Assignment 1

Computational Structural Mechanics and Dynamics

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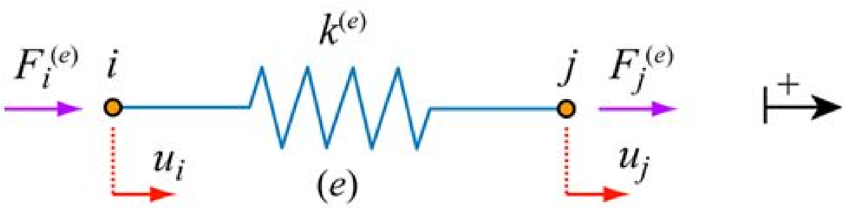
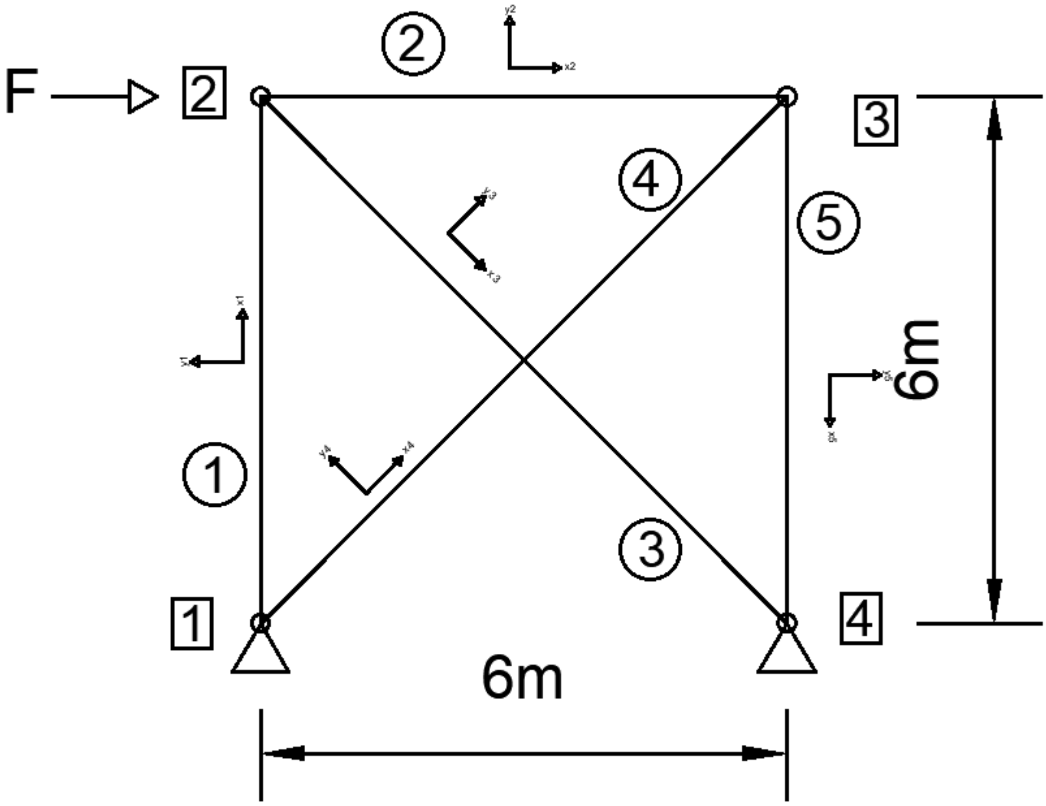


Contents

1 Class problem

Obtain the reaction forces on the supports and the displacements on the nodes of the following frame.



- $L = 6m$
- $A = 6cm^2$
- $E = 200GPa$
- $F = 80KN$

1.1 Solution

Stiffness method has to be applied for each element as follows:
Element 1

$$\begin{bmatrix} fx1 \\ fy1 \\ fx2 \\ fy2 \end{bmatrix} = \begin{bmatrix} 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 20000000.00 & 0.00 & -20000000.00 \\ 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & -20000000.00 & 0.00 & 20000000.00 \end{bmatrix} \begin{bmatrix} ux1 \\ uy1 \\ ux2 \\ uy2 \end{bmatrix}$$

Element 2

$$\begin{bmatrix} fx2 \\ fy2 \\ fx3 \\ fy3 \end{bmatrix} = \begin{bmatrix} 20000000.00 & 0.00 & -20000000.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 \\ -20000000.00 & 0.00 & 20000000.00 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.00 \end{bmatrix} \begin{bmatrix} ux2 \\ uy2 \\ ux3 \\ uy3 \end{bmatrix}$$

Element 3

$$\begin{bmatrix} fx2 \\ fy2 \\ fx4 \\ fy4 \end{bmatrix} = \begin{bmatrix} 7071067.81 & -7071067.81 & -7071067.81 & 7071067.81 \\ -7071067.81 & 7071067.81 & 7071067.81 & -7071067.81 \\ -7071067.81 & 7071067.81 & 7071067.81 & -7071067.81 \\ 7071067.81 & -7071067.81 & -7071067.81 & 7071067.81 \end{bmatrix} \begin{bmatrix} ux2 \\ uy2 \\ ux4 \\ uy4 \end{bmatrix}$$

Element 4

$$\begin{bmatrix} fx1 \\ fy1 \\ fx3 \\ fy3 \end{bmatrix} = \begin{bmatrix} 7071067.81 & 7071067.81 & -7071067.81 & -7071067.81 \\ 7071067.81 & 7071067.81 & -7071067.81 & -7071067.81 \\ -7071067.81 & -7071067.81 & 7071067.81 & 7071067.81 \\ -7071067.81 & -7071067.81 & 7071067.81 & 7071067.81 \end{bmatrix} \begin{bmatrix} ux1 \\ uy1 \\ ux3 \\ uy3 \end{bmatrix}$$

Element 5

$$\begin{bmatrix} fx3 \\ fy3 \\ fx4 \\ fy4 \end{bmatrix} = \begin{bmatrix} 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & 20000000.00 & 0.00 & -20000000.00 \\ 0.00 & 0.00 & 0.00 & 0.00 \\ 0.00 & -20000000.00 & 0.00 & 20000000.00 \end{bmatrix} \begin{bmatrix} ux3 \\ uy3 \\ ux4 \\ uy4 \end{bmatrix}$$

Once every elemental stiffness matrix has been computed, a global stiffness matrix has to be assembled in order to obtain a global system of equations:

Global system of equations

$$\begin{bmatrix} fx1 \\ fy1 \\ fx2 \\ fy2 \\ fx3 \\ fy3 \\ fx4 \\ fy4 \end{bmatrix} = \begin{bmatrix} 7071067.81 & 7071067.81 & 0.00 & 0.00 & -7071067.81 & -7071067.81 & 0.00 & 0.00 \\ 7071067.81 & 27071067.81 & 0.00 & -20000000.00 & -7071067.81 & -7071067.81 & 0.00 & 0.00 \\ 0.00 & 0.00 & 27071067.81 & -7071067.81 & -20000000.00 & 0.00 & -7071067.81 & 7071067.81 \\ 0.00 & -20000000.00 & -7071067.81 & 27071067.81 & 0.00 & 0.00 & 7071067.81 & -7071067.81 \\ -7071067.81 & -7071067.81 & -20000000.00 & 0.00 & 27071067.81 & 7071067.81 & 0.00 & 0.00 \\ -7071067.81 & -7071067.81 & 0.00 & 0.00 & 7071067.81 & 27071067.81 & 0.00 & -20000000.00 \\ 0.00 & 0.00 & -7071067.81 & 7071067.81 & 0.00 & 0.00 & 7071067.81 & -7071067.81 \\ 0.00 & 0.00 & 7071067.81 & -7071067.81 & 0.00 & -20000000.00 & -7071067.81 & 27071067.81 \end{bmatrix} \begin{bmatrix} ux1 \\ uy1 \\ ux2 \\ uy2 \\ ux3 \\ uy3 \\ ux4 \\ uy4 \end{bmatrix}$$

Applying Boundary Conditions, the global system of equations can be simplified as follows:

$$\begin{bmatrix} 80000 \\ 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 27071067.81 & -7071067.812 & -20000000 & 0 \\ -7071067.812 & 27071067.81 & 0 & 0 \\ -20000000 & 0 & 27071067.81 & 7071067.812 \\ 0 & 0 & 7071067.812 & 27071067.81 \end{bmatrix} \begin{bmatrix} ux2 \\ uy2 \\ ux3 \\ uy3 \end{bmatrix}$$

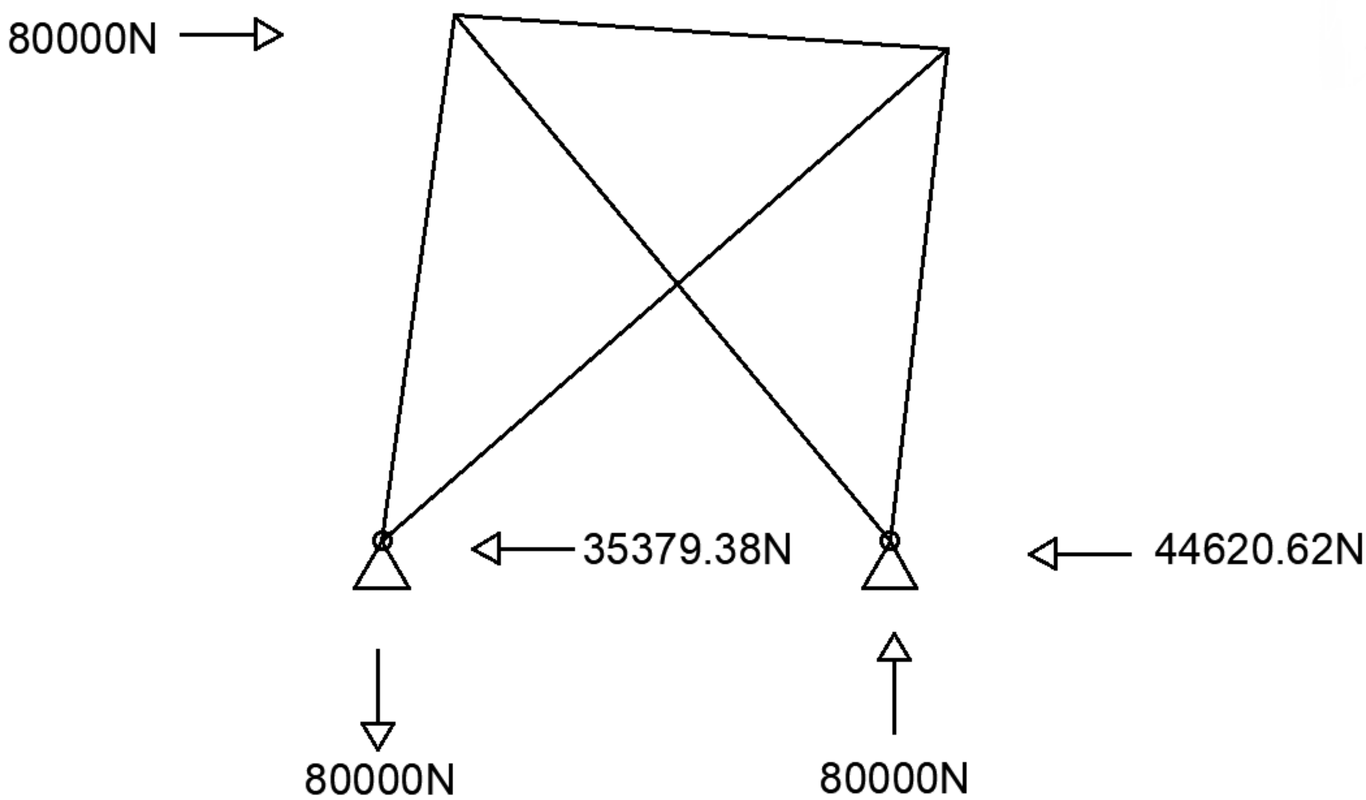
Solving the linear system of equations, the next results are obtained:

$$\begin{bmatrix} ux2 \\ uy2 \\ ux3 \\ uy3 \end{bmatrix} = \begin{bmatrix} 0.008541339m \\ 0.002231031m \\ 0.00677237m \\ -0.001768969m \end{bmatrix}$$

Once all the displacements are known, the reaction forces yields as follows:

$$\begin{bmatrix} fx1 \\ fy1 \\ fx4 \\ fy4 \end{bmatrix} = \begin{bmatrix} -35379.38N \\ -80000.00N \\ -44620.62N \\ 80000.00N \end{bmatrix}$$

The results are shown in the next figure:

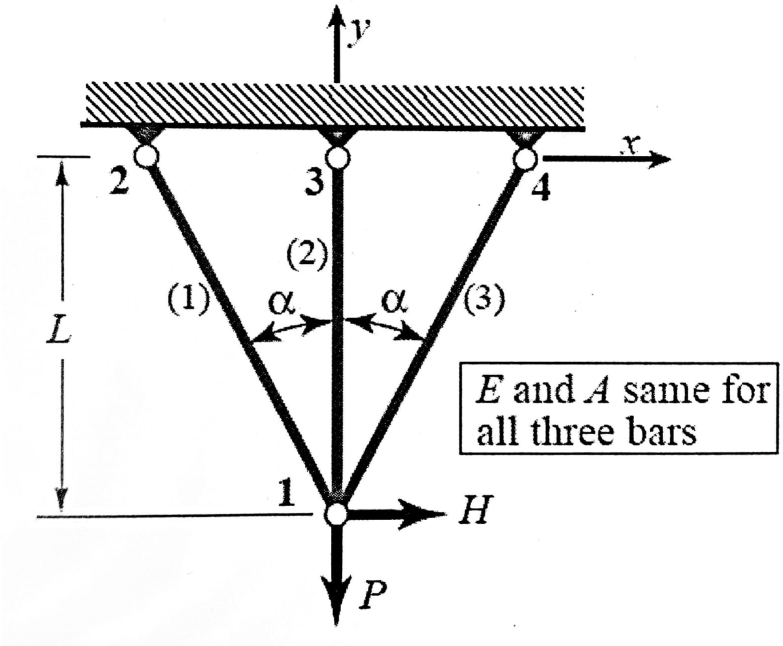


Note: The displacements are amplified x100.

2 Assignment 1

On "The Direct Stiffness Methid".

Consider the truss problem defined in the figure. All geometric and material properties: L,α, E and A, as well as the applied forces P and H, are to be kept as variables. This truss has 8 degrees of freedim, with six of them removable by the fixed-displacement conditions at nodes 2,3 and4. This structure is statically indeterminate as long as $\alpha \neq 0$.



2.1 (a)

Show the global system of equations Element 1

$$\begin{bmatrix} fx1 \\ fy1 \\ fx2 \\ fy2 \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} s^2c & -c^2s & -s^2c & sc^2 \\ -c^2s & c^3 & sc^2 & -c^3 \\ -s^2c & sc^2 & s^2c & -sc^2 \\ sc^2 & -c^3 & -sc^2 & c^3 \end{bmatrix} \begin{bmatrix} ux1 \\ uy1 \\ ux2 \\ uy2 \end{bmatrix}$$

Element 2

$$\begin{bmatrix} fx1 \\ fy1 \\ fx3 \\ fy3 \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} ux1 \\ uy1 \\ ux3 \\ uy3 \end{bmatrix}$$

Element 3

$$\begin{bmatrix} fx1 \\ fy1 \\ fx4 \\ fy4 \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} s^2c & sc^2 & -s^2c & -sc^2 \\ sc^2 & c^3 & -sc^2 & -c^3 \\ -s^2c & -sc^2 & s^2c & sc^2 \\ -sc^2 & -c^3 & sc^2 & c^3 \end{bmatrix} \begin{bmatrix} ux1 \\ uy1 \\ ux4 \\ uy4 \end{bmatrix}$$

Global system of equations:

$$\begin{bmatrix} fx1 \\ fy1 \\ fx2 \\ fy2 \\ fx3 \\ fy3 \\ fx4 \\ fy4 \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 2s^2c & 0 & -s^2c & sc^2 & 0 & 0 & -s^2c & -sc^2 \\ & 1+2c^3 & sc^2 & -c^3 & 0 & -1 & -sc^2 & -c^3 \\ & & s^2c & -sc^2 & 0 & 0 & 0 & 0 \\ & & & c^3 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ & S & Y & M & & 1 & 0 & 0 \\ & & & & & & s^2c & sc^2 \\ & & & & & & & c^3 \end{bmatrix} \begin{bmatrix} ux1 \\ uy1 \\ ux2 \\ uy2 \\ ux3 \\ uy3 \\ ux4 \\ uy4 \end{bmatrix}$$

Since it the elements only offers axial stiffness, the element that involves joint 3 has no horizontal stiffness (ux3), therefore all of its components are 0.

2.2 (b)

Apply the BCs and show the 2-equation modified stiffness system

Applying boundary conditions:

$$\begin{bmatrix} H \\ -P \\ fx2 \\ fy2 \\ fx3 \\ fy3 \\ fx4 \\ fy4 \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 2s^2c & 0 & -s^2c & sc^2 & 0 & 0 & -s^2c & -sc^2 \\ & 1+2c^3 & sc^2 & -c^3 & 0 & -1 & -sc^2 & -c^3 \\ & & s^2c & -sc^2 & 0 & 0 & 0 & 0 \\ & & & c^3 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ & S & Y & M & & 1 & 0 & 0 \\ & & & & & & s^2c & sc^2 \\ & & & & & & & c^3 \end{bmatrix} \begin{bmatrix} ux1 \\ uy1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Rows and columns 3-8 can be deleted, yielding as follows:

$$\frac{EA}{L} \begin{bmatrix} 2cs^2 & 0 \\ 0 & 1+2c^3 \end{bmatrix} \begin{bmatrix} ux1 \\ uy1 \end{bmatrix} = \begin{bmatrix} H \\ -P \end{bmatrix}$$

2.3 (c)

Solve for the displacemente ux1 and uy1. Check that the solution makes phyisical sense for the limit cases $\alpha \to 0$ and $\alpha \to \frac{\pi}{2}$. Why does ux1 "blows up if $H \neq 0$ and $\alpha \to 0$?

Solving the linear system of equations:

$$ux1 = \frac{HL}{2cs^2EA}$$

$$uy1 = \frac{-PL}{[2c^3+1]EA}$$

When $\alpha \to 0$:

- $ux1$ is undetermined, because non of the elements are offering an stiffness in the x direction.

- $uy1 = \frac{-PL}{3AE}$, this result makes sense because all the bars rely on the same axis and points, therefore, the force will be distributed on the three bars equaly.

When $\alpha \rightarrow \frac{\pi}{2}$:

- $ux1$ is undetermined, because elements 1 and 3 increase as α increases, therefore, in the limit of $\alpha \rightarrow \frac{\pi}{2}$, the length of the bars tend to ∞, causing the stiffness to tend to 0.

- $uy1 = \frac{-PL}{AE}$, since bars 1 and 3 tend to be completely horizontal, they will not offer vertical stifness, therefore, only bar 2 will be offering vertical stiffness.

2.4 (d)

Recover the axial forces in the three members. Partial answer:$F^{(3)} = -\frac{H}{2s} + \frac{Pc^2}{2c^3+1}$**. Why do** $F^(1)$ **and** $F^(3)$ **"blow up" if** $H \neq 0$ **and** $\alpha \rightarrow 0$**?**

$$fx1 = H$$

$$fy1 = -P$$

$$fx2 = \frac{-H}{2} + \frac{-Pc^2 s}{2c^3+1}$$

$$fy2 = \frac{Hc}{2s} + \frac{Pc^3}{2c^3+1}$$

$$fx3 = 0$$

$$fy3 = \frac{P}{2c^3+1}$$

$$fx4 = \frac{-H}{2} + \frac{Pc^2 s}{2c^3+1}$$

$$fy4 = \frac{-Hc}{2s} + \frac{Pc^3}{2c^3+1}$$

Therefore, the internal axial forces of the elements will be:

$$F^{(1)} = \frac{H}{2s} + \frac{Pc^2}{2c^3+1}$$

$$F^{(2)} = \frac{-P}{2c^3+1}$$

$$F^{(3)} = -\frac{H}{2s} + \frac{Pc^2}{2c^3+1}$$

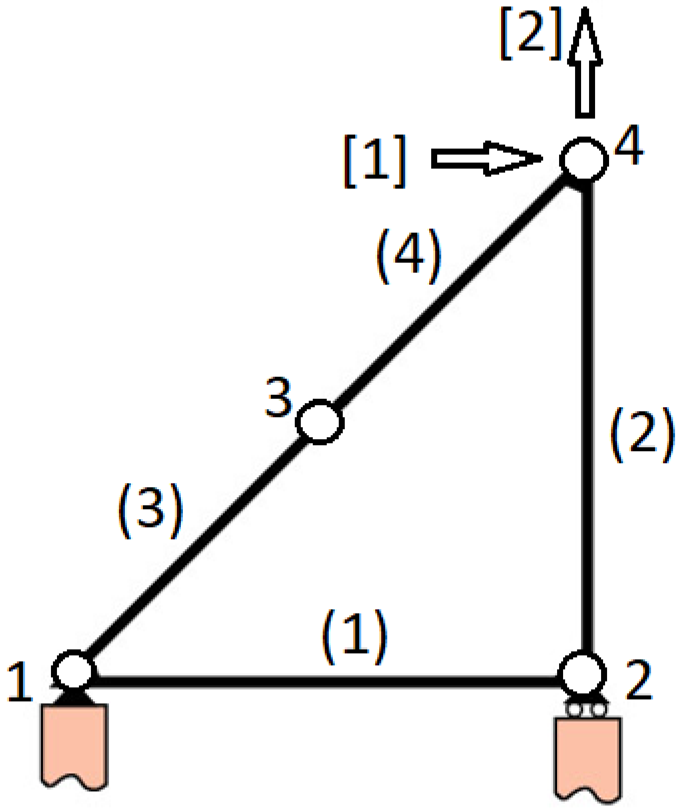
If $H \neq 0$ and $\alpha \rightarrow 0$, there will be no stiffness for the horizontal component.

3 Assignment 2

Dr. who proposes "improving" the result for the example truss of the 1^{st} lesson by putting one extra node, 4 at the midpoint of member (3) 1-3. so that it is subdivided in two different members: (3) 1-4 and (4) 3-4. His reasoning is that more is better. Try Dr. Who's suggestion by hand computations and verify that the solution "blows up" because the modified master stiffness is singular. Explain physically.

Properties:

- Member 1: L=10; EA=100
- Member 2: L=10; EA=100
- Member 3: L=$5\sqrt{2}$; EA=$200\sqrt{2}$
- Member 4: L=$5\sqrt{2}$; EA=$200\sqrt{2}$



Element 1

$$\begin{bmatrix} fx1 \\ fy1 \\ fx2 \\ fy2 \end{bmatrix} = \begin{bmatrix} 10 & 0 & -10 & 0 \\ 0 & 0 & 0 & 0 \\ -10 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} ux1 \\ uy1 \\ ux2 \\ uy2 \end{bmatrix}$$

Element 2

$$\begin{bmatrix} fx2 \\ fy2 \\ fx4 \\ fy4 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 10 & 0 & -10 \\ 0 & 0 & 0 & 0 \\ 0 & -10 & 0 & 10 \end{bmatrix} \begin{bmatrix} ux2 \\ uy2 \\ ux4 \\ uy4 \end{bmatrix}$$

Element 3

$$\begin{bmatrix} fx1 \\ fy1 \\ fx3 \\ fy3 \end{bmatrix} = \begin{bmatrix} 20 & 20 & -20 & -20 \\ 20 & 20 & -20 & -20 \\ -20 & -20 & 20 & 20 \\ -20 & -20 & 20 & 20 \end{bmatrix} \begin{bmatrix} ux1 \\ uy1 \\ ux3 \\ uy3 \end{bmatrix}$$

Element 4

$$\begin{bmatrix} fx3 \\ fy3 \\ fx4 \\ fy4 \end{bmatrix} = \begin{bmatrix} 20 & 20 & -20 & -20 \\ 20 & 20 & -20 & -20 \\ -20 & -20 & 20 & 20 \\ -20 & -20 & 20 & 20 \end{bmatrix} \begin{bmatrix} ux3 \\ uy3 \\ ux4 \\ uy4 \end{bmatrix}$$

Global system of equations:

$$\begin{bmatrix} fx1 \\ fy1 \\ fx2 \\ fy2 \\ fx3 \\ fy3 \\ fx4 \\ fy4 \end{bmatrix} = \begin{bmatrix} 30 & 20 & -10 & 0 & -20 & -20 & 0 & 0 \\ & 20 & 0 & 0 & -20 & -20 & 0 & 0 \\ & & 10 & 0 & 0 & 0 & 0 & 0 \\ & & & 10 & 0 & 0 & 0 & -10 \\ & & & & 40 & 40 & -20 & -20 \\ & S & Y & M & & 40 & -20 & -20 \\ & & & & & & 20 & 20 \\ & & & & & & & 30 \end{bmatrix} \begin{bmatrix} ux1 \\ uy1 \\ ux2 \\ uy2 \\ ux3 \\ uy3 \\ ux4 \\ uy4 \end{bmatrix}$$

Applying boundary conditions:

$$\begin{bmatrix} fx1 \\ fy1 \\ 0 \\ fy2 \\ 0 \\ 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 30 & 20 & -10 & 0 & -20 & -20 & 0 & 0 \\ & 20 & 0 & 0 & -20 & -20 & 0 & 0 \\ & & 10 & 0 & 0 & 0 & 0 & 0 \\ & & & 10 & 0 & 0 & 0 & -10 \\ & & & & 40 & 40 & -20 & -20 \\ & S & Y & M & & 40 & -20 & -20 \\ & & & & & & 20 & 20 \\ & & & & & & & 30 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ ux2 \\ 0 \\ ux3 \\ uy3 \\ ux4 \\ uy4 \end{bmatrix}$$

Simplifying:

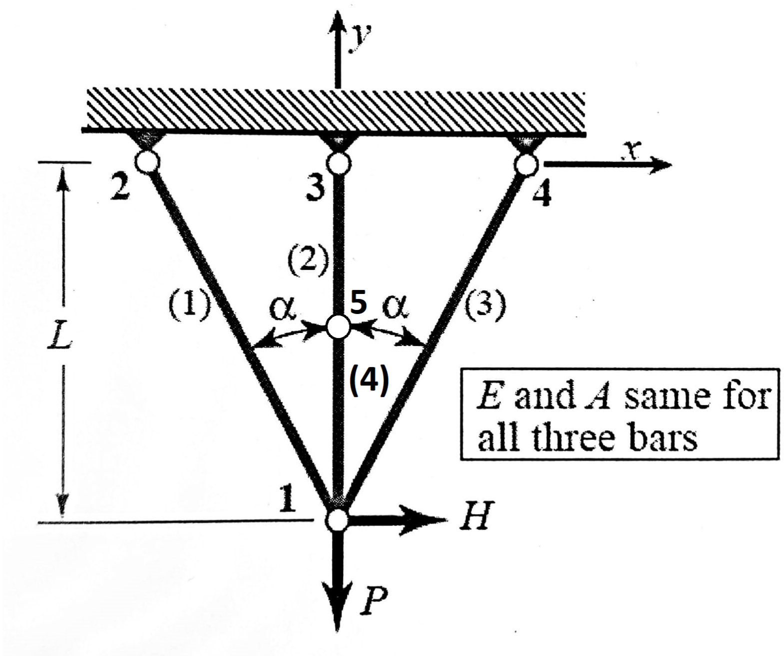
$$\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 2 \end{bmatrix} = \begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 40 & 40 & -20 & -20 \\ 0 & 40 & 40 & -20 & -20 \\ 0 & -20 & -20 & 20 & 20 \\ 0 & -20 & -20 & 20 & 30 \end{bmatrix} \begin{bmatrix} ux2 \\ ux3 \\ uy3 \\ ux4 \\ uy4 \end{bmatrix}$$

The determinant of this stiffness matrix is 0, therefore the matrix is singular.

To prove that whenever we add a node between two bars that rely on the same axis, the solution "blows up", I am presenting two proposed cases from the assignment one where after applying boundary conditions and simplifying the system of equations we get singular matrix stiffness as well.

4 Proposed problem 1

Note: This problem only aims to check the discussions shown in the last section.



Element 1

$$\begin{bmatrix} fx1 \\ fy1 \\ fx2 \\ fy2 \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} s^2c & -c^2s & -s^2c & sc^2 \\ -c^2s & c^3 & sc^2 & -c^3 \\ -s^2c & sc^2 & s^2c & -sc^2 \\ sc^2 & -c^3 & -sc^2 & c^3 \end{bmatrix} \begin{bmatrix} ux1 \\ uy1 \\ ux2 \\ uy2 \end{bmatrix}$$

Element 2

$$\begin{bmatrix} fx5 \\ fy5 \\ fx3 \\ fy3 \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} ux5 \\ uy5 \\ ux3 \\ uy3 \end{bmatrix}$$

Element 3

$$\begin{bmatrix} fx1 \\ fy1 \\ fx4 \\ fy4 \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} s^2c & sc^2 & -s^2c & -sc^2 \\ sc^2 & c^3 & -sc^2 & -c^3 \\ -s^2c & -sc^2 & s^2c & sc^2 \\ -sc^2 & -c^3 & sc^2 & c^3 \end{bmatrix} \begin{bmatrix} ux1 \\ uy1 \\ ux4 \\ uy4 \end{bmatrix}$$

Element 4

$$\begin{bmatrix} fx1 \\ fy1 \\ fx5 \\ fy5 \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 2 \end{bmatrix} \begin{bmatrix} ux1 \\ uy1 \\ ux5 \\ uy5 \end{bmatrix}$$

Global system of equations:

$$
\begin{bmatrix} fx1 \\ fy1 \\ fx2 \\ fy2 \\ fx3 \\ fy3 \\ fx4 \\ fy4 \\ fx5 \\ fy5 \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 2s^2c & 0 & -s^2c & sc^2 & 0 & 0 & -s^2c & -sc^2 & 0 & 0 \\ & 2+2c^3 & sc^2 & -c^3 & 0 & 0 & -sc^2 & -c^3 & 0 & -2 \\ & & s^2c & -sc^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & c^3 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & 2 & 0 & 0 & 0 & 0 \\ & S & Y & M & & & s^2c & sc^2 & 0 & 0 \\ & & & & & & & c^3 & 0 & 0 \\ & & & & & & & & 0 & 0 \\ & & & & & & & & & 4 \end{bmatrix} \begin{bmatrix} ux1 \\ uy1 \\ ux2 \\ uy2 \\ ux3 \\ uy3 \\ ux4 \\ uy4 \\ ux5 \\ uy5 \end{bmatrix}
$$

Applying boundary conditions:

$$
\begin{bmatrix} H \\ -P \\ fx2 \\ fy2 \\ fx3 \\ fy3 \\ fx4 \\ fy4 \\ 0 \\ 0 \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 2s^2c & 0 & -s^2c & sc^2 & 0 & 0 & -s^2c & -sc^2 & 0 & 0 \\ & 2+2c^3 & sc^2 & -c^3 & 0 & 0 & -sc^2 & -c^3 & 0 & -2 \\ & & s^2c & -sc^2 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & c^3 & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 & 0 & 0 \\ & & & & & 2 & 0 & 0 & 0 & 0 \\ & S & Y & M & & & s^2c & sc^2 & 0 & 0 \\ & & & & & & & c^3 & 0 & 0 \\ & & & & & & & & 0 & 0 \\ & & & & & & & & & 4 \end{bmatrix} \begin{bmatrix} ux1 \\ uy1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ ux5 \\ uy5 \end{bmatrix}
$$

Therefore, it will be simplified as follows:

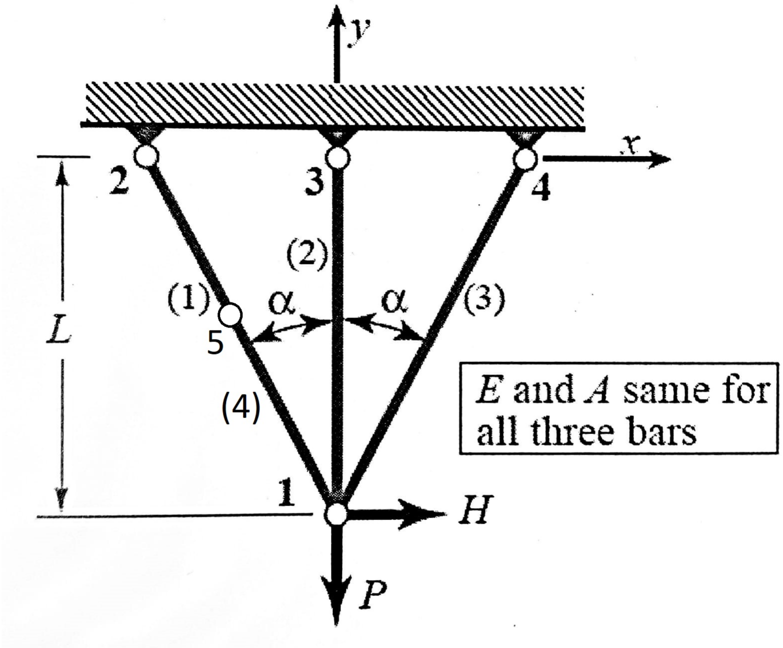
$$
\frac{EA}{L} \begin{bmatrix} 2s^2c & 0 & 0 & 0 \\ 0 & 2+2c^3 & 0 & -2 \\ 0 & 0 & 0 & 0 \\ 0 & -2 & 0 & 4 \end{bmatrix} \begin{bmatrix} ux1 \\ uy1 \\ ux5 \\ uy5 \end{bmatrix} = \begin{bmatrix} H \\ -P \\ 0 \\ 0 \end{bmatrix}
$$

Notice that after applying the boundary conditions, the matrix is singular, therefore, it has infinite solutions.

5 Proposed problem 2

Note: This problem only aims to check the discussions shown in the last section.

Instead of adding a node in the central bar, let the new node be in the first bar as follows:



Element 1

$$\begin{bmatrix} fx5 \\ fy5 \\ fx2 \\ fy2 \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 2s^2c & -2c^2s & -2s^2c & 2sc^2 \\ -2c^2s & 2c^3 & 2sc^2 & -2c^3 \\ -2s^2c & 2sc^2 & 2s^2c & -2sc^2 \\ 2sc^2 & -2c^3 & -2sc^2 & 2c^3 \end{bmatrix} \begin{bmatrix} ux5 \\ uy5 \\ ux2 \\ uy2 \end{bmatrix}$$

Element 2

$$\begin{bmatrix} fx1 \\ fy1 \\ fx3 \\ fy3 \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} ux1 \\ uy1 \\ ux3 \\ uy3 \end{bmatrix}$$

Element 3

$$\begin{bmatrix} fx1 \\ fy1 \\ fx4 \\ fy4 \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} s^2c & sc^2 & -s^2c & -sc^2 \\ sc^2 & c^3 & -sc^2 & -c^3 \\ -s^2c & -sc^2 & s^2c & sc^2 \\ -sc^2 & -c^3 & sc^2 & c^3 \end{bmatrix} \begin{bmatrix} ux1 \\ uy1 \\ ux4 \\ uy4 \end{bmatrix}$$

Element 4

$$\begin{bmatrix} fx1 \\ fy1 \\ fx5 \\ fy5 \end{bmatrix} = \frac{EA}{L} \begin{bmatrix} 2s^2c & -2c^2s & -2s^2c & 2sc^2 \\ -2c^2s & 2c^3 & 2sc^2 & -2c^3 \\ -2s^2c & 2sc^2 & 2s^2c & -2sc^2 \\ 2sc^2 & -2c^3 & -2sc^2 & 2c^3 \end{bmatrix} \begin{bmatrix} ux1 \\ uy1 \\ ux5 \\ uy5 \end{bmatrix}$$

Global system of equations:

$$
\begin{bmatrix} fx1 \\ fy1 \\ fx2 \\ fy2 \\ fx3 \\ fy3 \\ fx4 \\ fy4 \\ fx5 \\ fy5 \end{bmatrix} = \frac{EA}{L}
\begin{bmatrix}
3s^2c & -c^2s & 0 & 0 & 0 & 0 & -s^2c & -sc^2 & -2s^2c & 2sc^2 \\
 & 1+3c^3 & 0 & 0 & 0 & -1 & -sc^2 & -c^3 & 2sc^2 & -2c^3 \\
 & & 2s^2c & -2sc^2 & 0 & 0 & 0 & 0 & -2s^2c & 2sc^2 \\
 & & & 2c^3 & 0 & 0 & 0 & 0 & 2sc^2 & -2c^3 \\
 & & & & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & 1 & 0 & 0 & 0 & 0 \\
 & S & Y & M & & & s^2c & sc^2 & 0 & 0 \\
 & & & & & & & c^3 & 0 & 0 \\
 & & & & & & & & 4s^2c & -4sc^2 \\
 & & & & & & & & & 4c^3
\end{bmatrix}
\begin{bmatrix} ux1 \\ uy1 \\ ux2 \\ uy2 \\ ux3 \\ uy3 \\ ux4 \\ uy4 \\ ux5 \\ uy5 \end{bmatrix}
$$

Applying boundary conditions:

$$
\begin{bmatrix} H \\ -P \\ fx2 \\ fy2 \\ fx3 \\ fy3 \\ fx4 \\ fy4 \\ 0 \\ 0 \end{bmatrix} = \frac{EA}{L}
\begin{bmatrix}
3s^2c & -c^2s & 0 & 0 & 0 & 0 & -s^2c & -sc^2 & -2s^2c & 2sc^2 \\
 & 1+3c^3 & 0 & 0 & 0 & -1 & -sc^2 & -c^3 & 2sc^2 & -2c^3 \\
 & & 2s^2c & -2sc^2 & 0 & 0 & 0 & 0 & -2s^2c & 2sc^2 \\
 & & & 2c^3 & 0 & 0 & 0 & 0 & 2sc^2 & -2c^3 \\
 & & & & 0 & 0 & 0 & 0 & 0 & 0 \\
 & & & & & 1 & 0 & 0 & 0 & 0 \\
 & S & Y & M & & & s^2c & sc^2 & 0 & 0 \\
 & & & & & & & c^3 & 0 & 0 \\
 & & & & & & & & 4s^2c & -4sc^2 \\
 & & & & & & & & & 4c^3
\end{bmatrix}
\begin{bmatrix} ux1 \\ uy1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ ux5 \\ uy5 \end{bmatrix}
$$

Therefore, it will be simplified as follows:

$$
\frac{EA}{L}
\begin{bmatrix}
3s^2c & -c^2s & -2s^2c & 2sc^2 \\
-c^2s & 1+3c^3 & 2sc^2 & -2c^3 \\
-2s^2c & 2sc^2 & 4s^2c & -4sc^2 \\
2sc^2 & -2c^3 & -4sc^2 & 4c^3
\end{bmatrix}
\begin{bmatrix} ux1 \\ uy1 \\ ux5 \\ uy5 \end{bmatrix} =
\begin{bmatrix} H \\ -P \\ 0 \\ 0 \end{bmatrix}
$$

The stiffness matrix determinant is 0, therefore a singular matrix was obtained as well.

6 Discussion

- Assignment 1: The solution of the problem is bounded for $0 < \alpha < \frac{\pi}{2}$, because for $\alpha \to 0$ there is no horizontal stiffness from the elements, therefore it is completely free to move in the x direction, and for the case of $\alpha \to \frac{\pi}{2}$ nodes 2 and 4 will not be able to have a support, therefore no horizontal stiffness will exist as in the first case.

- Assignment 2: Adding a node between to bars that rely on the same axis and only have axial stiffness, will lead you to indeterminacy problems speaking in mathematical terms due to the singularity of the stiffness matrix, which comes from the physical explanation that since these bars have no perpendicular stiffness, therefore, the node is free to move in the perpendicular axis, causing the system to "blow up" whenever they face a complete perpendicular force without having other element that condition the displacement of the node, such as a support or another bar in the perpendicular direction.