Computational Structural Mechanics and Dynamics - Homework 4

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1 Introduction

This report describes the solution of Assignment 4 in the subject Computational Structural Mechanics and Dynamics.

Contents

2 Exercise 1

All calculations in this exercise is with the use of the program Matlab, but the conceptual thinking along with the results will be presented here. The script will be attached at the end of the assignment.

2.1 1.1

To calculate the stiffness matrix for a axisymetric triangle I use the formula.

$$K = 2\pi \int_A B^T E B r dr dz \tag{1}$$

Where the matrix B is a function of r and z expressed in the form.

$$\mathbf{B}^e = \frac{1}{2A}\begin{bmatrix} z_{23} & 0 & z_{31} & 0 & z_{12} & 0 \\ 0 & r_{32} & 0 & r_{13} & 0 & r_{21} \\ 2A\,\zeta_1/r & 0 & 2A\,\zeta_2/r & 0 & 2A\,\zeta_3/r & 0 \\ r_{32} & z_{23} & r_{13} & z_{31} & r_{21} & z_{12} \end{bmatrix}$$

Which is derived from the strain-displacement relation, by derivation of the shape function given in the form.

$$\begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \zeta_3 \end{bmatrix} = \frac{1}{2A}\begin{bmatrix} r_2 z_3 - r_3 z_2 & z_2 - z_3 & r_3 - r_2 \\ r_3 z_1 - r_1 z_3 & z_3 - z_1 & r_1 - r_3 \\ r_1 z_2 - r_2 z_1 & z_1 - z_2 & r_2 - r_1 \end{bmatrix}\begin{bmatrix} 1 \\ r \\ z \end{bmatrix}$$

I have in this exercised derived the stiffness matrix by evaluating B for a centroidal point ($\bar{r}$,$\bar{z}$) where $\bar{r}$ and $\bar{z}$ is.

$$r = \bar{r} = \frac{r_i + r_j + r_m}{3} \qquad z = \bar{z} = \frac{z_i + z_j + z_m}{3}$$

Giving an expression for the stiffness matrix as.

$$K = 2\pi A \bar{B^T} E \bar{B} \bar{r} \tag{2}$$

Thereby, replacing the values for the variable r and z with the values for the values for $\bar{r}$ and $\bar{z}$. Doing the matrix multiplications in Matlab I get the stiffness matrix as.

$$\text{K} = E\pi \begin{bmatrix} \frac{b}{6} & 0 & -\frac{b}{2} & 0 & \frac{b}{6} & 0 \\ 0 & \frac{b}{3} & \frac{a}{3} & -\frac{b}{3} & -\frac{a}{3} & 0 \\ -\frac{b}{2} & \frac{a}{3} & \frac{a^2}{3b}+\frac{5b}{6} & -\frac{a}{3} & \frac{b}{6}-\frac{a^2}{3b} & 0 \\ 0 & -\frac{b}{3} & -\frac{a}{3} & \frac{2a^2}{3b}+\frac{b}{3} & \frac{a}{3} & -\frac{2a^2}{3b} \\ \frac{b}{6} & -\frac{b}{3} & \frac{b}{6}-\frac{a^2}{3b} & \frac{a}{3} & \frac{a^2}{3b}+\frac{b}{6} & 0 \\ 0 & 0 & 0 & -\frac{2a^2}{3b} & 0 & \frac{2a^2}{3b} \end{bmatrix}$$

2.2 1.2

As seen from the stiffness matrix - if you sum up the values of columns (and rows) 2, 4 and 6 you get the sum 0. The reason for this is that the columns corresponds to the translation in z-direction. Meaning that if there is an equal free body motion in z-direction, then the force is equal to zero, and we have no additional strain.

On the other hand, if you sum up the values for columns 1,3 and 5 you don't get 0 because the columns corresponds with radius. We know that if the radius changes the area will have a change of strain because of the radial strain where you divide the displacement on r.

2.3 1.3

Considering we only have the body force acting on the triangle $b = [0, -g]^T$, we could write the force vector as.

$$f = 2\pi \int_A [N]^T b r dr dz \tag{3}$$

Where [N] is the shape functions all ready calculated and used in the first exercise.
$N_1 = 1 - r/a$
$N_2 = r/a - z/b$
$N_3 = z/b$

Inserting the shape functions into equation (3) and again calculating by evaluating for the centroidal point ($\bar{r}$,$\bar{z}$) we get that the force vector is expressed as.

$$\mathrm{f} = 2\pi A \begin{bmatrix} 0 \\ -\dfrac{ag}{9} \\ 0 \\ -\dfrac{ag}{9} \\ 0 \\ -\dfrac{ag}{9} \end{bmatrix}$$

```
clear;

syms a b E r z

%Points
r1=0;
z1=0;
r2=a;
z2=0;
r3=a;
z3=b;

%Values in the B-matrix.
A = b*a/2;
z23 = z2-z3;
z31 = z3-z1;
z12 = z1-z2;
r32 = r3-r2;
r13 = r1-r3;
r21 = r2-r1;

%Centroidal method
r_line = (r1+r2+r3)/3;
z_line = (z1+z2+z3)/3;

%Calculating values in the B-matrix.
X1 = (r2*z3-r3*z2)/r_line + (z2-z3) + (r3-r2)*z_line/r_line;
X2 = (r1*z3-r3*z1)/r_line + (z3-z1) + (r1-r3)*z_line/r_line;
X3 = (r1*z2-r2*z1)/r_line + (z1-z2) + (r2-r1)*z_line/r_line;

B = 1/(2*A)*[z23 0 z31 0 z12 0; 0 r32 0 r13 0 r21; X1 0 X2 0 X3 0; r32 z23 r13 z31 r21 z12];
B_trans = transpose(B);

E = [E 0 0 0; 0 E 0 0; 0 0 E 0; 0 0 0 E/2];

K = 2*pi*r_line*A*B_trans*E*B;

%Checking if the sum of columns 2,4 and 6 is equal to 0.
K(1,2)+K(1,4)+K(1,6);
K(2,2)+K(2,4)+K(2,6);
K(3,2)+K(3,4)+K(3,6);
K(4,2)+K(4,4)+K(4,6);
K(5,2)+K(5,4)+K(5,6);
K(6,2)+K(6,4)+K(6,6);

%Calculation of the force vector.

syms g

%Shape functions
N_1 = 1-r_line/a;
N_2 = r_line/a-z_line/b;
N_3 = z_line/b;

f_1 = [0; 2*pi*A*N_1*r_line*(-g)];
f_2 = [0; 2*pi*A*N_2*r_line*(-g)];
f_3 = [0; 2*pi*A*N_3*r_line*(-g)];

f = [f_1;f_2;f_3];
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