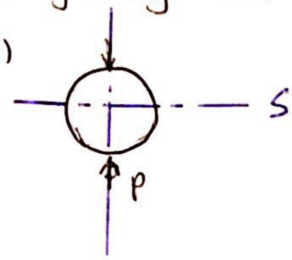
Assignment 2 Shushu Qin

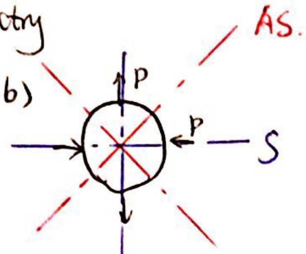
2.1 FEM Modelling.

1. S: Symmetry AS: Antisymmetry

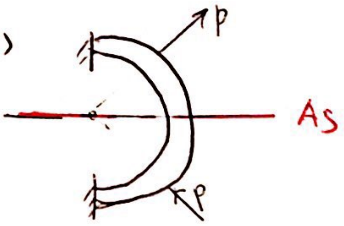
(a)



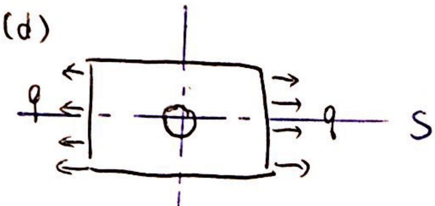
(b)



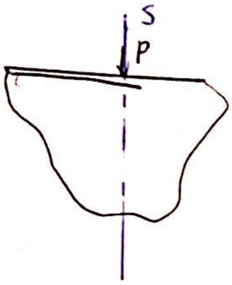
(c)



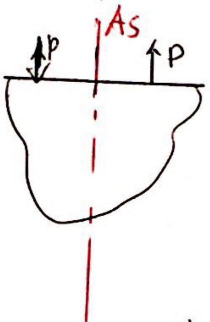
(d)



(e)



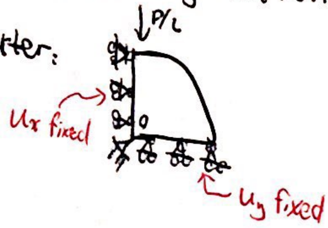
(f)



2. (a) We can cut the complete structure into one half or one quarter. The boundary conditions are applied so that there is no displacement perpendicular to the symmetry line. Node 0 is fixed to avoid free body motion.

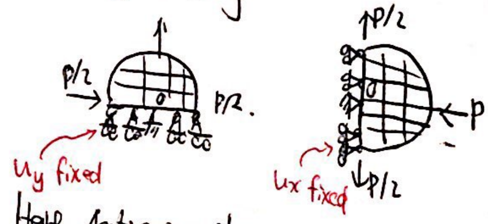
Half:

Quarter:

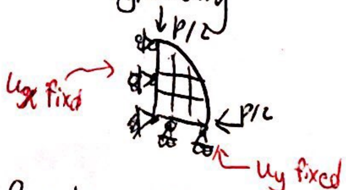


(b) The structure can be cut into half or quarter along the symmetry, antisymmetry line. Node 0 is fixed to avoid free body motion

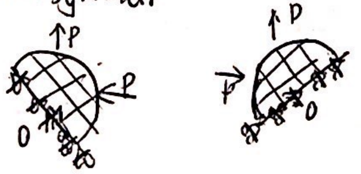
Half Symmetry



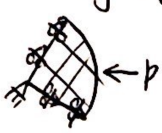
Quarter Symmetry



Half Antisymmetr

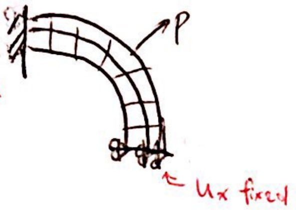


Quarter antisymmety

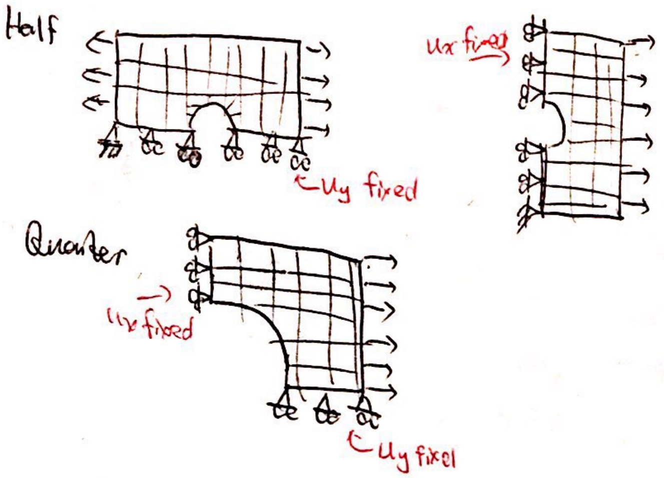


no displacement parellel to the antisymmety line

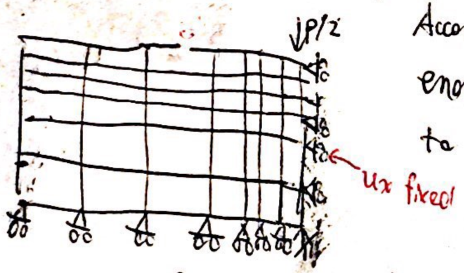
(C) The structure can be cut into half. No displacement parellel to the anti-symmetry line.



(d) The structure can be cut into half or quarter along the symmetry line.

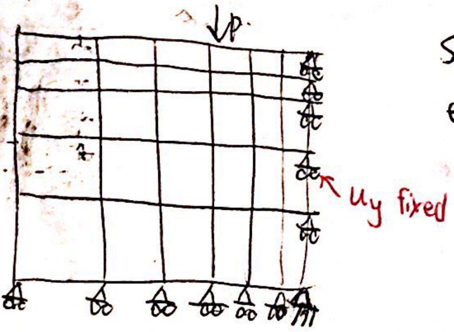


(e) The structure can be cut into half along the symmetry line



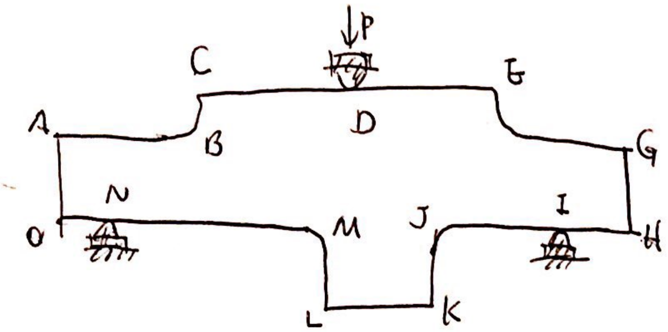
According to Saint-Venant's principle, if the half plate is large enough, the results from simulation will be similar to the analytical one

(f) The structure can be cut into half along the antisymmetry line



Same reason as (e), when the plate is large enough, the results will be accurate

2.2



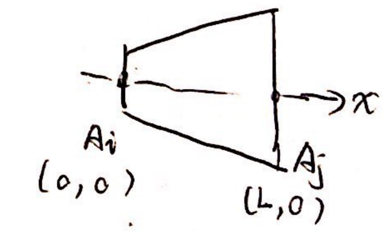
The trouble spots are

D: Vicinity of concentrated loads

B, M, J, F: entrant corners.

N, I: Vicinity of concentrated reactions

2.3 Variational Formulation



$A = A_i(1-\xi) + A_j\xi$

$q(x) = \rho A w^2 x$

natural coordinate $\xi = \frac{x}{\ell}$.

The consistent force vector is calcuted from

$W^e = \int_0^L q u\,dx = \int_0^1 q \underline{N}^T \underline{u}^e \ell\,d\xi = (\underline{u}^e)^T \int_0^1 q \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} L\,d\xi = (\underline{u}^e)^T \underline{f}^e$ $\underline{u}^e$ is arbitrary

$$\underline{f}^e = \int_0^1 q \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} L\,d\xi$$

$$= \int_0^1 \rho A w^2 x \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} L\,d\xi$$

$$= \int_0^1 \rho A w^2 L^2 \xi \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} d\xi$$

$$= \rho w^2 L^2 \int_0^1 \{A_i(1-\xi) + A_j\xi\} \xi \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} d\xi$$

$$= \rho w^2 L^2 \begin{bmatrix} \frac{A_i}{12} + \frac{A_j}{12} \\ \frac{A_i}{12} + \frac{A_j}{4} \end{bmatrix}$$

If $A_i = A_j = A$, $\underline{f}^e = \rho w^2 L^2 \begin{bmatrix} \frac{A}{6} \\ \frac{A}{3} \end{bmatrix}$