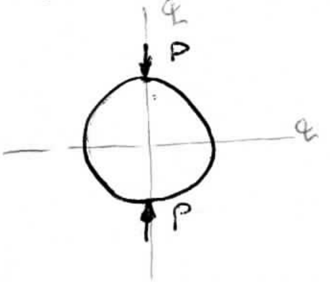
David Encalada

Assignment 2.1

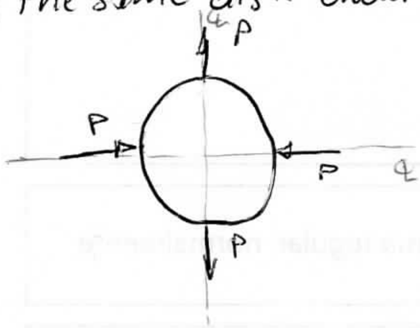
1- Identify the symmetry and antisymmetry lines the two-dimensional problems illustrated in the figure. They are

a) a circular disk under diametrically opposite point forces



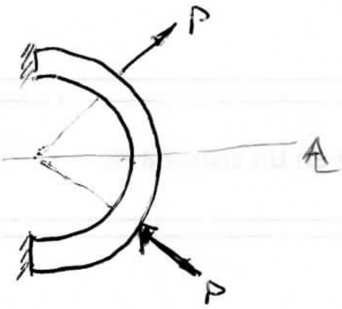
symmetry

b) the same disk under two diametrically opposite force pairs



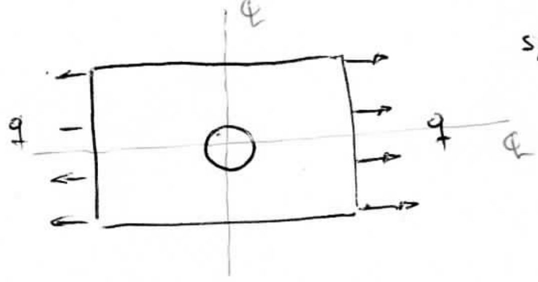
symmetry

c) A clamped semiannulus under a force pair oriented as shown



antisymmetry

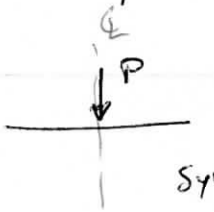
d) A stretched rectangular plate with a central circular hole



symmetry

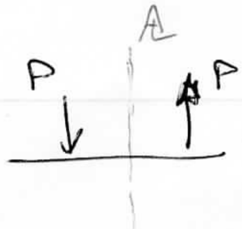
e) Half-planes under concentrated loads

e)



symmetry

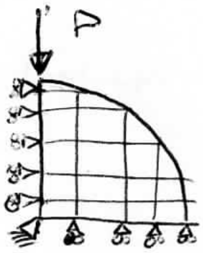
f)



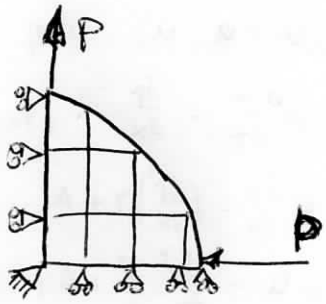
antisymmetry

2. Having identified those symmetry/antisymmetry lines, State whether it is possible to cut the complete structure to one half or one quarter before laying out a finite element mesh. Then draw a coarse FE mesh indicating with rollers or fixed supports, which kind of displacement BC you would specify on the symmetry or antisymmetry lines.

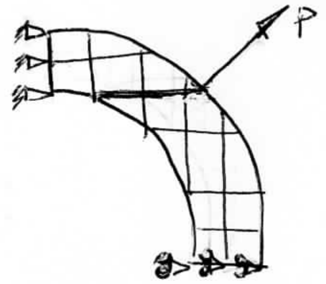
a)
quarter



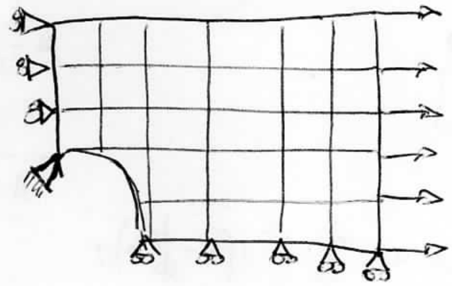
b)
quarter



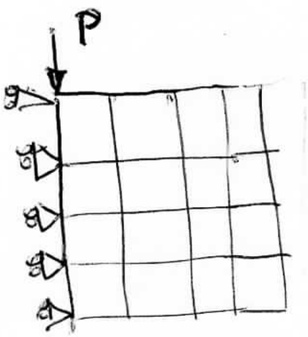
c)
half



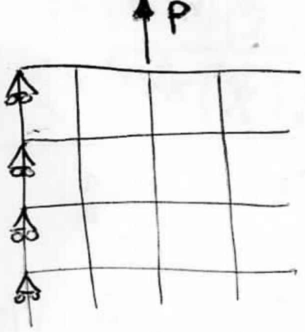
d)
quarter



e)
half

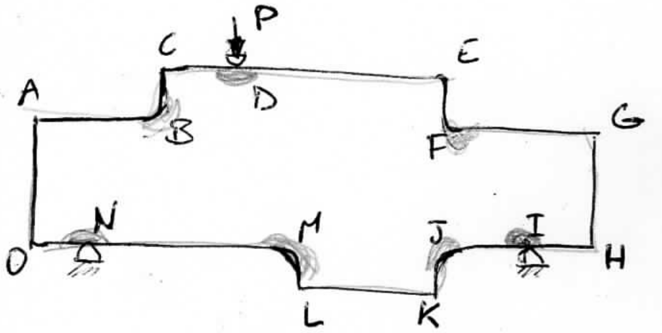


f)
half



Assignment 2.2

1. The plate structure shown in the figure is loaded and deforms in the plane of the paper. The applied load at D and the supports at I and N extend over a fairly narrow area. List what you think are the likely "trouble spots" that would require a locally finer finite element mesh to capture high stress gradient. Identify those spots by its letter and a reason.



N, I } concentrated reactions

D concentrated loads

B, M } Entrant corners

F, J } change in thickness

Assignment 2.3

On "Variational Formulation"

1.- A tapered bar element of length l and areas A_i and A_j with interpolated as

$$A = A_i(1-\xi) + A_j\xi$$

and constant density ρ rotate on a plane of uniform angular velocity ω (rad/s) about node i. Taking axis x along the rotating bar with origin at node i, the centrifugal axial force is $q(x) = \rho A \omega^2 x$ along the leght in which x is the longitudinal coordinate $x = x^e$

Find the consistent node forces as functions of ρ, A_i, A_j, ω and l, and specialize the result to the prismatic bar $A = A_i = A_j$

$$f^e = \int_{x_i}^{x_j} q(x) \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} dx = \rho\omega^2 \int_0^1 (A_i(1-\xi) + A_j\xi)\,\xi l \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l\, d\xi = \rho\omega^2 l^2 \int_0^1 \begin{bmatrix} A_i(\xi - 2\xi^2 + \xi^3) + A_j(\xi^2 - \xi^3) \\ A_i(\xi^2 - \xi^3) + A_j\xi^3 \end{bmatrix}$$

$$f^e = \rho\omega^2 l^2 \begin{bmatrix} A_i\left(\frac{1}{2} - \frac{2}{3} + \frac{1}{4}\right) + A_j\left(\frac{1}{3} - \frac{1}{4}\right) \\ A_i\left(\frac{1}{3} - \frac{1}{4}\right) + A_j\left(\frac{1}{4}\right) \end{bmatrix} \qquad f^e = \rho\omega^2 l^2 \begin{bmatrix} \frac{A_i}{12} + \frac{A_j}{12} \\ \frac{A_i}{12} + \frac{A_j}{4} \end{bmatrix}$$

$$\boxed{\begin{aligned} f_i &= \rho \omega^2 l^2 \left(\frac{A_i}{12} + \frac{A_j}{12} \right) \\ f_j &= \rho \omega^2 l^2 \left(\frac{A_i}{12} + \frac{A_j}{4} \right) \end{aligned}}$$

If $A = A_i = A_j$

$$\boxed{\begin{aligned} f_i &= \frac{1}{6} \rho \omega^2 l^2 A \\ f_j &= \frac{1}{3} \rho \omega^2 l^2 A \end{aligned}}$$