UPC - BARCELONA TECH
MSc Computational Mechanics
Spring 2018

Coputations Solid Mechanics Dynamics

Assignment 2

Due 19/02/2018

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Assignment 2.1

On "FEM Modelling: Introduction":

1. Identify the symmetry and antisymmetry lines in the two-dimensional problems illustrated in the figure. They are:

(a) a circular disk under two diametrically opposite point forces (the famous "Brazilian test" for concrete)
(b) the same disk under two diametrically opposite force pairs
(c) a clamped semiannulus under a force pair oriented as shown
(d) a stretched rectangular plate with a central circular hole.
(e) and (f) are half-planes under concentrated loads.

2. Having identified those symmetry/antisymmetry lines, state whether it is possible to cut the complete structure to one half or one quarter before laying out a finite element mesh. Then draw a coarse FE mesh indicating, with rollers or fixed supports, which kind of displacement BCs you would specify on the symmetry or antisymmetry lines.

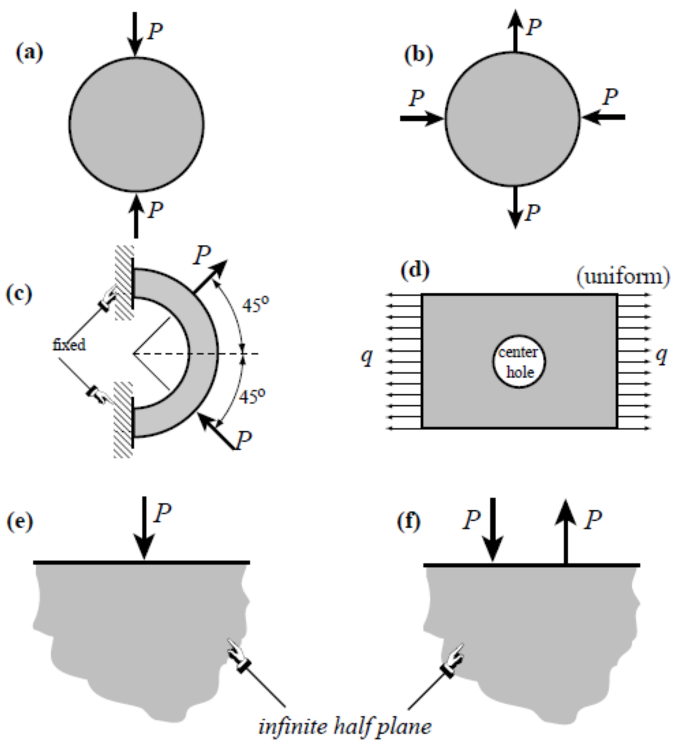


Figure 2.1.- Problems for assignment 2.1

Assignment 2.2

On “FEM Modelling: Introduction”:

1. The plate structure shown in the figure is loaded and deforms in the plane of the paper. The applied load at *D* and the supports at *I* and *N* extend over a fairly narrow area. List what you think are the likely “trouble spots” that would require a locally finer finite element mesh to capture high stress gradients. Identify those spots by its letter and a reason.

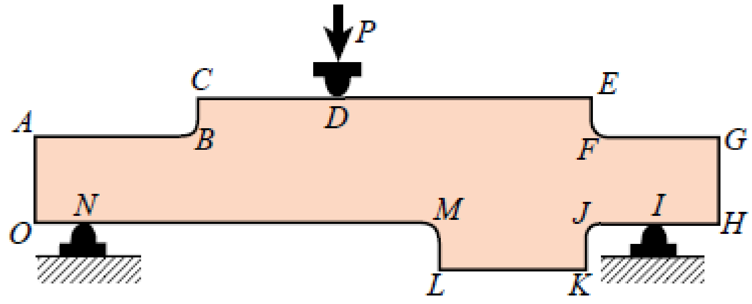


Figure 2.2.- Inplane bent plate

Assignment 2.3

On “Variational Formulation”:

1. A tapered bar element of length l and areas A_i and A_j with A interpolated as

$$A = A_i(1-\xi) + A_j\xi$$

and constant density ρ rotates on a plane at uniform angular velocity ω (rad/sec) about node i. Taking axis x along the rotating bar with origin at node i , the centrifugal axial force is $q(x)=\rho A\omega^2 x$ along the length in which x is the longitudinal coordinate $x=x^e$.

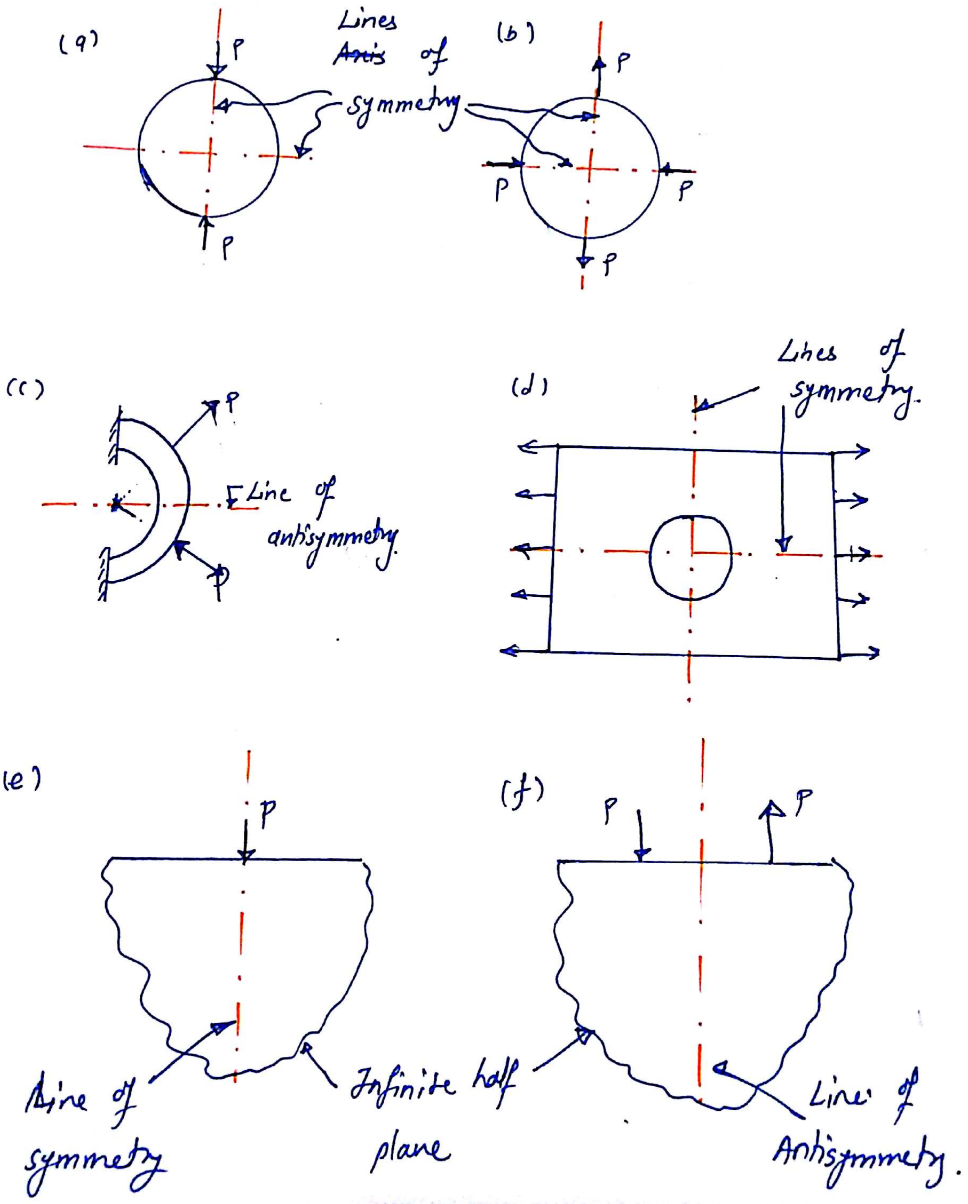
Find the consistent node forces as functions of ρ, A_i, A_j, ω and l, and specialize the result to the prismatic bar $A = A_i = A_j$.

Date of Assignment: 12 / 02 / 2018
Date of Submission: 19 / 02 / 2018

The assignment must be submitted as a pdf file named **As2-Surname.pdf** to the CIMNE virtual center.

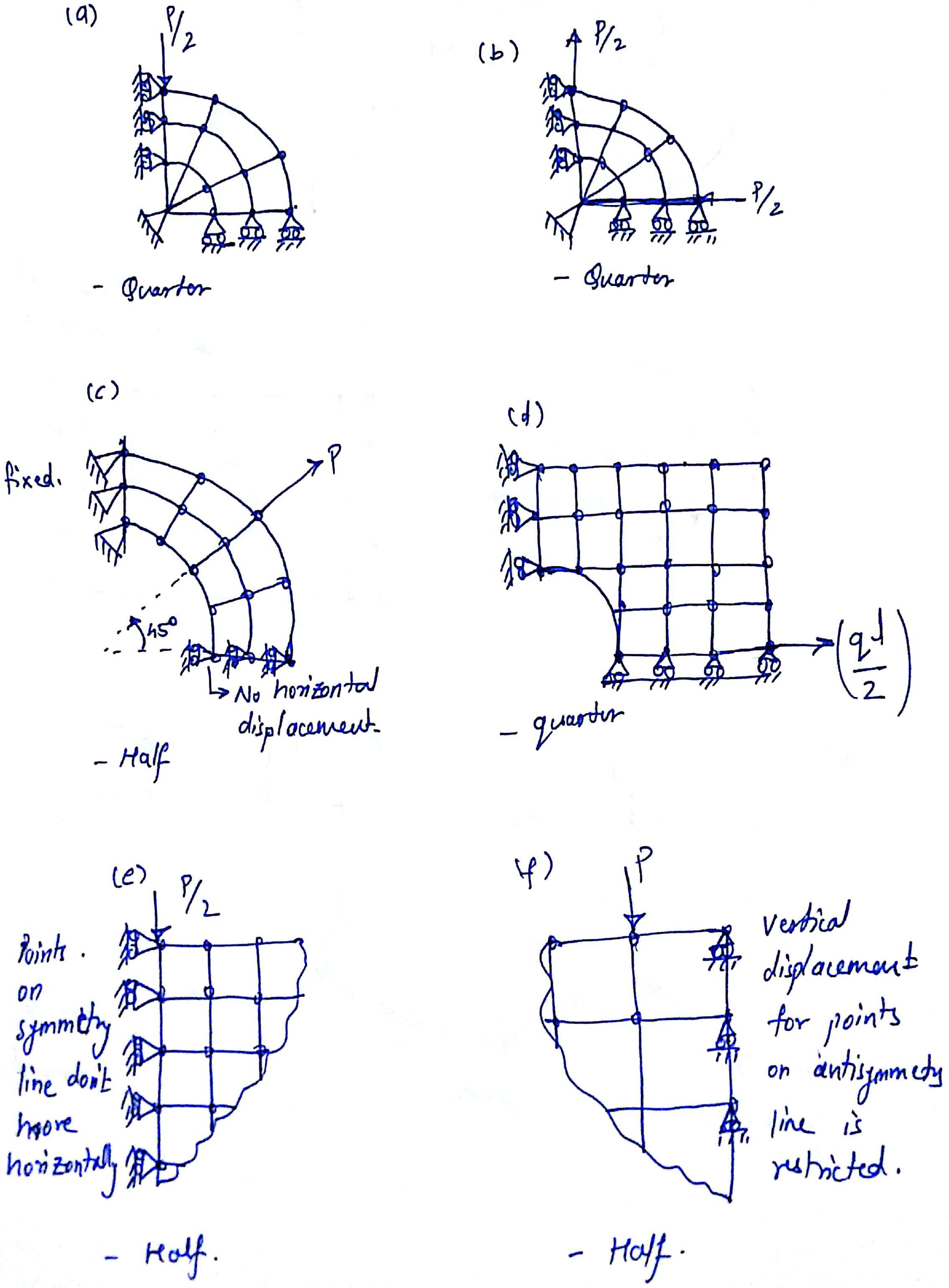
1 Identify the symmetry and antisymmetry lines in the two dimensional prolems illustrated in the assignment sheet

The symmentry and antsymmetry lines are shown with the orange dash-dot lines as shown in the following figure.



2 Mesh

Having identified the symmetry and anti symmetry lines, the complete structure is cut into half or quarter accordingly. And then the finite element mesh is layed out. Also according to the geometry and forces, fixed or roller suppoerts and respective forces are shown in the figure.

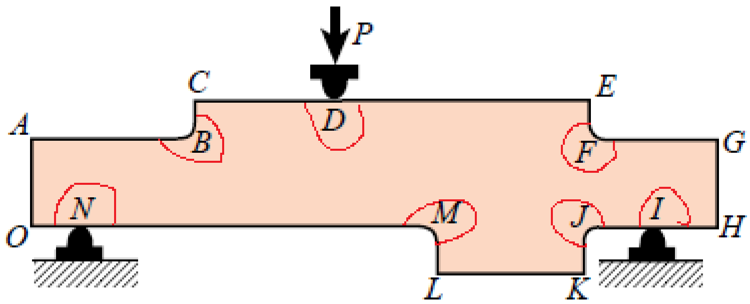


For rectangular elements with a circular hole in middle, a uniformly divided load is applied along the length. This UDL can be represented as a point load applied at the center is when multiplied by length.

3 Local Fine Mesh

As shown in the figure, local fine mesh is required at following points, also respective reasons are stated:

1. D: because of point load
2. N, I: Pointed supports
3. B, F, J, M: Due to sudden contraction



4 Question on Variation formulation

To find: Consistent nodal force
The consistent nodal forces are given as follows:

$$f^{(e)} = \int_0^l q \begin{bmatrix} 1-\zeta \\ \zeta \end{bmatrix} dx$$

But we Know that q is a function of x, as given in the question, we substitute the value of q

$$f^{(e)} = \int_0^l \rho A \omega^2 x \begin{bmatrix} 1-\zeta \\ \zeta \end{bmatrix} dx$$

$$A = A_i(1-\zeta) + A_j\zeta$$

$$f^{(e)} = \int_0^l \rho \omega^2 x (A_i(1-\zeta) + A_j\zeta) \begin{bmatrix} 1-\zeta \\ \zeta \end{bmatrix} dx$$

$$\zeta = \frac{x - x_1}{l} = \frac{x-0}{l}$$

$$x = \zeta l$$

$$dx = l d\zeta$$

Substituting the value of x and dx in the above equation, we get

$$f^{(e)} = \int_0^1 \rho \omega^2 l \zeta (A_i(1-\zeta) + A_j\zeta) \begin{bmatrix} 1-\zeta \\ \zeta \end{bmatrix} d\zeta$$

Simplifying above equation, we get

$$f^{(e)} = \int_0^1 \rho\omega^2 l\zeta[(1-\zeta)(A_i(1-\zeta)+A_j\zeta)+\zeta(A_i(1-\zeta)+A_j\zeta)]d\zeta$$

Solving the above integral separately as follows: For $f_1^{(e)}$

$$f_1^{(e)} = \int_0^1 \rho\omega^2 l\zeta[(1-\zeta)(A_i(1-\zeta)+A_j\zeta)]d\zeta$$

after simplifying and solving the integral we get

$$f_1^{(e)} = \rho\omega^2 l^2 \left[\frac{A_i + A_j}{12}\right]$$

For $f_2^{(e)}$

$$f_2^{(e)} = \int_0^1 \rho\omega^2 l\zeta[\zeta(A_i(1-\zeta) + A_j\zeta)]d\zeta$$

after simplifying and solving the integral we get

$$f_2^{(e)} = \rho\omega^2 l^2 \left[\frac{A_i}{12} + \frac{A_j}{4}\right]$$

$$f^{(e)} = f_1^{(e)} + f_2^{(e)}$$

Substituting respective values we obtain the consistent nodal force equation as follows

$$f^{(e)} = \rho\omega^2 l^2 \begin{bmatrix} \dfrac{A_i + A_j}{12} \\ \dfrac{A_i}{12} + \dfrac{A_j}{4} \end{bmatrix}$$

The above equation can be further simplified as follows:

$$f^{(e)} = \frac{\rho\omega^2 l^2}{12} \begin{bmatrix} A_i + A_j \\ A_i + 3A_j \end{bmatrix}$$

For the special case of $A = A_i = A_j$, the consistent nodal force can be written as follows:

$$f^{(e)} = \frac{\rho\omega^2 l^2 A}{6} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$