Computational Structural Mechanics and Dynamics — February 19, 2019
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Assignment 2.1

2.1.1 Identify the symmetry and antisymmetry lines in the two-dimensional problems illustrated in the figure

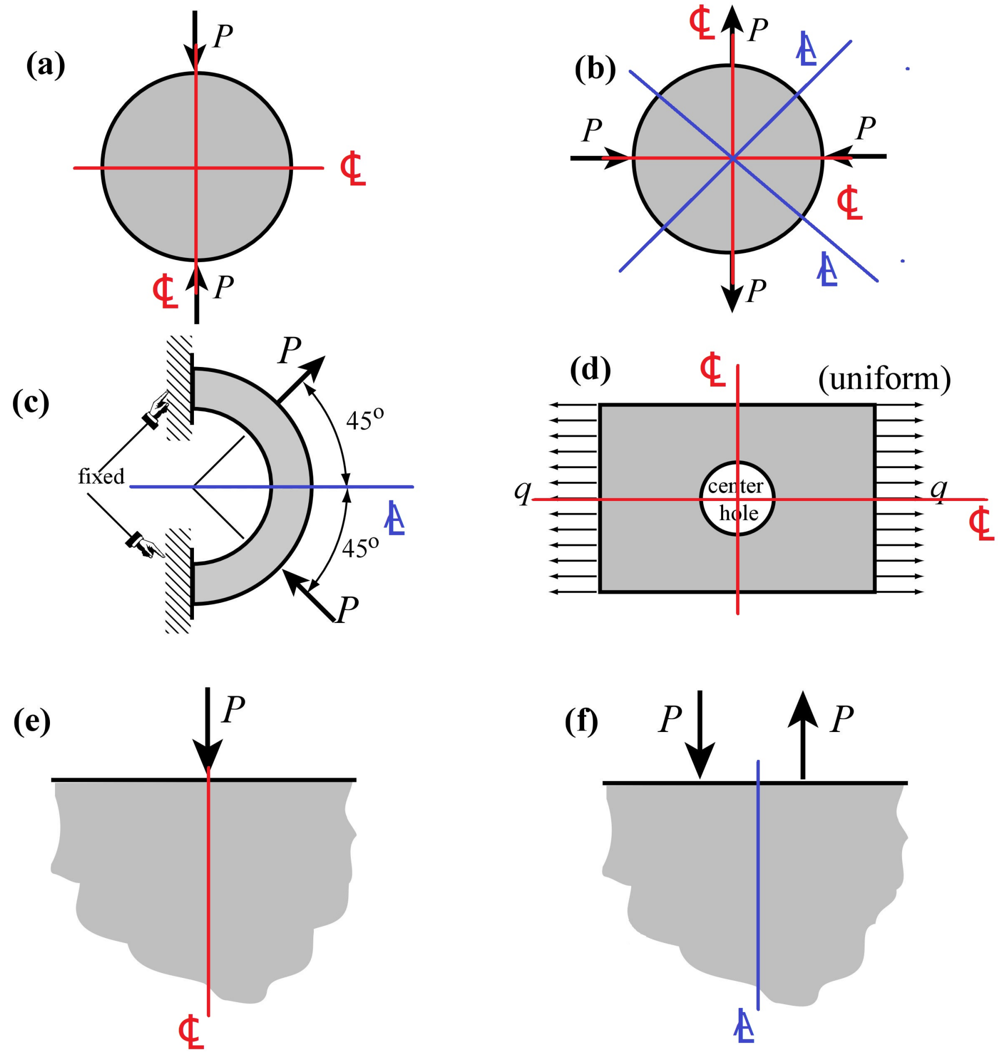

Figure 2.1.1.1: Symmetry and antisymmetry lines.

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2.1.2 Having identified those symmetry/antisymmetry lines, state whether it is possible to cut the complete structure to one half or one quarter before laying out a finite element mesh. Then draw a coarse FE mesh indicating, with rollers or fixed supports, which kind of displacement BCs you would specify on the symmetry or antisymmetry lines.

Problem domains may be reduced if we use appropriate supports for these symmetry and antisymmetry conditions.

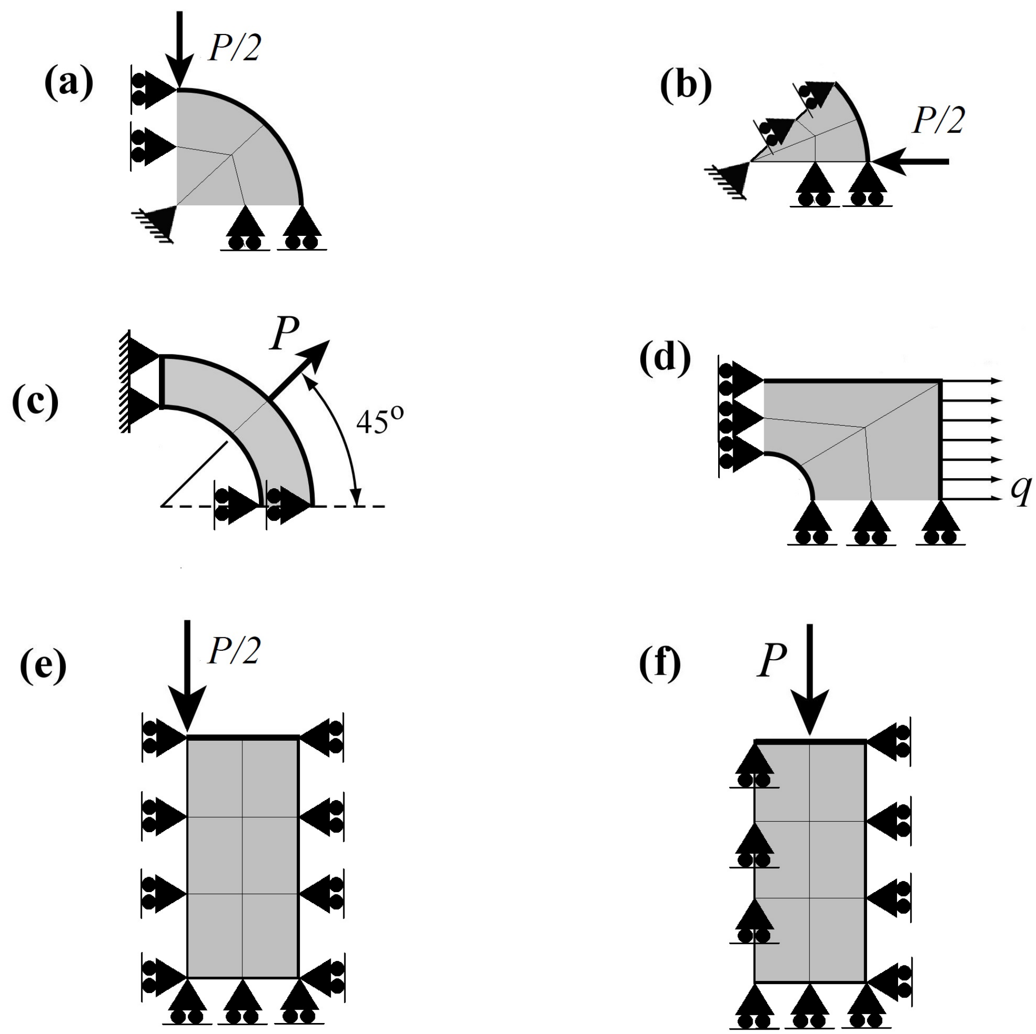

Figure 2.1.2.1: Reduced problem domains.

2.2.1 The plate structure shown in the figure is loaded and deforms in the plane of the paper. The applied load at D and the supports at I and N extend over a fairly narrow area. List what you think are the likely trouble spots that would require a locally finer finite element mesh to capture high stress gradients. Identify those spots by its letter and a reason.

Trouble spots are: B, F, J, M (entrant corners), N, D, I (concentrated forces).

The entrant corners are a trouble spot because they are a region where isostatics (principal stress trajectories) gather up together.

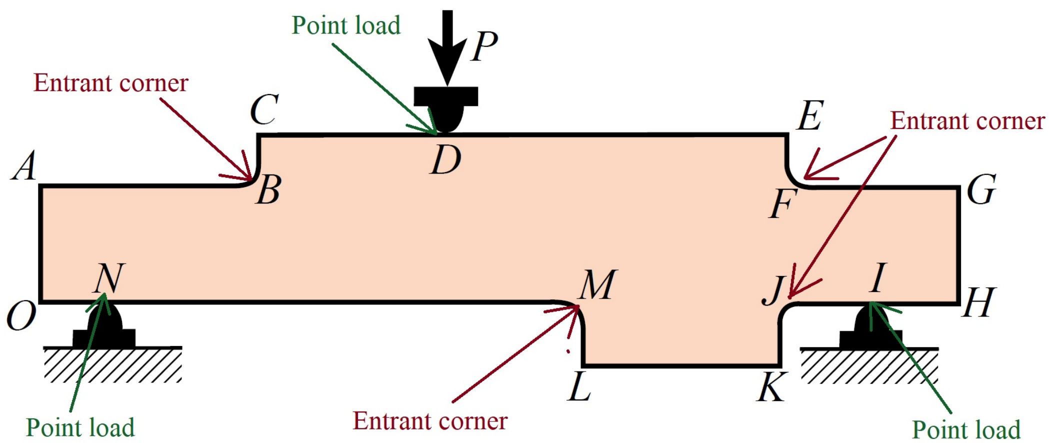

Figure 2.2.1.1: Trouble spots.

2.2.2 A tapered bar element of length l and areas Ai and Aj with A interpolated as:

$$A = A_i(1-\xi) + A_j\xi$$

and constant density ρ rotates on a plane at uniform angular velocity ω (rad/sec) about node i. Taking axis x along the rotating bar with origin at node i , the centrifugal axial force is $q(x) = \rho A \omega^2 x$ along the length in which x is the longitudinal coordinate $x = x^e$.

The consistent node force vector f^e comes from the element contribution to the external work potential W:

$$W^e = \int_{x_1}^{x_2} q\, u\, dx = \int_0^1 q\, N^T\, u^e\, l\, d\xi = \int_0^1 q \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l d\xi = (u^e)^T f^e$$

in which $\xi = \dfrac{(x-x_1)}{l}$ Consequently,

$$f^e = \int_{x_1}^{x_2} q \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} dx = \int_0^1 q \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l d\xi$$

Substituting the value of q and A into the force node vector:

$$f^e = \int_0^1 \rho[A_i(1-\xi) + A_j\xi]\omega^2(\xi l) \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l d\xi$$

$$f^e = \rho\omega^2 l^2 \begin{bmatrix} \dfrac{1}{12}A_i + \dfrac{1}{12}A_j \\ \\ \dfrac{1}{12}A_i + \dfrac{1}{4}A_j \end{bmatrix}$$

If we now consider $A = A_i = A_j$, the problem reduces to:

$$f^e = \rho\omega^2 l^2 \begin{bmatrix} \dfrac{1}{6}A \\ \\ \dfrac{1}{3}A \end{bmatrix}$$