UNIVERSITAT POLITÈCNICA DE CATALUNYA
BARCELONATECH

Universitat Politècnica de Catalunya

Escola Tècnica Superior d'Enginyers de Camins, Canals i Ports

MASTER EN INGENIERÍA ESTRUCTURAL Y DE LA CONSTRUCCIÓN

Asignatura:

COMPUTATIONAL STRUCTURAL MECHANICS AND DYNAMICS

Assignment 2

On "FEM Modelling: Introduction"

And "Variational Formulation"

By

Sierra, Pablo Leonel

Assignment 2.1:

On "FEM Modelling: Introduction":

Exercise 1

Identify the symmetry and antisymmetry lines in the two-dimensional problems illustrated in the figure. They are:

(a) a circular disk under two diametrically opposite point forces (the famous "Brazilian test" for concrete)

(b) the same disk under two diametrically opposite force pairs

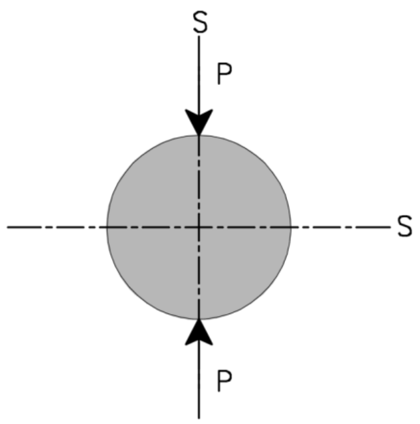
(c) a clamped semiannulus under a force pair oriented as shown

(d) a stretched rectangular plate with a central circular hole.

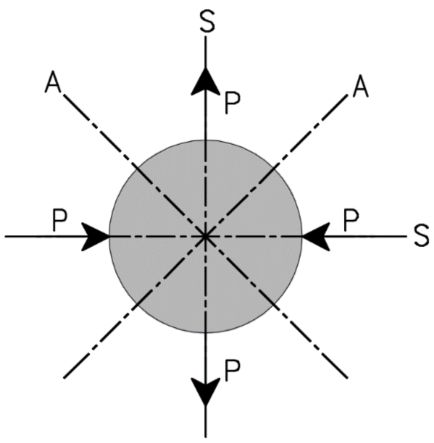
(e) and (f) are half-planes under concentrated loads.

In the figures the solutions. S: symmetry line, A: antisymmetric line

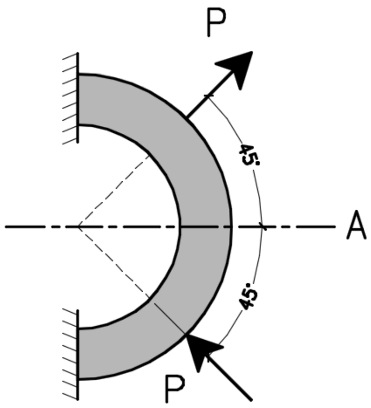
(a)



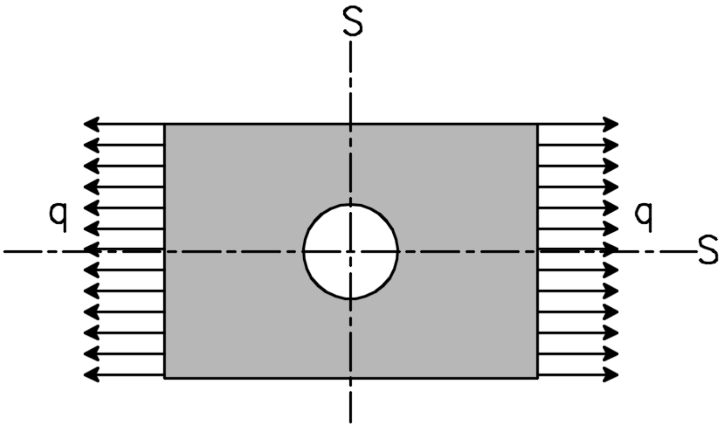
(b)



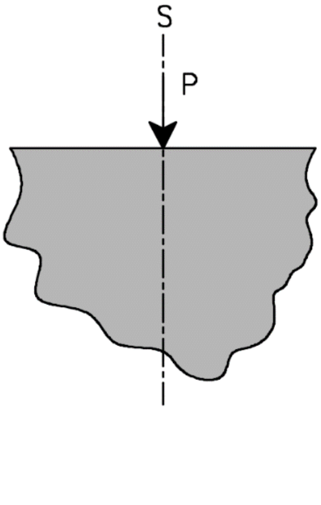
(c)



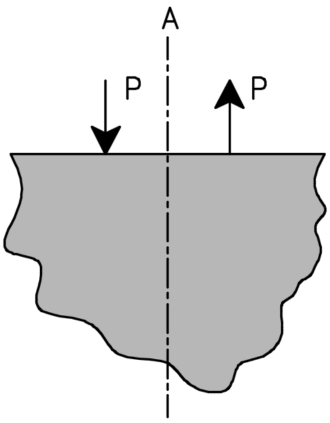
(d)



(e)



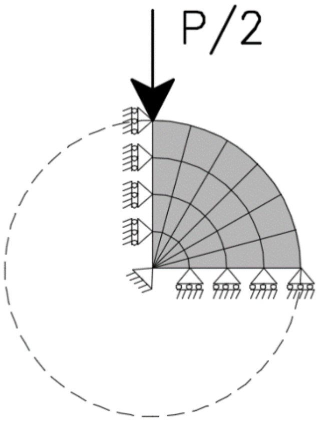
(f)



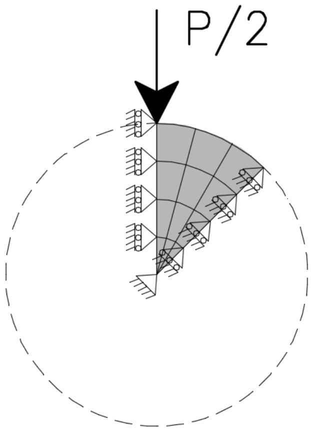
Exercise 2

Having identified those symmetry/antisymmetry lines, state whether it is possible to cut the complete structure to one half or one quarter before laying out a finite element mesh. Then draw a coarse FE mesh indicating, with rollers or fixed supports, which kind of displacement BCs you would specify on the symmetry or antisymmetry lines.

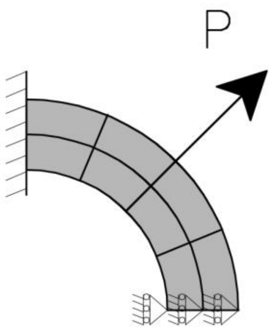
(a) On double symmetric axes



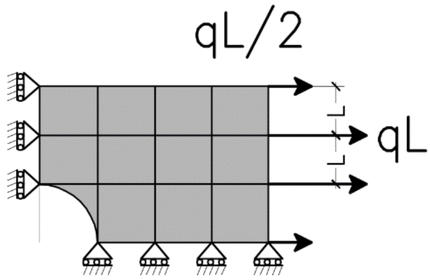
(b) Using one symmetric and one anti-symmetric axes



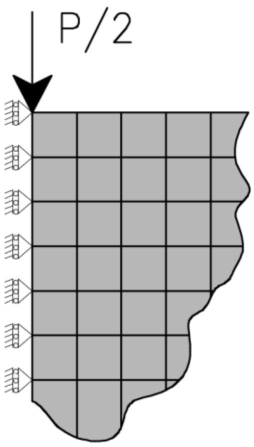
(c) On single anti-symmetric axes



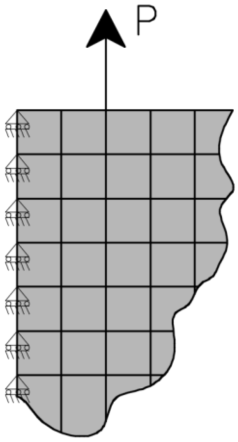
(d) On double symmetric axes



(e) On single symmetric load distribution



(f) On single anti-symmetric load distribution



Assignment 2.2:

On “FEM Modelling: Introduction”:

Exercise 1

The plate structure shown in the figure is loaded and deforms in the plane of the paper. The applied load at D and the supports at I and N extend over a fairly narrow area. List what you think are the likely “trouble spots” that would require a locally finer finite element mesh to capture high stress gradients. Identify those spots by its letter and a reason.

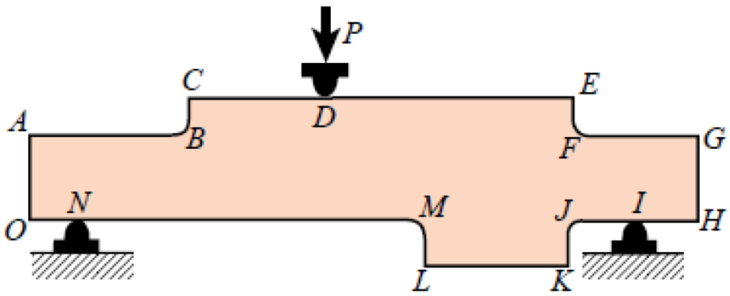


Figure 2.2.- Inplane bent plate

Trouble spots:

- Points D, N and I: there are concentred loads on that points.
- Points B, F, M and J: abrupt change of section of the piece.

Assignment 2.3:

On “Variational Formulation”:

Exercise 1

A tapered bar element of length l and areas A_i and A_j with A interpolated as

$$A = A_i(1 - \xi) + A_j\xi$$

and constant density ρ rotates on a plane at uniform angular velocity ω (rad/sec) about node i. Taking axis x along the rotating bar with origin at node i, the centrifugal axial force is $q(x) = \rho A \omega^2 x$ along the length in which x is the longitudinal coordinate $x = x^e$.

Find the consistent node forces as functions of ρ, A_i, A_j, ω and l, and specialize the result to the prismatic bar A = A_i= A_j.

First step is change variables on $q(x)$

$$x^e = x - x_i$$
$$\xi = \frac{x - x_i}{l} \rightarrow x^e = \xi l$$

Replacing on $q(x)$ equation

$$q(x^e) => q(\xi) = \rho A(\xi)\omega^2 \xi l$$

So, using the form equations in slides, the equation for consistent node forces is

$$\boldsymbol{f_{ext}} = \int_0^1 q \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l d\xi = \int_0^1 \rho\big(A_i(1-\xi) + A_j\xi\big)\omega^2 \xi \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} l^2 d\xi$$

Solving for each component

$$f_{ext\,i} = \int_0^1 \rho\big(A_i(1-\xi) + A_j\xi\big)\omega^2 \xi(1-\xi) l^2 d\xi = \frac{1}{12}\rho\omega^2 l^2 \big(A_i + A_j\big)$$

$$f_{ext\,j} = \int_0^1 \rho\big(A_i(1-\xi) + A_j\xi\big)\omega^2 \xi^2 l^2 d\xi = \frac{1}{12}\rho\omega^2 l^2 \big(A_i + 3\,A_j\big)$$

$$\boldsymbol{f_{ext}} = \frac{1}{12}\rho\omega^2 l^2 \begin{pmatrix} A_i + A_j \\ A_i + 3A_j \end{pmatrix}$$

If A = A_i= A_j

$$\boldsymbol{f_{ext}} = \frac{1}{12}\rho\omega^2 l^2 \begin{pmatrix} 2A \\ 4A \end{pmatrix} = \frac{1}{6}\rho\omega^2 l^2 \begin{pmatrix} A \\ 2A \end{pmatrix}$$