Master's Degree Numerical Methods in Engineering

Universitat Politècnica de Catalunya BarcelonaTech

Computational Structural Mechanics and Dynamics

Assignment 2: FEM Modeling Introduction

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1 Symmetry

On "FEM Modelling: Introduction":

1. Identify the symmetry and antisymmetry lines in the two dimensional problems illustrated in the figure. They are:
(a) A circular disk under two diametrically opposite point forces (the famous "Brazilian test" for concrete.
(b) The same disk under two diametrically opposite force pairs.
(c) A clamped semiannulus under force pairs illustrated below.
(d) A stretched rectangular plate with a central circular hole.
(e) and (f) are half-planes under concentrated loads.

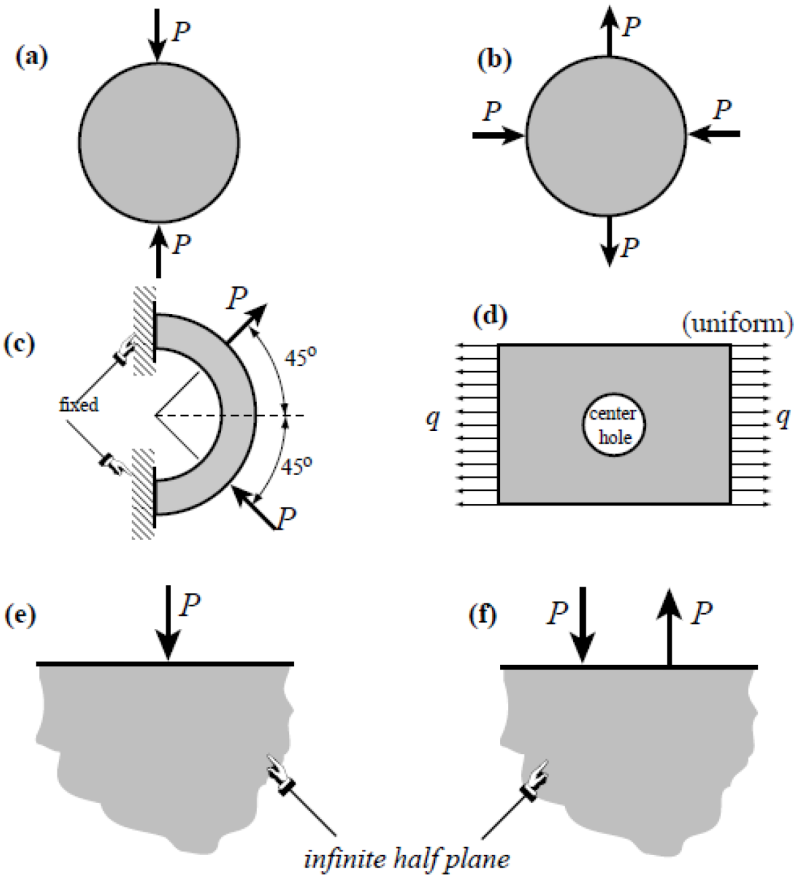

Figure 1: Problems for assignment 2.1

2. Having identified those symmetry/antisymmetry lines, state whether it is possible to cut the complete structure to one half or one quarter before laying out a finite element mesh. Then draw a coarse FE mesh indicating, with rollers or fixed supports, which kind of displacement BCs you would specify on the symmetry or antisymmetry lines.

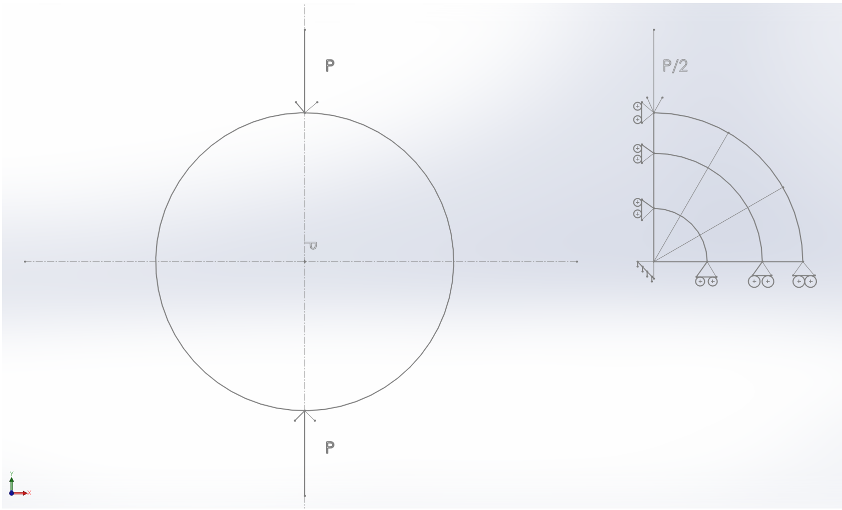

Figure 2: The circle has four symmetry lines

The circle in Figure 2. has four symmetry lines, one horizontal, one vertical and two on 45° and 135°.

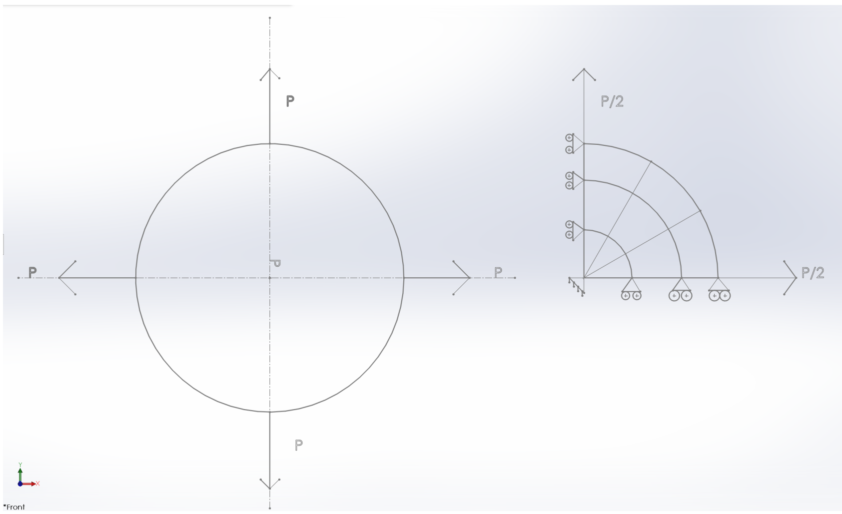

Figure 3: This circle has two symmetry lines

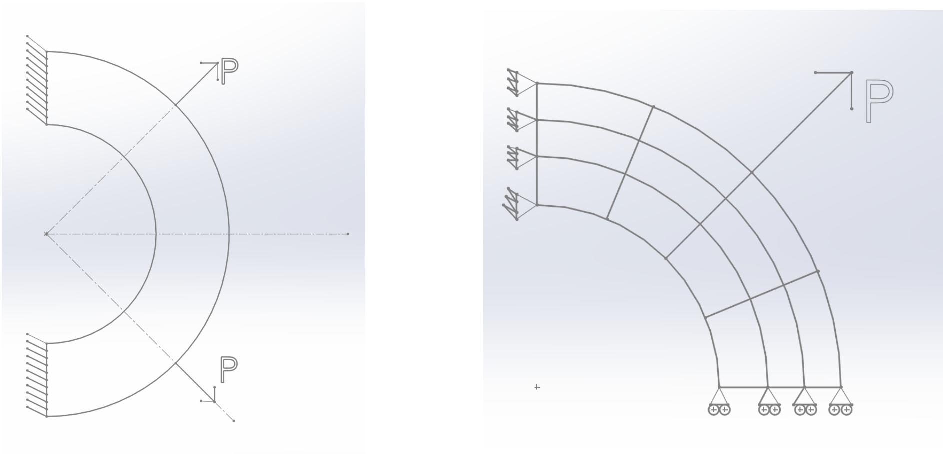

Figure 4: The symmetry line and the reduced model

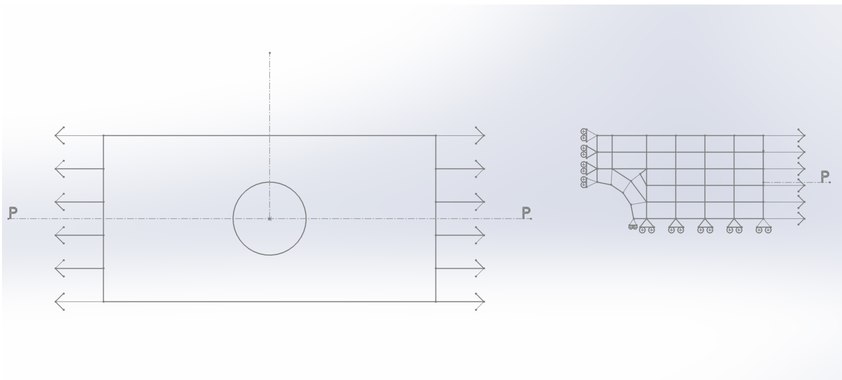

Figure 5: The rectangular shape has two symmetry lines

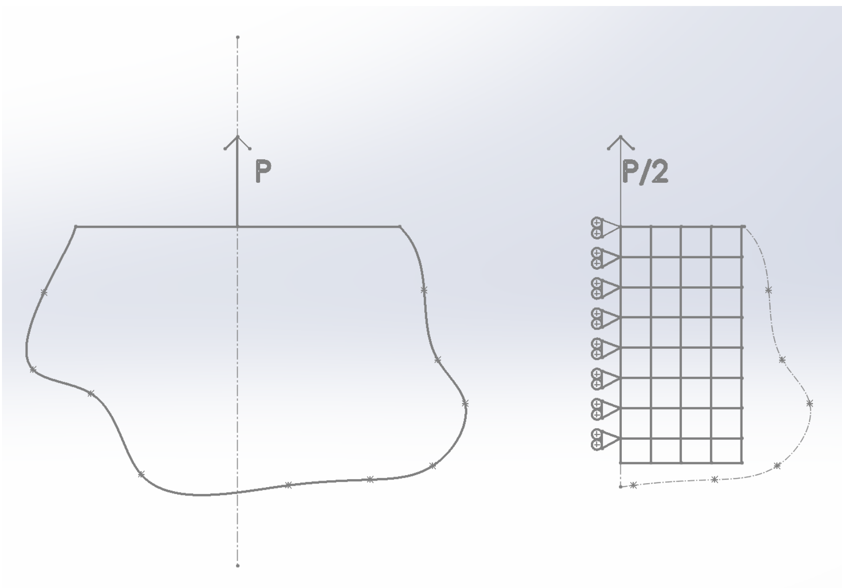

Figure 6: The infinite half plate has only one symmetry line

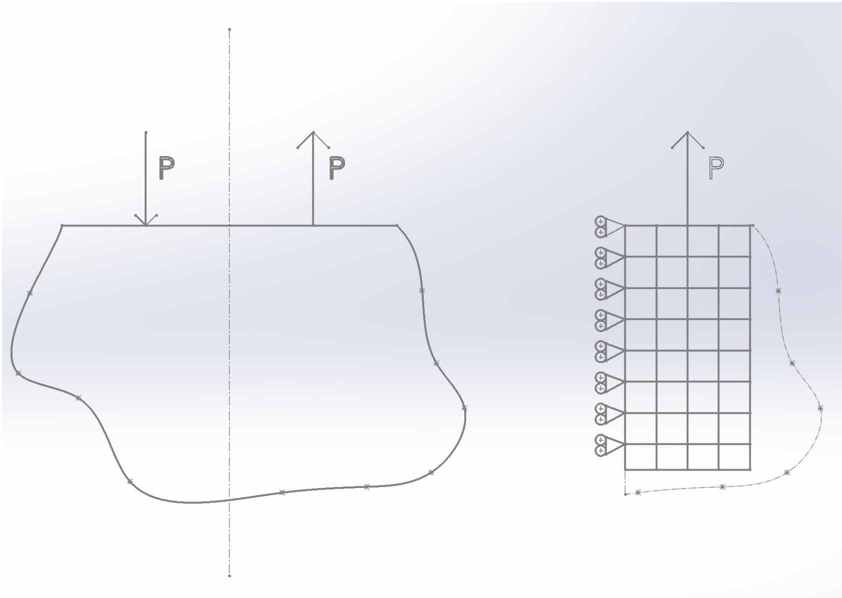

Figure 7: This infinite half plane has one antisymmetry line and a rolling boundary condition in the x direction

3. Explain the difference between “Verification” and “Validation” in the context of the FEM-Modelling procedure.

In every Finite-Element analysis we need the validation and verification process, to see if the reached answer represents the phenomena that we wanted or not. The difference between "Validation" and "verification" is that verification checks to see if the final answer is converging to the answer of the model or not. Verification is to see if the model that we have used for our FEM analysis is really representing the physical phenomena or not. So validation is only checking to see if we are getting the correct answer of the model and verification checks to see if we are using the correct model for our problem.

4. On “Variational Formulation”:
A tapered bar element of length l and areas A_i and A_j with A interpolated as:

$$A = A_i(1 - \eta) + A_j\eta$$

and constant density ρ rotates on a plane at uniform angular velocity $\omega(rad/s)$ about node i.

Taking axis x along the rotating bar with origin at node i , the centrifugal axial force is

$$\rho(x) = \rho A\omega^2 x$$

along the length in which x is the longitudinal coordinate $x = x_e$.

Find the consistent node forces as functions of ρ, A_i, A_j, ω and l, and specialize the result to the prismatic bar $A = A_i = A_j$.

Using the variational formulation for a linear element we know that the nodal force can be computed as:

$$f^e = \int_0^1 q \begin{pmatrix} 1-\eta \\ \eta \end{pmatrix} l d\eta$$

For the local coordinates of η which can be defined in the x direction as $\eta = x - x_i/l$ so if we transform the x coordinates to the local one for the axial force we will have:

$$q = \rho A \omega^2 x = \rho A \omega^2 \eta l$$

so the nodal force formula will have the form:

$$f^e = \int_0^1 \rho(A_i(1-\eta) + A_j\eta)\omega^2 \eta l \begin{pmatrix} 1-\eta \\ \eta \end{pmatrix} l d\eta$$

$$= \rho\omega^2 l^2 \int_0^1 (A_i(1-\eta) + A_j\eta)\eta \begin{pmatrix} 1-\eta \\ \eta \end{pmatrix} d\eta$$

$$= \rho\omega^2 l^2 \int_0^1 \begin{pmatrix} A_i\eta(1-\eta)^2 + A_j\eta^2(1-\eta) \\ A_i\eta^2(1-\eta) + A_j\eta^3 \end{pmatrix} d\eta$$

$$= \rho\omega^2 l^2 \begin{pmatrix} 1/12A_i + 1/12A_j \\ 1/12A_i + 1/4A_j \end{pmatrix}$$

For the case of $A = A_i = A_j$ the system will reduce to:

$$f^e = \rho\omega^2 l^2 \begin{pmatrix} 1/6A \\ 1/3A \end{pmatrix}$$