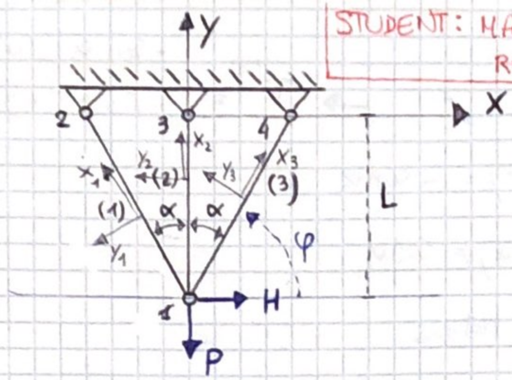
STUDENT: MARCELLO RUBINO



Element Stiffness Matrix

$$\bar{\underline{K}}^{(1)} = \bar{\underline{K}}^{(3)} = \frac{EA \cdot \cos\alpha}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\bar{\underline{K}}^{(2)} = \frac{EA}{L}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Generic Transformation Matrix

$$\underline{T}^{(e)} = \begin{bmatrix} \cos\varphi & \sin\varphi & 0 & 0 \\ -\sin\varphi & \cos\varphi & 0 & 0 \\ 0 & 0 & \cos\varphi & \sin\varphi \\ 0 & 0 & -\sin\varphi & \cos\varphi \end{bmatrix}$$

Transformation Matrices

- $\varphi^{(1)} = \frac{\pi}{2} + \alpha \rightarrow \underline{T}^{(1)} = \begin{bmatrix} -\sin\alpha & \cos\alpha & 0 & 0 \\ -\cos\alpha & -\sin\alpha & 0 & 0 \\ 0 & 0 & -\sin\alpha & \cos\alpha \\ 0 & 0 & -\cos\alpha & -\sin\alpha \end{bmatrix}$

Knowing that: $\cos\left(\frac{\pi}{2}+\alpha\right) = -\sin\alpha$
$\sin\left(\frac{\pi}{2}+\alpha\right) = \cos\alpha$

- $\varphi^{(2)} = \frac{\pi}{2} \rightarrow \underline{T}^{(2)} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$

- $\varphi^{(3)} = \frac{\pi}{2} - \alpha \longrightarrow \underline{T}^{(3)} = \begin{bmatrix} \sin\alpha & \cos\alpha & 0 & 0 \\ -\cos\alpha & \sin\alpha & 0 & 0 \\ 0 & 0 & \sin\alpha & \cos\alpha \\ 0 & 0 & -\cos\alpha & \sin\alpha \end{bmatrix}$

Knowing that : $\cos\left(\frac{\pi}{2} - \alpha\right) = \sin\alpha$

$\sin\left(\frac{\pi}{2} - \alpha\right) = \cos\alpha$

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Transposes Matrices of Transformation Matrices

- $\underline{T}^{(1)^T} = \begin{bmatrix} -\sin\alpha & -\cos\alpha & 0 & 0 \\ \cos\alpha & -\sin\alpha & 0 & 0 \\ 0 & 0 & -\sin\alpha & -\cos\alpha \\ 0 & 0 & \cos\alpha & -\sin\alpha \end{bmatrix}$

- $\underline{T}^{(2)^T} = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix}$

- $\underline{T}^{(3)^T} = \begin{bmatrix} \sin\alpha & -\cos\alpha & 0 & 0 \\ \cos\alpha & \sin\alpha & 0 & 0 \\ 0 & 0 & \sin\alpha & -\cos\alpha \\ 0 & 0 & \cos\alpha & \sin\alpha \end{bmatrix}$

Globalized Element Stiffness Matrices , $c = \cos\alpha$, $s = \sin\alpha$

- $\underline{K}^{(1)} = \underline{T}^{(1)^T} \underline{\bar{K}}^{(1)} \underline{T}^{(1)}$

$$\begin{bmatrix} f_{x_1}^{(1)} \\ f_{y_1}^{(1)} \\ f_{x_2}^{(1)} \\ f_{y_2}^{(1)} \end{bmatrix} = \begin{bmatrix} cs^2 & -c^2s & -cs^2 & c^2s \\ -c^2s & c^3 & c^2s & -c^3 \\ -cs^2 & c^2s & cs^2 & -c^2s \\ c^2s & -c^3 & -c^2s & c^3 \end{bmatrix} \cdot \frac{EA}{L} \cdot \begin{bmatrix} u_{x_1}^{(1)} \\ u_{y_1}^{(1)} \\ u_{x_2}^{(1)} \\ u_{y_2}^{(1)} \end{bmatrix}$$

- $\underline{K}^{(2)} = \underline{T}^{(2)^T} \underline{\bar{K}}^{(2)} \underline{T}^{(2)}$

$$\begin{bmatrix} f_{x_1}^{(2)} \\ f_{y_1}^{(2)} \\ f_{x_3}^{(2)} \\ f_{y_3}^{(2)} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \cdot \frac{EA}{L} \cdot \begin{bmatrix} u_{x_1}^{(2)} \\ u_{y_1}^{(2)} \\ u_{x_3}^{(2)} \\ u_{y_3}^{(2)} \end{bmatrix}$$

- $\underline{K}^{(3)} = \underline{T}^{(3)^T} \, \underline{\bar{K}}^{(3)} \, \underline{T}^{(3)}$

$$\begin{bmatrix} f_{x_1}^{(3)} \\ f_{y_1}^{(3)} \\ f_{x_4}^{(3)} \\ f_{y_4}^{(3)} \end{bmatrix} = \begin{bmatrix} cs^2 & c^2 s & -cs^2 & -c^2 s \\ c^2 s & c^3 & -c^2 s & -c^3 \\ -cs^2 & -c^2 s & cs^2 & c^2 s \\ -c^2 s & -c^3 & c^2 s & c^3 \end{bmatrix} \cdot \frac{EA}{L} \cdot \begin{bmatrix} u_{x_1}^{(3)} \\ u_{y_1}^{(3)} \\ u_{x_4}^{(3)} \\ u_{y_4}^{(3)} \end{bmatrix}$$

- Augmentation of the globalized stiffness matrices and dropping of the member index from the nodal displacements

(1) ELEMENT

$$\begin{bmatrix} f_{x_1}^{(1)} \\ f_{y_1}^{(1)} \\ f_{x_2}^{(1)} \\ f_{y_2}^{(1)} \\ f_{x_3}^{(1)} \\ f_{y_3}^{(1)} \\ f_{x_4}^{(1)} \\ f_{y_4}^{(1)} \end{bmatrix} = \begin{bmatrix} cs^2 & -c^2 s & -cs^2 & c^2 s & 0 & 0 & 0 & 0 \\ -c^2 s & c^3 & c^2 s & -c^3 & 0 & 0 & 0 & 0 \\ -cs^2 & c^2 s & cs^2 & -c^2 s & 0 & 0 & 0 & 0 \\ c^2 s & -c^3 & -c^2 s & c^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \frac{EA}{L} \cdot \begin{bmatrix} u_{x_1}^{\cancel{(1)}} \\ u_{y_1}^{\cancel{(1)}} \\ u_{x_2}^{\cancel{(1)}} \\ u_{y_2}^{\cancel{(1)}} \\ u_{x_3}^{\cancel{(1)}} \\ u_{y_3}^{\cancel{(1)}} \\ u_{x_4}^{\cancel{(1)}} \\ u_{y_4}^{\cancel{(1)}} \end{bmatrix}$$

using compatib.

(2) ELEMENT

$$\begin{bmatrix} f_{x_1}^{(2)} \\ f_{y_1}^{(2)} \\ f_{x_2}^{(2)} \\ f_{y_2}^{(2)} \\ f_{x_3}^{(2)} \\ f_{y_3}^{(2)} \\ f_{y_4}^{(2)} \\ f_{y_4}^{(2)} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \cdot \frac{EA}{L} \cdot \begin{bmatrix} u_{x_1}^{\cancel{(2)}} \\ u_{y_1}^{\cancel{(2)}} \\ u_{x_2}^{\cancel{(2)}} \\ u_{y_2}^{\cancel{(2)}} \\ u_{x_3}^{\cancel{(2)}} \\ u_{y_3}^{\cancel{(2)}} \\ u_{x_4}^{\cancel{(2)}} \\ u_{y_4}^{\cancel{(2)}} \end{bmatrix}$$

• ELEMENT (3)

$$\begin{bmatrix} f_{x_1}^{(3)} \\ f_{y_1}^{(3)} \\ f_{x_2}^{(3)} \\ f_{y_2}^{(3)} \\ f_{x_3}^{(3)} \\ f_{y_3}^{(3)} \\ f_{x_4}^{(3)} \\ f_{y_4}^{(3)} \end{bmatrix} = \begin{bmatrix} cs^2 & c^2s & 0 & 0 & 0 & 0 & -cs^2 & -c^2s \\ c^2s & c^3 & 0 & 0 & 0 & 0 & -c^2s & -c^3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -cs^2 & -c^2s & 0 & 0 & 0 & 0 & cs^2 & c^2s \\ -c^2s & -c^3 & 0 & 0 & 0 & 0 & c^2s & c^3 \end{bmatrix} \cdot \frac{EA}{L} \cdot \begin{bmatrix} u_{x_1}^{\cancel{(3)}} \\ u_{y_1}^{\cancel{(3)}} \\ u_{x_2}^{\cancel{(3)}} \\ u_{y_2}^{\cancel{(3)}} \\ u_{x_3}^{\cancel{(3)}} \\ u_{y_3}^{\cancel{(3)}} \\ u_{x_4}^{\cancel{(3)}} \\ u_{y_4}^{\cancel{(3)}} \end{bmatrix}$$

(a)

• Global Equilibrium $\rightarrow \underline{f} = \underline{f}^{(1)} + \underline{f}^{(2)} + \underline{f}^{(3)} = (\underline{K}^{(1)} + \underline{K}^{(2)} + \underline{K}^{(3)})\,\underline{u} = \underline{K}\,\underline{u}$

$$\underbrace{\begin{bmatrix} H \\ -P \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}}_{\underline{f}} = \frac{EA}{L} \underbrace{\begin{bmatrix} 2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\ 0 & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ -cs^2 & c^2s & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\ c^2s & -c^3 & -c^2s & c^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ -cs^2 & -c^2s & 0 & 0 & 0 & 0 & cs^2 & c^2s \\ -c^2s & -c^3 & 0 & 0 & 0 & 0 & c^2s & c^3 \end{bmatrix}}_{\underline{K}} \cdot \underbrace{\begin{bmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \\ u_{x_3} \\ u_{y_3} \\ u_{x_4} \\ u_{y_4} \end{bmatrix}}_{\underline{u}}$$

The 5 dof is the horizontal dof in the node 3. This node is connected to element (2) which is vertical. As such, there can't be any internal forces associated to this dof, otherwise the internal equilibrium in member (2) can't be satisfied. Therefore the 5th row and 5th column of the matrix $\underline{K}$ contain only zeros.

(b)

Applying the BCs, some rows and colums are deleted (3, 4, 5, 6, 7, 8)

The system becomes resolvable and simple : $\hat{\underline{K}}\,\hat{\underline{u}} = \hat{\underline{f}}$

$$\frac{EA}{L} \cdot \begin{bmatrix} 2cs^2 & 0 \\ 0 & 1+2c^3 \end{bmatrix} \begin{Bmatrix} u_{x_1} \\ u_{y_1} \end{Bmatrix} = \begin{Bmatrix} H \\ -P \end{Bmatrix} \quad \text{or:} \quad \begin{cases} \dfrac{EA\,2cs^2}{L} \cdot u_{x_1} = H \\ \dfrac{EA(1+2c^3)}{L} \cdot u_{y_1} = -P \end{cases}$$

(c)

$$u_{x_1} = \frac{LH}{2cs^2 EA} \qquad u_{y_1} = \frac{-LP}{(2c^3+1)EA}$$

$$\lim_{\alpha \to 0} \underset{*}{\frac{LH}{EA}} \cdot \frac{1}{2\cos\alpha \cdot (\sin^2\alpha)} = +\infty \qquad \lim_{\alpha \to 0} -\frac{LP}{EA} \cdot \frac{1}{2\cos^3\alpha + 1} = -\frac{LP}{3EA}$$

No sense

$$\lim_{\alpha \to \frac{\pi}{2}} \frac{LH}{EA} \cdot \frac{1}{2\cos\alpha \cdot (\sin^2\alpha)} = +\infty \qquad \lim_{\alpha \to \frac{\pi}{2}} -\frac{LP}{EA} \cdot \frac{1}{2\cos^3\alpha + 1} = -\frac{LP}{EA}$$

No sense

* u_{x_1} blow up to ∞ if $H \neq 0$ and $\alpha \to 0$ because the structure tends to become labile and for horizontal forces the equilibrium can't be satisfied. The three trusses tends to become only one truss that behaves like a pendulum.

(d)

displacements vector: $$\underline{u} = \begin{bmatrix} \frac{LH}{2cs^2 EA} \\ \frac{-LP}{(2c^3+1)EA} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Remaining in global system, let's write the element displacement:

$$\underline{u}^{(1)} = \begin{bmatrix} \frac{LH}{2cs^2EA} \\ \frac{-LP}{(2c^3+1)EA} \\ 0 \\ 0 \end{bmatrix} \qquad \underline{u}^{(2)} = \begin{bmatrix} \frac{LH}{2cs^2EA} \\ \frac{-LP}{(2c^3+1)EA} \\ 0 \\ 0 \end{bmatrix} \qquad \underline{u}^{(3)} = \begin{bmatrix} \frac{LH}{2cs^2EA} \\ \frac{-LP}{(2c^3+1)EA} \\ 0 \\ 0 \end{bmatrix}$$

Now, let's "rotate" the displacements to local system ($\bar{\underline{u}}^{(1)} = \underline{T}^{(1)} u^{(1)}$, etc...)

$$\bar{\underline{u}}^{(1)} = \begin{bmatrix} -\frac{LH}{2cs\,EA} - \frac{LPc}{(2c^3+1)EA} \\ -\frac{LH}{2s^2EA} + \frac{LPs}{(2c^3+1)EA} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} u_{x_i}^{(1)} \\ u_{y_i}^{(1)} \\ u_{x_j}^{(1)} \\ u_{y_j}^{(1)} \end{bmatrix} ; \quad \bar{\underline{u}}^{(2)} = \begin{bmatrix} \frac{-LP}{(2c^3+1)EA} \\ \frac{-LH}{2cs^2EA} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} u_{x_i}^{(2)} \\ u_{y_i}^{(2)} \\ u_{x_j}^{(2)} \\ u_{y_j}^{(2)} \end{bmatrix}$$

$$\underline{\bar{u}}^{(3)} = \begin{bmatrix} \frac{LH}{2cs\,EA} - \frac{LPc}{(2c^3+1)\,EA} \\ -\frac{LH}{2s^2\,EA} - \frac{LPs}{(2c^3+1)\,EA} \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} u_{x_i}^{(3)} \\ u_{y_i}^{(3)} \\ u_{x_j}^{(3)} \\ u_{y_j}^{(3)} \end{bmatrix}$$

Calculation of elongations:

$$d^{(1)} = u_{x_j}^{(1)} - u_{x_i}^{(1)} = \frac{LH}{2cs\,EA} + \frac{LPc}{(2c^3+1)\,EA}$$

$$d^{(2)} = u_{x_j}^{(2)} - u_{x_i}^{(2)} = \frac{LP}{(2c^3+1)\,EA}$$

$$d^{(3)} = u_{x_j}^{(3)} - u_{x_i}^{(3)} = -\frac{LH}{2cs\,EA} + \frac{LPc}{(2c^3+1)\,EA}$$

The axial strains:

$$e^{(1)} = \frac{d^{(1)}}{L^{(1)}} = \frac{H}{2sEA} + \frac{Pc^2}{(2c^3+1)EA} \quad ; \quad e^{(2)} = \frac{d^{(2)}}{L^{(2)}} = \frac{P}{(2c^3+1)EA}$$

$$e^{(3)} = \frac{d^{(3)}}{L^{(3)}} = -\frac{H}{2sEA} + \frac{Pc^2}{(2c^3+1)EA}$$

The axial stresses:

$$\sigma^{(1)} = e^{(1)}E^{(1)} = \frac{H}{2sA} + \frac{Pc^2}{(2c^3+1)\,A} \quad ; \quad \sigma^{(2)} = e^{(2)}E^{(2)} = \frac{P}{(2c^3+1)\,A}$$

$$\sigma^{(3)} = e^{(3)}E^{(3)} = -\frac{H}{2sA} + \frac{Pc^2}{(2c^3+1)A}$$

The axial forces:

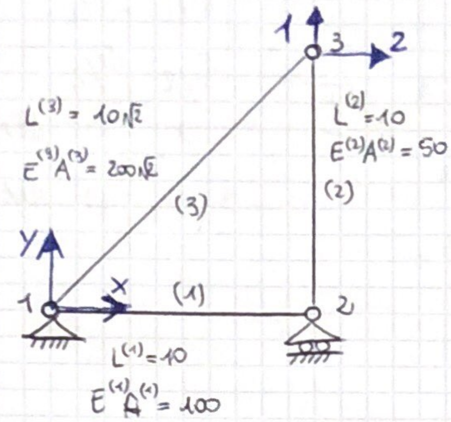
$$F^{(1)} = \sigma^{(1)}A^{(1)} = \frac{H}{2s} + \frac{Pc^2}{2c^3+1} \quad ; \quad F^{(2)} = \sigma^{(2)}A^{(2)} = \frac{P}{2c^3+1}$$

$$F^{(3)} = \sigma^{(3)}A^{(3)} = -\frac{H}{2s} + \frac{Pc^2}{2c^3+1}$$

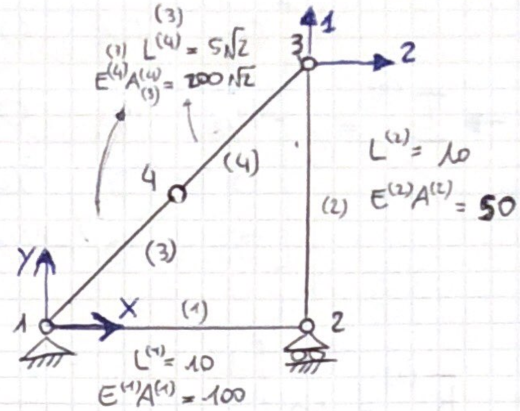
If $H \neq 0$ and $\alpha \to 0$ $F^{(1)}$ and $F^{(3)}$ "blow up" $\left(\lim_{\alpha\to 0} \frac{\pm H}{2\cdot \sin\alpha} + \frac{P\cos^2\alpha}{2\cos^3\alpha+1} = \pm\infty\right)$ because the structure becomes labile and no axial force in the elements can balance H.

Assignment 2 - Dr Who's proposal

from :



to:



- Globalized Stiffness Matrices of element (1) and (2)

ELEMENT 1

$$\begin{bmatrix} f_{x_1}^{(1)} \\ f_{y_1}^{(1)} \\ f_{x_2}^{(1)} \\ f_{y_2}^{(1)} \end{bmatrix} = 10 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{x_1}^{(1)} \\ u_{y_1}^{(1)} \\ u_{x_2}^{(1)} \\ u_{y_2}^{(1)} \end{bmatrix}$$

ELEMENT 2

$$\begin{bmatrix} f_{x_2}^{(2)} \\ f_{y_2}^{(2)} \\ f_{x_3}^{(2)} \\ f_{y_3}^{(2)} \end{bmatrix} = 5 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix} \begin{bmatrix} u_{x_2}^{(2)} \\ u_{y_2}^{(2)} \\ u_{x_3}^{(2)} \\ u_{y_3}^{(2)} \end{bmatrix}$$

- Globalized Stiffness Matrices of element (3) and (4)

ELEMENT 3

$$\varphi^{(3)} = \frac{\pi}{4} \rightarrow \underline{T}^{(3)} = \begin{bmatrix} \cos\varphi^{(3)} & \sin\varphi^{(3)} & 0 & 0 \\ -\sin\varphi^{(3)} & \cos\varphi^{(3)} & 0 & 0 \\ 0 & 0 & \cos\varphi^{(3)} & \sin\varphi^{(3)} \\ 0 & 0 & -\sin\varphi^{(3)} & \cos\varphi^{(3)} \end{bmatrix}$$

$$\rightarrow \underline{T}^{(3)} = \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 & 0 \\ 0 & 0 & \sqrt{2}/2 & \sqrt{2}/2 \\ 0 & 0 & -\sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix} ; \quad \underline{T}^{(3)^T} = \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 & 0 \\ 0 & 0 & \sqrt{2}/2 & -\sqrt{2}/2 \\ 0 & 0 & \sqrt{2}/2 & \sqrt{2}/2 \end{bmatrix}$$

$$\underline{\bar{K}}^{(3)} = 40 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\Downarrow$$

$$\begin{bmatrix} f_{x_1}^{(3)} \\ f_{y_1}^{(3)} \\ f_{x_4}^{(3)} \\ f_{y_4}^{(3)} \end{bmatrix} = 40 \begin{bmatrix} 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} u_{x_1}^{(3)} \\ u_{y_1}^{(3)} \\ u_{x_4}^{(3)} \\ u_{y_4}^{(3)} \end{bmatrix}$$

the same matrix can be written for element (4) [4-3]

$$\begin{bmatrix} f_{x_4}^{(4)} \\ f_{y_4}^{(4)} \\ f_{x_3}^{(4)} \\ f_{y_3}^{(4)} \end{bmatrix} = 40 \begin{bmatrix} 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{bmatrix} \begin{bmatrix} u_{x_4}^{(4)} \\ u_{y_4}^{(4)} \\ u_{x_3}^{(4)} \\ u_{y_3}^{(4)} \end{bmatrix}$$

- Augmentation of the globalized stiffness matrices and dropping of the member index from the nodal displacements

$$\begin{cases} \underline{f}^{(1)} = \underline{K}^{(1)} \underline{u} \\ \underline{f}^{(2)} = \underline{K}^{(2)} \underline{u} \\ \underline{f}^{(3)} = \underline{K}^{(3)} \underline{u} \\ \underline{f}^{(4)} = \underline{K}^{(4)} \underline{u} \end{cases}$$

ELEMENT (1)

$$\begin{bmatrix} f_{x_1}^{(1)} \\ f_{y_1}^{(1)} \\ f_{x_2}^{(1)} \\ f_{y_2}^{(1)} \\ f_{x_3}^{(1)} \\ f_{y_3}^{(1)} \\ f_{x_4}^{(1)} \\ f_{y_4}^{(1)} \end{bmatrix} = \begin{bmatrix} 10 & 0 & -10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -10 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \\ u_{x_3} \\ u_{y_3} \\ u_{x_4} \\ u_{y_4} \end{bmatrix}$$

ELEMENT (2)

$$\begin{bmatrix} f_{x_1}^{(2)} \\ f_{y_1}^{(2)} \\ f_{x_2}^{(2)} \\ f_{y_2}^{(2)} \\ f_{x_3}^{(2)} \\ f_{y_3}^{(2)} \\ f_{x_4}^{(2)} \\ f_{y_4}^{(2)} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -5 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \\ u_{x_3} \\ u_{y_3} \\ u_{x_4} \\ u_{y_4} \end{bmatrix}$$

ELEMENT (3)

$$\begin{bmatrix} f_{x_1}^{(3)} \\ f_{y_1}^{(3)} \\ f_{x_2}^{(3)} \\ f_{y_2}^{(3)} \\ f_{x_3}^{(3)} \\ f_{y_3}^{(3)} \\ f_{x_4}^{(3)} \\ f_{y_4}^{(3)} \end{bmatrix} = \begin{bmatrix} 20 & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\ 20 & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -20 & -20 & 0 & 0 & 0 & 0 & 20 & 20 \\ -20 & -20 & 0 & 0 & 0 & 0 & 20 & 20 \end{bmatrix} \begin{bmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \\ u_{x_3} \\ u_{y_3} \\ u_{x_4} \\ u_{y_4} \end{bmatrix}$$

ELEMENT (4)

$$\begin{bmatrix} f_{x_1}^{(4)} \\ f_{y_1}^{(4)} \\ f_{x_2}^{(4)} \\ f_{y_2}^{(4)} \\ f_{x_3}^{(4)} \\ f_{y_3}^{(4)} \\ f_{x_4}^{(4)} \\ f_{y_4}^{(4)} \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 20 & -20 & -20 \\ 0 & 0 & 0 & 0 & 20 & 20 & -20 & -20 \\ 0 & 0 & 0 & 0 & -20 & -20 & 20 & 20 \\ 0 & 0 & 0 & 0 & -20 & -20 & 20 & 20 \end{bmatrix} \begin{bmatrix} u_{x_1} \\ u_{y_1} \\ u_{x_2} \\ u_{y_2} \\ u_{x_3} \\ u_{y_3} \\ u_{x_4} \\ u_{y_4} \end{bmatrix}$$

- Global Equilibrium

$$\begin{bmatrix} f_{x_1} \\ f_{y_1} \\ 0 \\ f_{y_2} \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} 30 & 20 & -10 & 0 & 0 & 0 & -20 & -20 \\ 20 & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\ -10 & 0 & 10 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & -5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 20 & 20 & -20 & -20 \\ 0 & 0 & 0 & -5 & 20 & 25 & -20 & -20 \\ -20 & -20 & 0 & 0 & -20 & -20 & 40 & 40 \\ -20 & -20 & 0 & 0 & -20 & -20 & 40 & 40 \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ u_{x_2} \\ 0 \\ u_{x_3} \\ u_{y_3} \\ u_{x_4} \\ u_{y_4} \end{bmatrix}$$

- Reduced Master Stiffness Equations (consideration of the BCs)

$$\begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 20 & 20 & -20 & -20 \\ 0 & 20 & 25 & -20 & -20 \\ 0 & -20 & -20 & 40 & 40 \\ 0 & -20 & -20 & 40 & 40 \end{bmatrix} \begin{bmatrix} u_{x_2} \\ u_{x_3} \\ u_{y_3} \\ u_{x_4} \\ u_{y_4} \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

$$\underline{\hat{K}} \qquad \underline{\hat{u}} = \underline{\hat{f}}$$

The reduced master stiffness matrix contains two rows and columns that are the same. Therefore, without making any calculation, it's possible to see that this matrix is SINGULAR ($\det \hat{K} = \varnothing$!). The reason is because the structure is labile; a possible mechanism can be the following one: PAGE 11