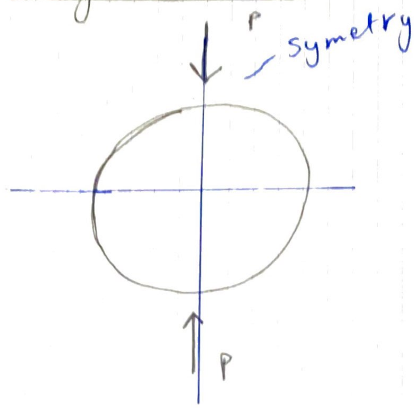
Nicolas Grønland

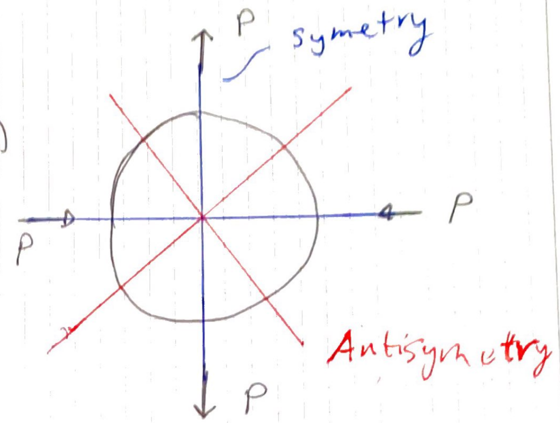
Assignment 2.1

1)

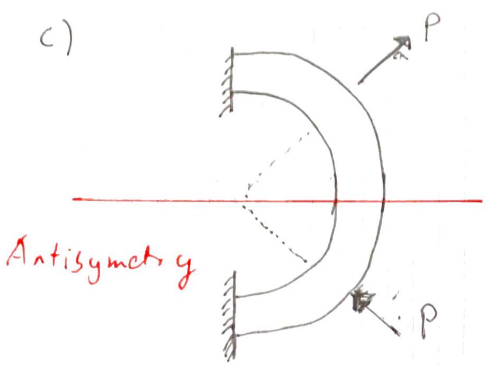
a)



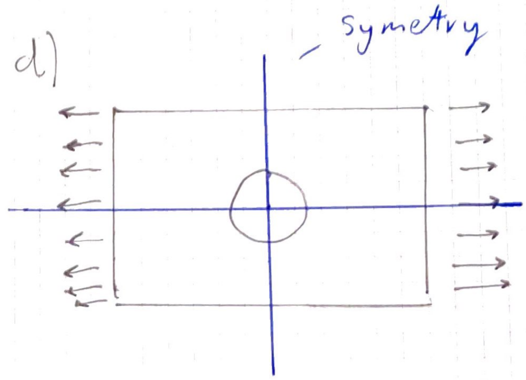

b)



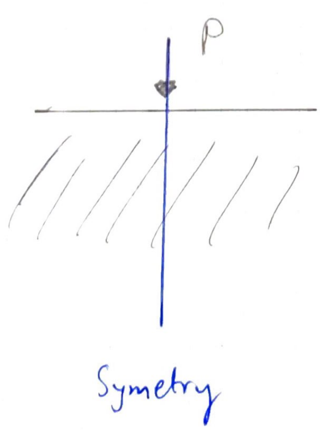

c)



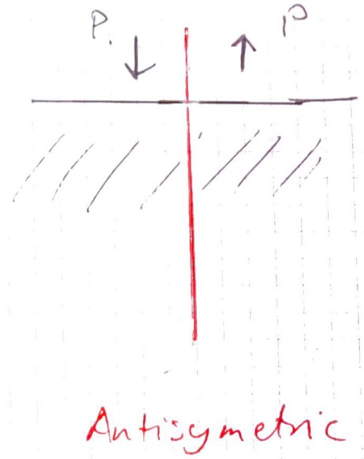

d)




e)



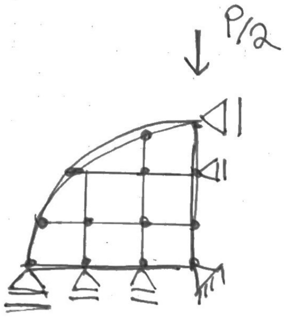

f)



2)

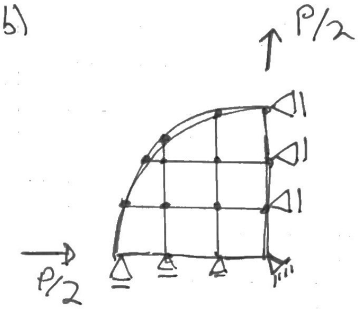
Quarter

a)



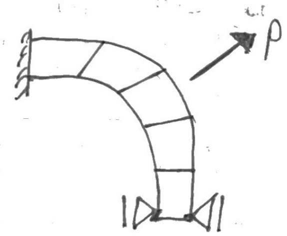

Quarter

b)



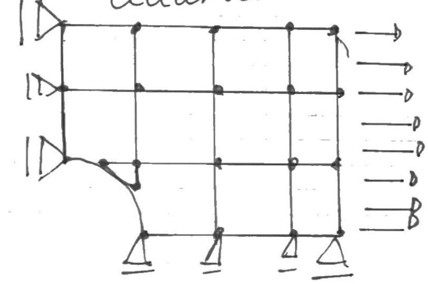

c)

Half

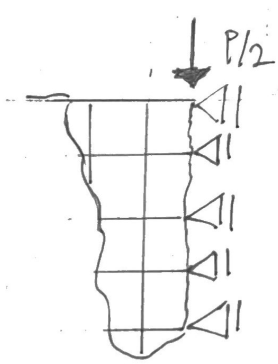



d)

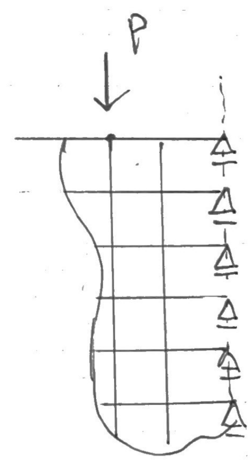
Quarter



e)




f)




For c) the force on the other side of the a-symetry line will prevent horizontal movement.

For f) we have the same argument, but here vertical movement is prevented.

## Assignment 2.2.

- Verification is about the checking process that should be done when using FEM. This means:
  - Making sure it works
  - Getting the math right
  - Providing an accurate analysis.

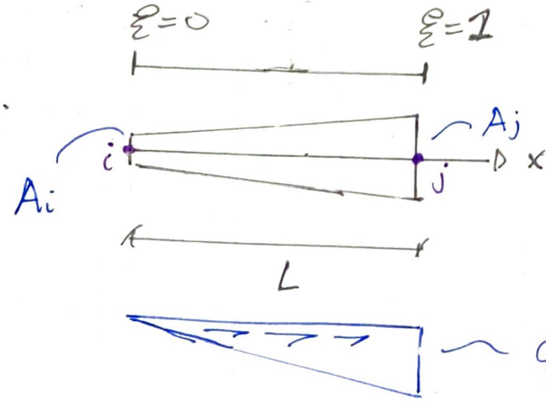
⇒ Does your analysis accurately represent the mathematical problem that you have formulated

- On the other hand validation is about checking if the analysis coincide with avalible data.

  ⇒ Is the analysis a correct represen-tation of the physics and the actual physical problem.

## Assignment 2.3

1.




$A = A_i(1-\xi) + A_j \xi$

$\xi = \frac{x}{L} \Rightarrow x = \xi L$

$q(x) = \rho \cdot A(x) \cdot \omega^2 . x$

Transforming everything to natural coordinates:

$$A = A_i\,(1-\xi) + A_j \cdot \xi$$

$$q(\xi) = \rho \cdot \omega^2 \cdot A(\xi) \cdot \xi \cdot L$$

$$\frac{dx}{d\xi} = L \quad \Rightarrow \quad dx = d\xi \cdot L$$

Consistent nodal force vector:

$$f_{ext} = \int_0^L q(x) \cdot N(x)\, dx$$

Defining shape functions based on natural coordinates:

$$N_i\,(\xi) = 1 - \xi$$

$$N_j\,(\xi) = \xi$$

$$\Rightarrow f_{ext} = \int_0^L q(x)\, \mathbb{N}(x)\, dx$$

$$= \int_0^1 q(\xi) \cdot \mathbb{N}(\xi) \cdot L \cdot d\xi$$

$$= \int_0^1 \rho \cdot \omega^2 \cdot L \cdot \xi \left( A_i (1-\xi) + A_j \cdot \xi \right) \cdot \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} \cdot L \cdot d\xi$$

$$= \rho \cdot \omega^2 \cdot L^2 \int_0^1 \left( A_i (\xi - \xi^2) + A_j \cdot \xi^2 \right) \begin{bmatrix} 1-\xi \\ \xi \end{bmatrix} d\xi$$

$$= \rho \cdot \omega^2 \cdot L^2 \int_0^1 \begin{bmatrix} A_i (\xi - \xi^2)(1-\xi) + A_j \xi^2 (1-\xi) \\ A_i (\xi^2 - \xi^3) + A_j \xi^3 \end{bmatrix} d\xi$$

$$= \rho \omega^2 \cdot L^2 \int_0^1 \begin{bmatrix} A_i (\xi - \xi^2 - \xi^2 + \xi^3) + A_j (\xi^2 - \xi^3) \\ A_i (\xi^2 - \xi^3) + A_j \xi^3 \end{bmatrix} d\xi$$

$$= \rho \cdot \omega^2 \cdot L^2 \begin{bmatrix} A_i \left( 1/2 - 2 \cdot 1/3 + 1/4 \right) + A_j (1/3 - 1/4) \\ A_i (1/3 - 1/4) + A_j \cdot 1/4 \end{bmatrix}$$

$$= \rho \cdot \omega^2 \cdot L^2 \begin{bmatrix} A_i \cdot \frac{1}{12} + A_j \cdot \frac{1}{12} \\ A_i \cdot \frac{1}{12} + A_j \cdot \frac{1}{4} \end{bmatrix} = \begin{bmatrix} f_{ext,1} \\ f_{ext,2} \end{bmatrix}$$

Specializing the result to $A_i = A_j = A$ :

$$\Rightarrow f_{ext} = \rho \cdot w^2 \cdot L^2 \cdot A \begin{bmatrix} 1/12 + 1/12 \\ 1/12 + 1/4 \end{bmatrix}$$

$$= \rho \cdot w^2 \cdot L^2 \cdot A \begin{bmatrix} 1/6 \\ 1/3 \end{bmatrix}$$