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Assignment 5.1

From assignment 4.1 we know that

$$J = \frac{dx}{d\xi} = \frac{l}{2}(1-4\xi\alpha) \qquad -\frac{1}{2}<\alpha<\frac{1}{2}$$

$$1-4\xi\alpha=0$$

$$\xi = \frac{1}{4\alpha} \qquad -1 \le \xi \le 1$$

$$-1 \le \frac{1}{4\alpha} \le 1 \qquad \alpha>0$$

$$-4\alpha \le 1 \le 4\alpha$$

$$-4\alpha \le 1 \qquad -\alpha \le \frac{1}{4} \to \alpha \ge -\frac{1}{4}$$

$$4\alpha \ge 1 \to \alpha \ge \frac{1}{4}$$

$\alpha<0$

$$-4\alpha \ge 1 \ge 4\alpha$$

$$-4\alpha \ge 1 \to \alpha \le -\frac{1}{4}$$

$$4\alpha \le 1 \to \alpha \le \frac{1}{4}$$

$\alpha<0$ and $\alpha \le -\frac{1}{4}$ and $\alpha \le \frac{1}{4} \to \alpha \le -\frac{1}{4}$

Hence if $|\alpha| \ge \frac{1}{4}$ there will be at least one point (ξ) in wich $J=0$.

At the end points $\xi=1$ and $\xi=-1$:

$$e = \frac{du}{dx} = \frac{du}{d\xi}\cdot\frac{d\xi}{dx} = \frac{du}{d\xi}\cdot\frac{1}{J}$$

If $\alpha = \frac{1}{4} \to$ at $\xi=1$, $J=0$ and

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$$\frac{du}{d\varepsilon} = u_1 \cdot \frac{d\left(\frac{1}{2}(\varepsilon^2 - \varepsilon)\right)}{d\varepsilon} + u_2 \frac{d\left(\frac{1}{2}(\varepsilon^2 + \varepsilon)\right)}{d\varepsilon} + u_3 \frac{d(-\varepsilon^2 + 1)}{d\varepsilon}$$

$$\frac{du}{d\varepsilon} = u_1\left(\varepsilon - \frac{1}{2}\right) + u_2\left(\varepsilon + \frac{1}{2}\right) + u_3(-2\varepsilon)$$

At $\varepsilon = 1$

$$\frac{du}{d\varepsilon} = \frac{u_1}{2} + \frac{3}{2}u_2 - 2u_3$$

At $\varepsilon = -1$

$$\frac{du}{d\varepsilon} = -\frac{3}{2}u_1 - \frac{1}{2}u_2 + 2u_3$$

→ finite number

$$e(1) = \left(\frac{u_1}{2} + \frac{3}{2}u_2 - 2u_3\right) \cdot \frac{-1}{J}$$

if $\alpha \to \frac{1}{4}$, $J \to 0$ and $e \to \infty$

$$e(-1) = -\frac{3}{2}u_1 - \frac{1}{2}u_2 + 2u_3$$

if $\alpha \to \frac{-1}{4}$, $J \to 0$ and $e \to \infty$

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$N_4 = \eta\,\varepsilon(\varepsilon-1)(\eta+1)$

$N_7 = (\varepsilon+1)(\varepsilon-1)(\eta+1)\eta$

$$\frac{\partial N_3}{\partial \eta} = \frac{\partial}{\partial \eta}\left(\frac{1}{4}(1+\varepsilon)(1+\eta)\eta\varepsilon\right) = \frac{(2\eta+1)(\varepsilon+1)\varepsilon}{4}$$

$$\frac{\partial N_4}{\partial \varepsilon} = \frac{\partial}{\partial \varepsilon}\left(\frac{-1}{4}(\varepsilon-1)(\eta+1)\eta\,\varepsilon\right) = -\frac{(2\varepsilon-1)(\eta+1)\eta}{4}$$

$$\frac{\partial N_4}{\partial \eta} = \frac{-1(2\eta+1)(\varepsilon-1)\varepsilon}{4}$$

$$\frac{\partial N_5}{\partial \varepsilon} = \frac{\partial}{\partial \varepsilon}\left[\frac{-1}{2}(1-\varepsilon^2)(1-\eta)\cdot\eta\right] = \varepsilon(1-\eta)\eta$$

$$\frac{\partial N_5}{\partial \eta} = \frac{-1}{2}(1-2\eta)\cdot(1-\varepsilon^2)$$

$$\frac{\partial N_6}{\partial \varepsilon} = \frac{\partial}{\partial \varepsilon}\left(\frac{1}{2}(1+\varepsilon)(1-\eta^2)\varepsilon\right) = \frac{(2\varepsilon+1)(1-\eta^2)}{2}$$

$$\frac{\partial N_6}{\partial \eta} = -1\cdot(1+\varepsilon)\varepsilon\eta$$

$$\frac{\partial N_7}{\partial \varepsilon} = \frac{\partial}{\partial \varepsilon}\left[-\frac{1}{2}(\varepsilon^2-1)(\eta+1)\eta\right] = -1\cdot(\eta+1)\eta\varepsilon$$

$$\frac{\partial N_7}{\partial \eta} = -\frac{(2\eta+1)(\varepsilon^2-1)}{2}$$

$$\frac{\partial N_8}{\partial \varepsilon} = \frac{\partial}{\partial \varepsilon}\left[\frac{1}{2}(\varepsilon-1)\varepsilon(1-\eta^2)\right] = \frac{(2\varepsilon-1)(1-\eta^2)\varepsilon}{2}$$

$N_3 = \frac{1}{4}\varepsilon(\varepsilon+1)\eta(\eta-1)$

$$\frac{\partial N_6}{\partial \eta} = -1\cdot(\varepsilon-1)\varepsilon\eta \qquad \varepsilon =$$

$$\frac{\partial N_8}{\partial \varepsilon} = \frac{\partial}{\partial \varepsilon}\left[(1-\varepsilon^2)(1-\eta^2)\right] = -2\varepsilon(1-\eta^2)$$

$$\frac{\partial N_8}{\partial \eta} = -2(1-\varepsilon^2)\cdot\eta$$

$$\frac{\partial x}{\partial \varepsilon}(\varepsilon,-1) = \left[-\frac{(2\varepsilon-1)\eta(\eta-1)}{4} - \frac{(2\varepsilon+1)(1-\eta)\eta}{4} + \right.$$

$$+ \frac{(2\varepsilon+1)\cancelto{0}{(\eta+1)}\eta}{4} + \frac{(2\varepsilon-1)\cancelto{0}{(\eta+1)}\eta}{4} + 2\alpha\varepsilon(1-$$

$$\left. + \frac{(2\varepsilon+1)\cancelto{0}{(1-\eta^2)}}{2} - \frac{(2\varepsilon-1)\cancelto{0}{(1-\eta^2)\varepsilon}}{2}\right] \cdot \frac{e}{2}$$

$$= \frac{\partial x}{\partial \varepsilon}(\varepsilon,-1) = \left[-\frac{(2\varepsilon-1)}{2} + \frac{(2\varepsilon+1)}{2} - 4\alpha\varepsilon\right]\cdot\frac{e}{2}$$

$$= \frac{e}{2}\left[1-4\alpha\varepsilon\right]$$

$$\frac{\partial y}{\partial \varepsilon}(\varepsilon,-1) = \frac{e}{2}\left[-1\cdot\frac{(2\varepsilon-1)}{2} - \frac{(2\varepsilon+1)}{2} + 2\varepsilon\right]$$

$$= \frac{e}{2}\left[-\varepsilon+\frac{1}{2}-\varepsilon-\frac{1}{2}+2\varepsilon\right]$$

$$= \frac{e}{2}\cdot 0$$

$$= 0$$

to compute J there's no need to know $\frac{\partial x}{\partial \eta}$

$$\frac{\partial y}{\partial \eta} = \sum_{1}^{3} y_i \frac{\partial \psi}{\partial \eta}$$

$$\frac{\partial y}{\partial \eta} = \frac{e}{2}\left[(-1)\frac{(2\eta-1)\,\varepsilon\,(\varepsilon-1)}{4} + (-1).(-1)\frac{(1-2\eta)(1+\varepsilon)\varepsilon}{4} \right.$$

$$+ \frac{(2\eta+1)(\varepsilon+1)\,\varepsilon}{4} - \frac{1.(2\eta+1)(\varepsilon-1)\,\varepsilon}{4} +$$

$$\left. (-1).\left(-\frac{1}{2}\right)(1-2\eta)(1-\varepsilon^2) - \frac{(2\eta+1)(\varepsilon^2-1)}{2} \right]$$

$$\frac{\partial y}{\partial \eta}(\varepsilon, -1) = \frac{e}{2}\left[\frac{3}{4}(\varepsilon).(\varepsilon-1) + \frac{3(1+\varepsilon)\varepsilon}{4} \right.$$

$$\left. + \frac{(-1)(\varepsilon+1)\varepsilon}{4} + \frac{(\varepsilon-1)}{4}\varepsilon + \frac{3}{2}(1-\varepsilon^2) + \frac{(\varepsilon^2-1)}{2} \right]$$

$$\frac{\partial y}{\partial \eta}(\varepsilon, -1) = \frac{e}{2}\left[(\varepsilon-1)\varepsilon + \frac{1}{2}(\varepsilon+1)\varepsilon + (1-\varepsilon^2) \right]$$

$$= \frac{e}{2}\left[\cancel{\varepsilon^2} - \varepsilon + \frac{1}{2}\varepsilon^2 + \frac{\varepsilon}{2} + 1 - \cancel{\varepsilon^2} \right]$$

$$= \frac{e}{2}\left[\frac{\varepsilon^2 - \varepsilon}{2} + 1 \right]$$

$$J = \frac{e}{2}\begin{bmatrix} 1-4\alpha\varepsilon & 0 \\ \frac{\partial x}{\partial \eta} & \frac{\varepsilon^2-\varepsilon+1}{2} \end{bmatrix}$$

$$\det J = (1-4\alpha\varepsilon)\left(\frac{\varepsilon^2}{2} - \frac{\varepsilon}{2} + 1\right)$$

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det $J = 0$ if

$\boxed{1 - 4\alpha\varepsilon = 0}$

$\frac{\varepsilon^2 - \varepsilon + 1}{2} = \frac{\varepsilon^2}{2} + 1 - \varepsilon$ with $-1 \leq \varepsilon \leq 1$

$\frac{\varepsilon^2 - \varepsilon + 1}{2} > 0$ for all ε / $-1 \leq \varepsilon \leq 1$

As it was discussed in assignment 5.1

for $|\alpha| \geq \frac{1}{4}$ there will be at least one point

point (ε) in wich $J = 0$.

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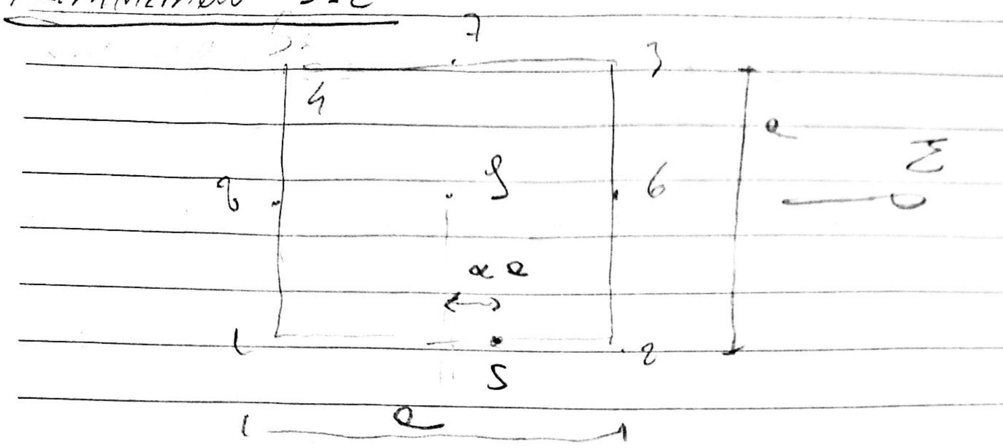
Assignment 5.2



$N_3 = \frac{1}{4}\eta \cdot \xi(\xi+1)\cdot(\eta+1)$

$N_5 = c \cdot \eta \cdot (\xi+1)(\xi-1)(\eta-1)$

$N_5(0,-1) = c \cdot 1 \cdot (1) \cdot (-1)(-2) = 2c \Rightarrow c = \frac{1}{2}$

$N_5 = \frac{-1}{2}\eta(\eta-1)(\xi^2-1)$

$$J = \begin{pmatrix} \frac{\partial x}{\partial \xi} & \frac{\partial y}{\partial \xi} \\ \frac{\partial x}{\partial \eta} & \frac{\partial y}{\partial \eta} \end{pmatrix} = PX = \begin{bmatrix} \frac{\partial N_1}{\partial \xi} & \frac{\partial N_2}{\partial \xi} & \cdots & \frac{\partial N_8}{\partial \xi} \\ \frac{\partial N_1}{\partial \eta} & \frac{\partial N_2}{\partial \eta} & \cdots & \frac{\partial N_8}{\partial \eta} \end{bmatrix} \cdot \frac{e}{2} \begin{bmatrix} -1 & -1 \\ 1 & -1 \\ 1 & 1 \\ -1 & 1 \\ 2\alpha & -1 \\ 1 & 0 \\ 0 & 1 \\ -1 & 0 \\ 0 & 0 \end{bmatrix}$$

$\frac{\partial N_1}{\partial \xi} = \frac{\partial}{\partial \xi}\left(\frac{1}{4}(\xi-1)(\eta-1)\xi\eta\right) = \frac{(2\xi-1)\eta(\eta-1)}{4}$

$\frac{\partial N_1}{\partial \eta} = \frac{(2\eta-1)\xi(\xi-1)}{4}$

$\frac{\partial N_2}{\partial \xi} = \frac{\partial}{\partial \xi}\left(\frac{-1}{4}(\xi+1)(1-\eta)\xi\eta\right) = -\frac{(2\xi+1)(1-\eta)\eta}{4}$

$\frac{\partial N_2}{\partial \eta} = \frac{-1(1-2\eta)(1+\xi)\cdot\xi}{4}$

$\frac{\partial N_3}{\partial \xi} = \frac{\partial}{\partial \xi}\left(\frac{1}{4}(\xi+1)(\eta+1)\xi\eta\right) = \frac{(2\xi+1)(\eta+1)\eta}{4}$