Master Of Science in Computational Mechanics
Computational Structural Mechanics and Dynamics

Chinmay Khisti (19 Februrary 2019)

Assignment 2.1

2.1.1. Identify the symmetry and antisymmetry lines in the two-dimensional problems illustrated in the figure. They are:
(a) a circular disk under two diametrically opposite point forces (the famous "Brazilian test" for concrete)
(b) the same disk under two diametrically opposite force pairs
(c) a clamped semiannulus under a force pair oriented as shown
(d) a stretched rectangular plate with a central circular hole.
(e) and (f) are half-planes under concentrated loads.

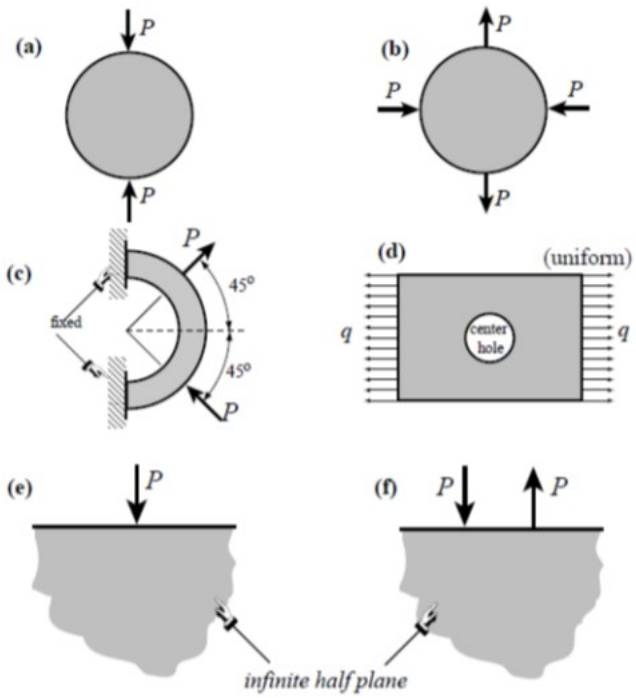

Figure 1: Problem

Symmetry for a 2D object can be stated if the object has a mirror image of its shape along a specific axis. Due to this property a part of the object can be solved to get the solution for whole object with consideration of various boundary conditions. this leads to reduction in time and computational cost of the problem. figures shown below states symmetric and antisymmetry lines for the given objects

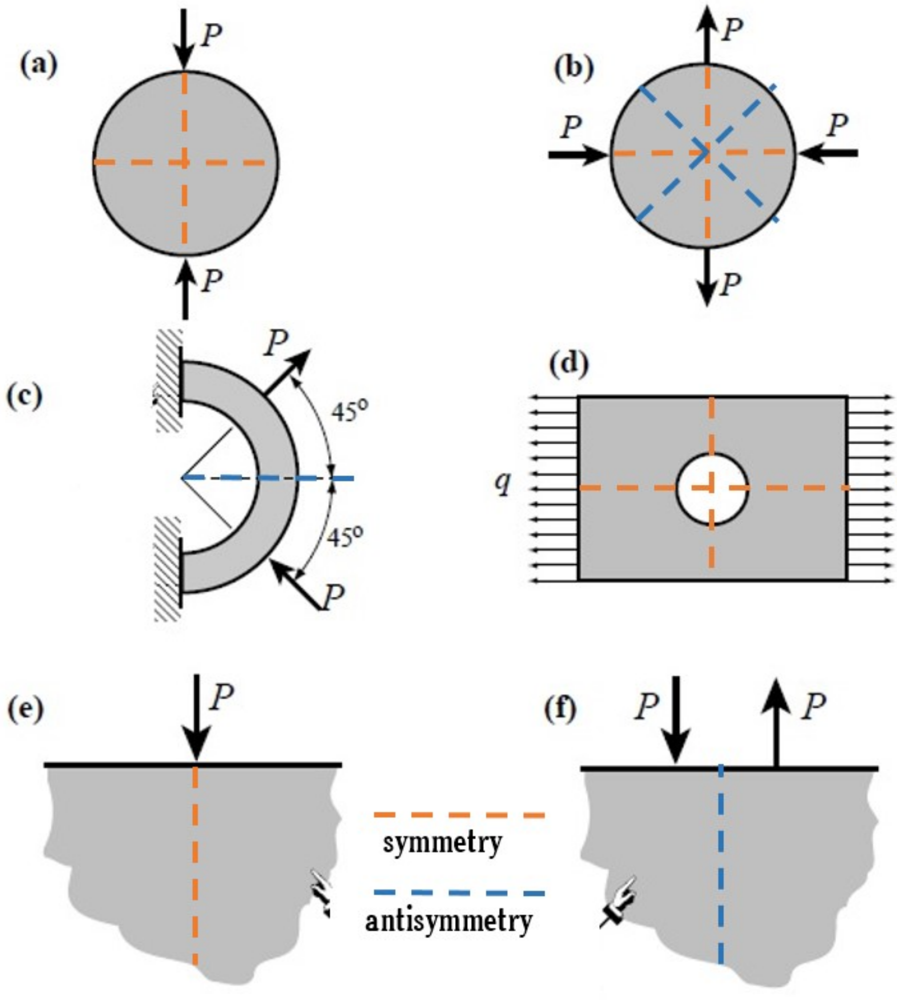

Figure 2: Solution for assignment 2.1.1

2.1.2. Having identified those symmetry/antisymmetry lines, state whether it is possible to cut the complete structure to one half or one quarter before laying out a finite element mesh. Then draw a coarse FE mesh indicating, with rollers or fixed supports, which kind of displacement BCs you would specify on the symmetry or antisymmetry lines.

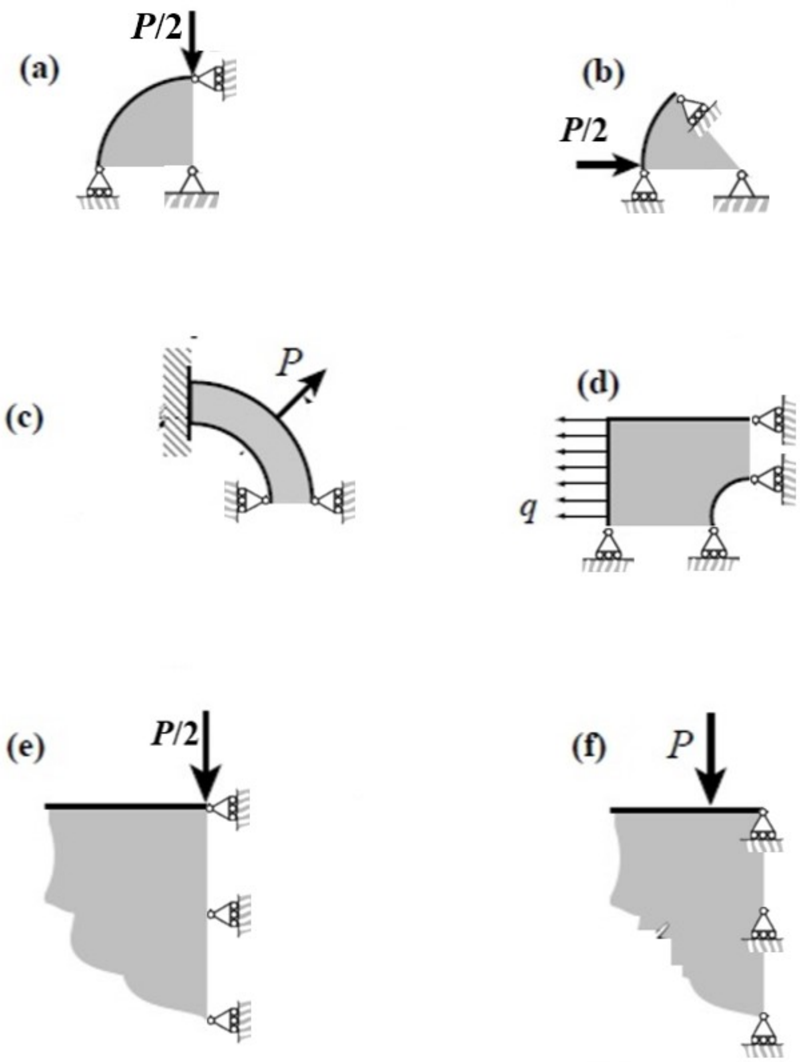

Figure 3: Solution for assignment 2.1.2

Assignment 2.2

The plate structure shown in the figure is loaded and deforms in the plane of the paper. The applied load at D and the supports at I and N extend over a fairly narrow area. List what you think are the likely “trouble spots” that would require a locally finer finite element mesh to capture high stress gradients. Identify those spots by its letter and a reason.

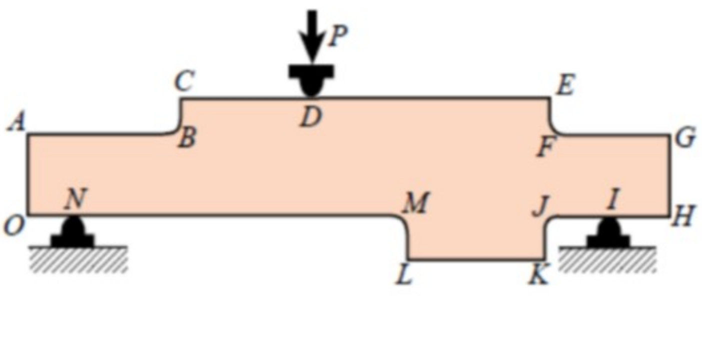

Figure 4: - Inplane bent plate

Bellow is the list of "trouble spots" and there reasons respectively to have finer mesh to capture high stress gradients. Stress propagates in the body through various boundary conditions so they must be included.

Node B: Curved edge
Node D: Loading point
Node F: Curved edge
Node I: Support
Node J: Curved edge near t the support
Node M: Curved edge
Node N: Support

Assignment 2.3

A tapered bar element of length l and areas Ai and Aj with A interpolated as

$$A = A_i(1-\xi) + A_j\xi \tag{1}$$

and constant density ρ rotates on a plane at uniform angular velocity ω (rad/sec) about node i. Taking axis x along the rotating bar with origin at node i , the centrifugal axial force is $q(x) = \rho A\omega^2 x$ along the length in which x is the longitudinal coordinate $x = x^e$.

Find the consistent node forces as functions of ρ, Ai, Aj, ω and l, and specialize the result to the prismatic bar $A = A_i = A_j$.
Answer:
As axial force is given along the x direction the consistent nodal force is given by,

$$W^e = \int_{x_1}^{x_2} qudx = (u^e)^T \int_0^1 q \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} ld\xi$$

where,

$$\xi = \frac{x - x_1}{l}$$

of which,

$$f^e = \int_0^1 q \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} ld\xi$$

substituting given values od A and q

$$f^e = \int_0^1 \rho\omega^2(\xi l)[A_i(1-\xi) + A_j\xi] \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} ld\xi$$

$$= \rho\omega^2 l^2 \int_0^1 \begin{bmatrix} A_i\xi(1-\xi^2) + A_j\xi^2(1-\xi) \\ \\ A_i\xi^2(1-\xi) + A_j\xi^3 \end{bmatrix} d\xi$$

$$= \rho\omega^2 l^2 \begin{bmatrix} \frac{1}{12}A_i + \frac{1}{12}A_j \\ \\ \frac{1}{12}A_i + \frac{1}{4}A_j \end{bmatrix}$$

By considering it as prismatic bar $A = A_i = A_j$ we get,

$$\begin{bmatrix} f^e = \begin{bmatrix} \frac{1}{6}A \\ \\ \frac{1}{3}A \end{bmatrix} \end{bmatrix}$$