UNIVERSITAT POLITÈCNICA DE CATALUNYA

MASTER IN COMPUTATION MECHANICS AND NUMERICAL METHODS IN ENGINEERING

COMPUTATIONAL STRUCTURAL MECHANICS AND DYNAMICS

Assignment 6

by

Renan Alessio

Barcelona, March of 2020

INDEX

1- Introduction

The goal of the assignment is to apply the concepts of the beam theories. Among the beam theories, the Euler-Bernoulli theory for thin beams and the Timoshenko theory for thick beams are considered. The Euler-Bernoulli beam element and the Timoshenko beam element with full and reduced integrations were considered in the present study.

2 – Assignment A

To consider the effect of the reduced integration in the shear stiffness matrix from the Timoshenko beam [1], the following shear stiffness matrix was implemented in the code:

```
const =   D_mats/len;

K_s = [   1    ,    len/2 ,    -1    ,    len/2 ;
        len/2 , len^2/4 , -len/2 , len^2/4 ;
         -1    ,  -len/2 ,     1    ,  -len/2 ;
        len/2 , len^2/4 , -len/2 ,  len^2/4 ];

K_s = K_s * const;
```

Figure 1. Implemented shear stiffness matrix to consider the reduced integration.

3 – Assignment B

Considering the data provided in the assignment [1], Tables 1 presents the values of maximum deflection and rotation for the different values of ratio a/L considering 64 2-noded Euler-Bernoulli beam elements. Tables 2 and 3 present the values of maximum deflection, rotation and shear for the different values of ratio a/L considering 64 2-noded Timoshenko beam elements with full and reduced integration.

Table 1. Maximum deflections and rotations for 2-noded Euler-Bernoulli element for different ratios a/L.

a	**a/L**	**Maximum deflection [mm]**	**Maximum rotation**
0.001	0.00025	-1904700000.00	1523800000.00
0.005	0.00125	-3047600.00	2438000.00
0.01	0.0025	-190476.19	152380.95
0.02	0.005	-11904.76	9523.81
0.05	0.0125	-304.76	243.81
0.1	0.025	-19.05	15.24
0.2	0.05	-1.19	0.95
0.4	0.1	-0.07	0.06

Table 2. Maximum deflections, rotations and shear for 2-noded Timoshenko beam element with full integration for different rations a/L.

a	**a/L**	**Maximum deflection [mm]**	**Maximum rotation**	**Maximum shear [N]**
0.001	0.00025	-1461449.26	1169102.13	8179.78
0.005	0.00125	-57400.98	45918.37	8031.89
0.01	0.0025	-13582.78	10865.53	7602.35
0.02	0.005	-2797.31	2237.61	6262.75
0.05	0.0125	-200.43	160.27	2804.74
0.1	0.025	-16.88	13.48	944.84
0.2	0.05	-1.16	0.92	259.98
0.4	0.1	-0.08	0.06	68.03

Table 3. Maximum deflections, rotations and shear for 2-noded Timoshenko beam element with reduced integration for different ratios a/L.

a	**a/L**	**Maximum deflection [mm]**	**Maximum rotation**	**Maximum shear [N]**
0.001	0.00025	-1904018258.39	1523437626.83	10656400.00
0.005	0.00125	-3046440.00	2437500.00	426257.00
0.01	0.0025	-190404.64	152343.75	106566.00
0.02	0.005	-11900.83	9521.48	26642.90
0.05	0.0125	-304.76	243.75	4264.52
0.1	0.025	-19.07	15.23	1067.61
0.2	0.05	-1.20	0.95	268.38
0.4	0.1	-0.08	0.06	68.57

According to Tables 1-3, for a ratio a/L of 0.00025 the mesh with 2-noded Timoshenko beam elements with full integration has a maximum deflection three orders smaller when compared to the other elements. Such behavior is due to shear locking effect, which happens to the Timoshenko beam element when it is fully integrated and applied to a slender beam [1]. When both conditions arise together, the shear stiffness matrix components increase greatly, neglecting the components of the curvature stiffness matrix and greatly decreasing the displacements. Since the beam is slender enough and bending is its main effect, the shear stiffness matrix influence on the final system of equations should vanish and the curvature stiffness matrix should control the system [1]. Therefore, when the 2-noded Timoshenko beam element with full integration is used to discretize a slender beam, spurious results are obtained unless a great number of elements is used [1]. Another example of the shear locking effect may be seen in the evolution of the maximum internal bending moment as the ratio a/L increases considering the mesh with the three types of beam elements. Figure 2 presents the evolution of the maximum internal bending moment for the three types of elements.

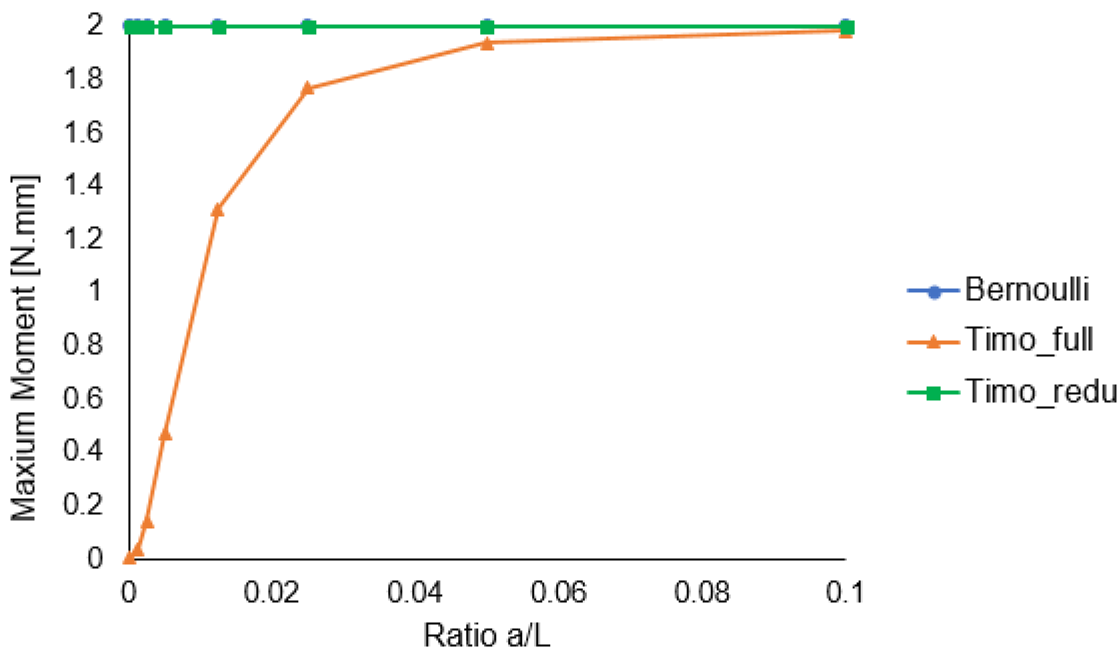

Figure 2. Evolution of maximum interval bending moment with increasing a/L for the three types of elements.

According to Figure 2, the Timoshenko beam element with full integration ("Timo_full") present a much smaller internal bending moment for smaller values of the ratio a/L. Such behavior is a consequence of the shear locking effect, which decreases the maximum rotations and consequently decreases the maximum internal bending moment [1].

The Euler-Bernoulli element ("Bernoulli") and the Timoshenko beam element with reduced integration ("Timo_redu") presented an accurate result for maximum internal bending moment for all values of a/L (Figure 2) since the analytical result is 2 N.mm. Such observation highlights how the reduced integration of the Timoshenko beam element diminishes the shear locking effect. The same behavior is seen when the values presented in Tables 1 and 3 are compared. The values for maximum deflection and rotation are in agreement for both elements. Therefore, the reduced integration of the Timoshenko beam element turns it suitable for applications in both thick and slender beams.

The maximum shear force presented in Tables 2 and 3 diverges considerably from the analytical value (2 N) for both configurations of the Timoshenko beam element for all values of a/L. Such result suggests that a higher number of elements should be considered to better capture the shear effects in the beam. Nevertheless, as the ratio a/L increases (the beam becomes thicker), the three elements present converging results for maximum bending moment, maximum deflection and maximum rotation. Such behavior is expected since the ratio a/L approaches the limit value (0.1) which distinguishes a slender beam from a thick beam. If the ratio a/L was increased beyond 0.1, the shear effect would become more important and the Euler-Bernoulli beam element would not be suitable for such scenario.

4 – Conclusion

The results presented in the current study showed that it is crucial to choose the proper beam element for the considered case. The Euler-Bernoulli beam element presented accurate results for the maximum internal bending moment for the present case since different configurations of slender beams were considered. The Timoshenko beam element with full integration presented spurious results as the beam became slenderer, highlighting the shear locking effect in such scenarios. The reduced integration of the Timoshenko beam element diminished the shear locking effect, enabling accurate results for the maximum internal bending moment for all a/L values considered. Nevertheless, both configurations of the Timoshenko beam element were unable to capture representative results for internal shear forces for the considered values of a/L. Such behavior suggests that a greater number of elements should be considered for each value of a/L to aim for better results regarding internal shear force.

5 – References

[1] – Presentation “TEMA 07_NEW”, Computational Structural Mechanics and Dynamics, Master of Science in Computational Mechanics, 2020.