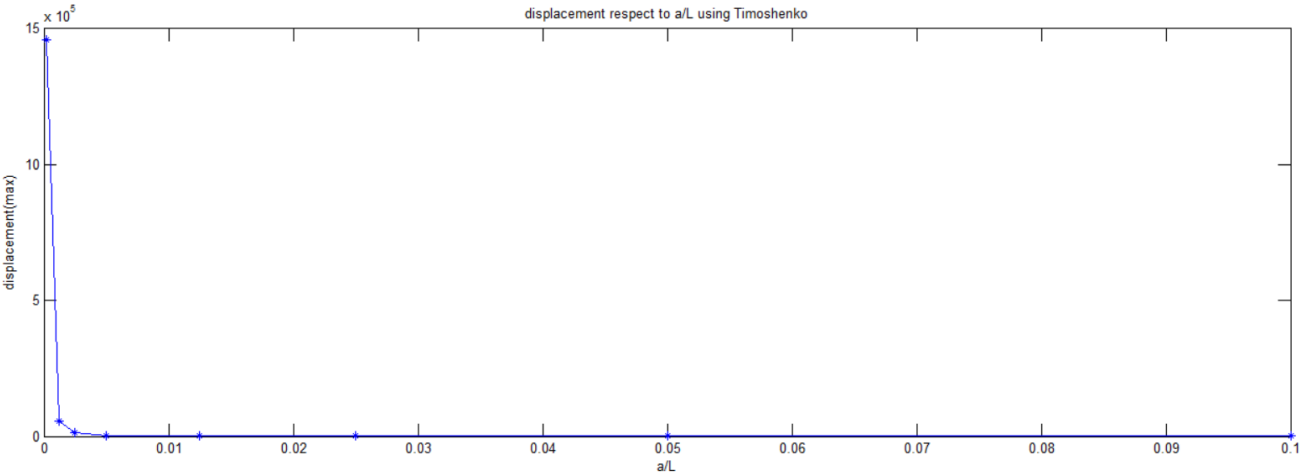

Figure 3. Maximum displacement respect to a/L using Timoshenko

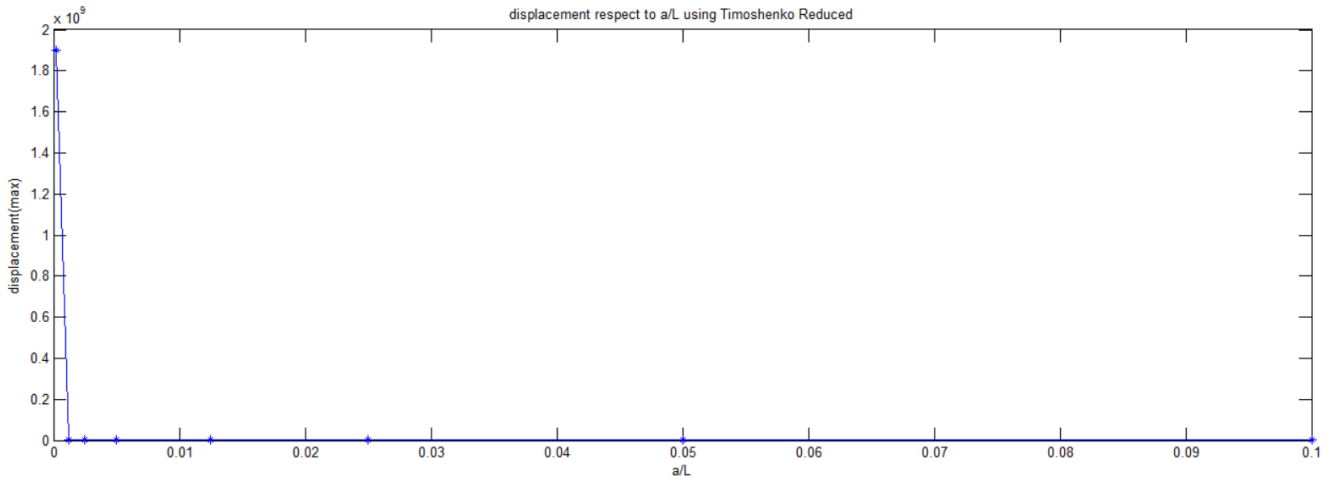

Figure 3. Maximum displacement respect to a/L using Timoshenko Reduced

From above pictures, we can find that when a/L is small, the results of these three are not reliable, which means that under this situation, the beam model is no longer accurate for calculating the displacement. We need use other model to compute the displacement when the a/L is small. But if a/L is in the range that we can consider the structure as beam model, the results will become reliable.

II. The maximum moment of three models with respect to a/L

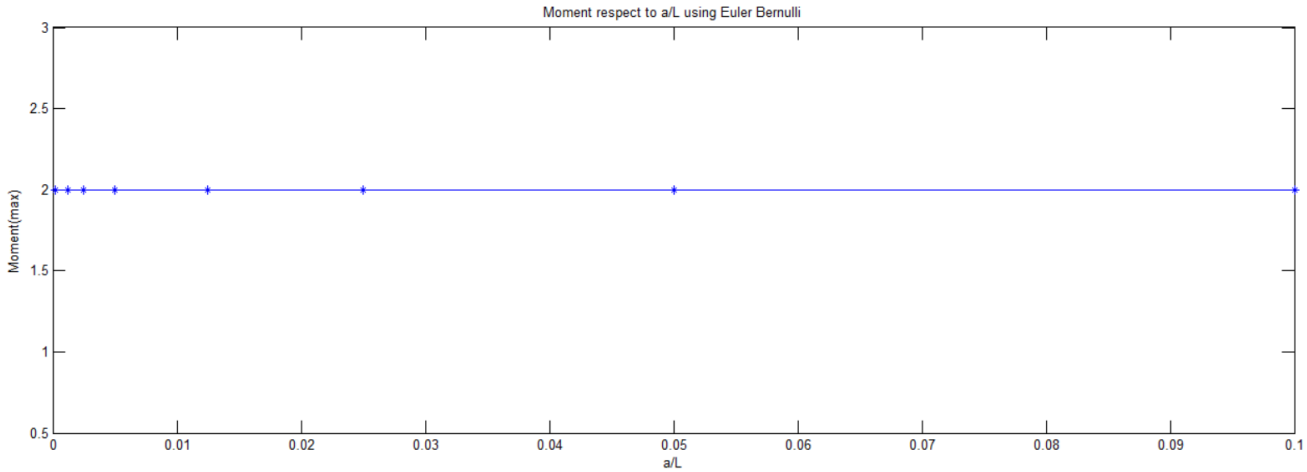

Figure 4. Maximum moment respect to a/L using Euler Bernulli

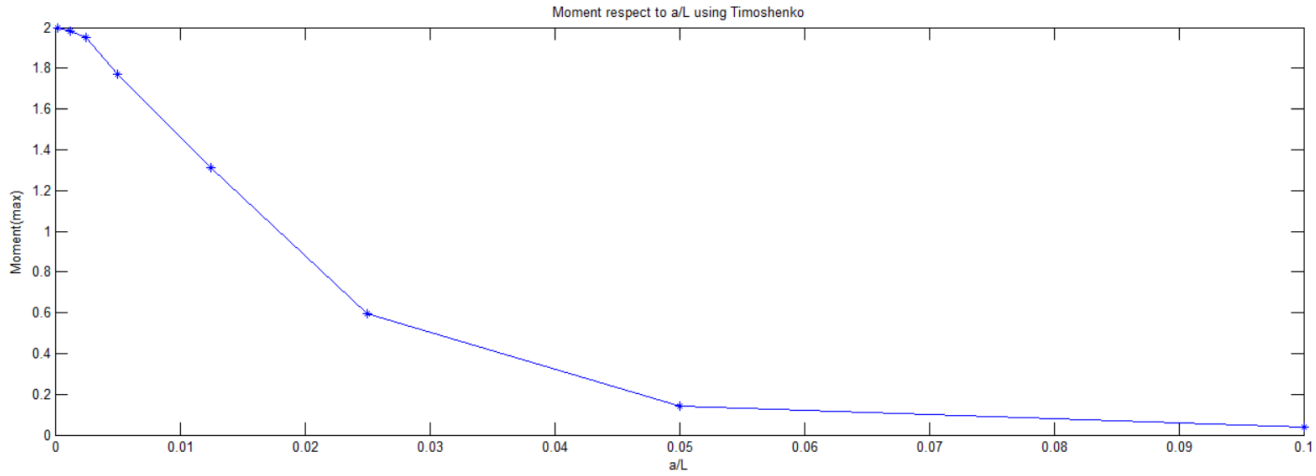

Figure 5. Maximum moment respect to a/L using Timoshenko

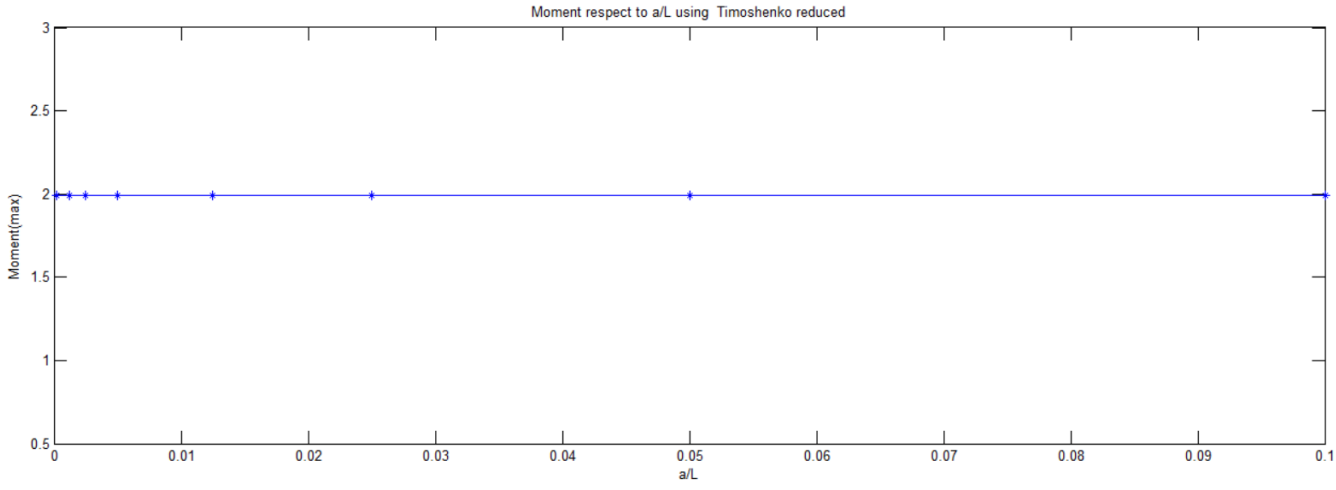

Figure 6. Maximum moment respect to a/L using Timoshenko Reduced

The moment results of Euler Bernulli and Timoshenko Reduced method are stable even though a/L keeps changing. The unaccuracy happened in calculating the displacement when a/L is small doesn't show in computing the moment by using above two methods.

But the situation of Timoshenko method is unsteady. The result keeps decreasing when a/L keeps decreasing. Consequently, Timoshenko method is unreliable when it coms to computing the moment with samll a/L.

III. The maximum shear stress of three models with respect to a/L

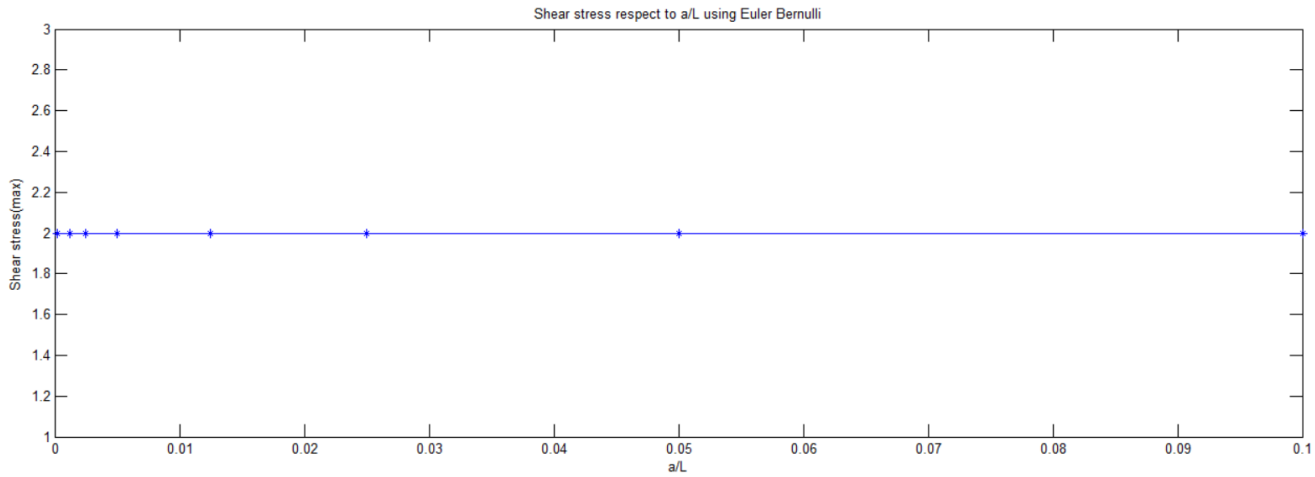

Figure 7. Maximum shear stress respect to a/L using Euler Bernulli

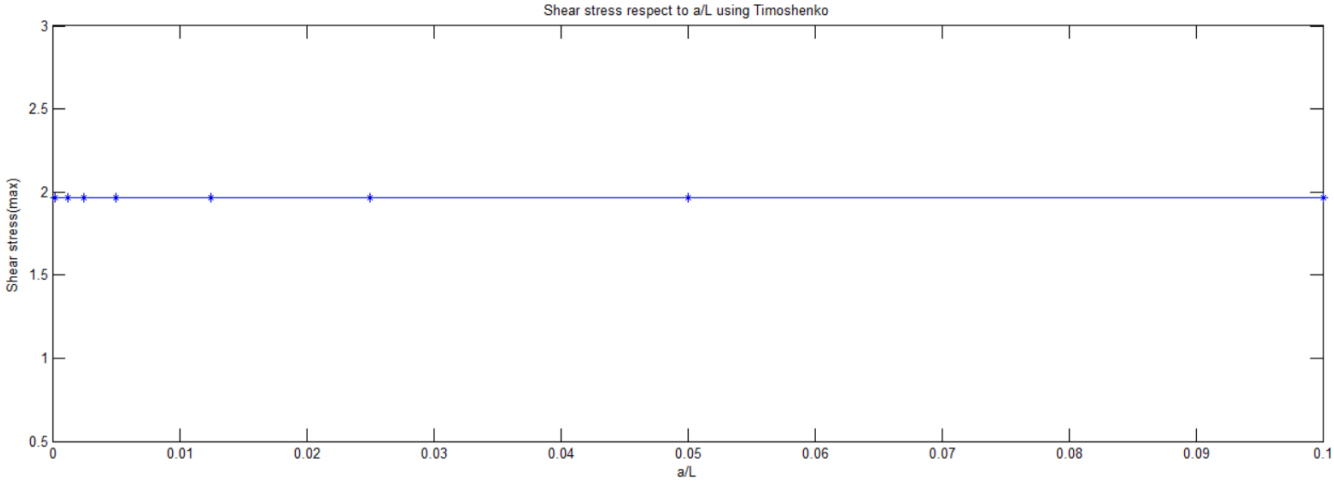

Figure 8. Maximum shear stress respect to a/L using Timoshenko

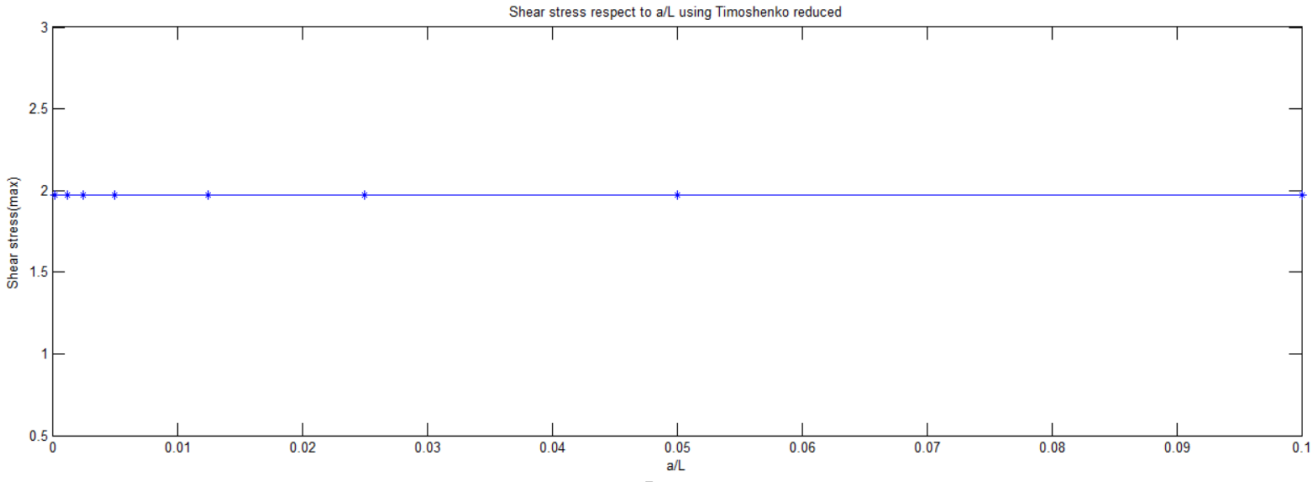

Figure 8. Maximum shear stress respect to a/L using Timoshenko Reduced

We can see, above three methods are all stable when we use them to calculate the shear stress. But there is little difference between Timoshenko method/ Timoshenko Reduced method and Euler Bernulli method. The results of Timoshenko method and Timoshenko

Reduced method are a little smaller than Euler Bernulli method. The reason is that Timoshenko method and Timoshenko Reduced method have considered the influence of the shear strain. So the result will be little different.