Comptutational Structural Mechanics and Dynamics

Assignment 6 – Beams

By Trond Jørgen Opheim

The assignment consists of two parts:

- Change the script using the Timoshenko 2-noded beam element with full integration to a script using reduced integration for the same element
- Solve the problem “simply supported beam with uniformly distributed load” with three different solutions, that is 2-noded Euler-Bernoulli element, 2-noded Timoshenko Full Integrate element and 2-nded Timoshenko Reduced Integration element.

1. Program in Matlab the Timoshenko 2-noded beam element with reduced integration for the shear stiffness matrix

As stated in the slides “Tema 07_NEW_High_Res.pdf” on CIMNE, is the reduced integration for 2-Noded Timoshenko element” done by changing the shear stiffness matrix in the script. Since we are only interested in the displacement and moment and shear distribution in this exercise, I do not change anything in the script for computing stresses.

$$K_{s,full}^{(e)} = \left(\frac{GA^*}{l}\right)^{(e)} \begin{bmatrix} 1 & \frac{l^{(e)}}{2} & -1 & \frac{l^{(e)}}{2} \\ \dots & \frac{\left(l^{(e)}\right)^2}{3} & -\frac{l^{(e)}}{2} & \frac{\left(l^{(e)}\right)^2}{6} \\ \dots & \dots & 1 & -\frac{l^{(e)}}{2} \\ Sym. & \dots & \dots & \frac{\left(l^{(e)}\right)^2}{6} \end{bmatrix}$$

$$K_{s,red} = \left(\frac{GA^*}{l}\right)^{(e)} \begin{bmatrix} 1 & \frac{l^{(e)}}{2} & -1 & \frac{l^{(e)}}{2} \\ \dots & \frac{\left(l^{(e)}\right)^2}{4} & -\frac{l^{(e)}}{2} & \frac{\left(l^{(e)}\right)^2}{4} \\ \dots & \dots & 1 & -\frac{l^{(e)}}{2} \\ Sym. & \dots & \dots & \frac{\left(l^{(e)}\right)^2}{4} \end{bmatrix}$$

2. Solve the following problem with a 64 element mesh with the

- 2-noded Euler Bernoulli element
- 2-noded Timoshenko Full Integrate element
- 2-noded Timoshenko Reduced Integration element

and compare maximum displacements, moments and shear for the different a/L relationship.

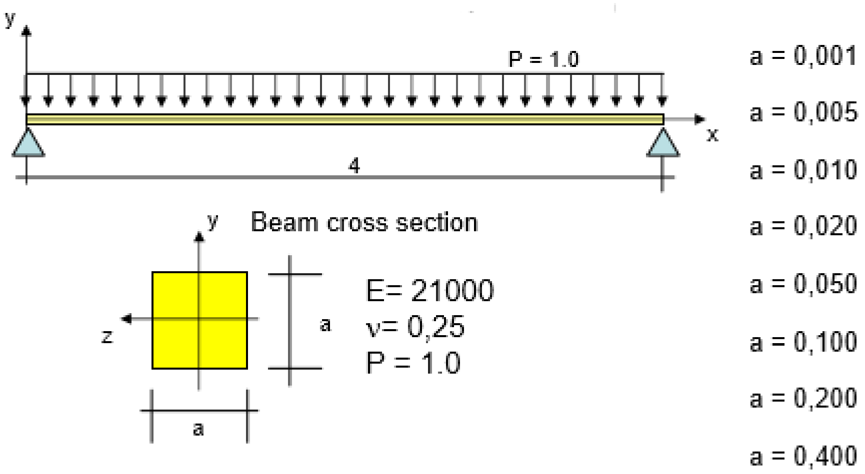

Figure 1 - Problem to be solved with geometry and material properties

To use the given scripts for solving the problem, I use the input file "SimpleSupUL_Beam_64" where all the material properties in the problem is assigned. By changing some variable names and adding a uniformly distributed load instead of point load, is the input file ready to be used in the scripts.

```
young   =   2.1000e+11 ;
poiss   =  2.500000000e-01 ;
denss   =   0.000000000e+00 ;
a = 0.4;                %here given for the side length to be 0.4
area    =   a*a ;
inertia=   1/12*a^4 ;
P = -1;


pointload = [ ];
uniload = P*ones(1,64);
```

After running the script are the results written into a file which is opened in GID. Here I read visualize the displacement and withdraw the interesting sizes, maximum displacement, moment and shear.

Results

Maximum displacement			
a	Euler-Bernoulli	Timo,full	Timo,red
0,001	1,90E+02	1,46E-01	1,90E+02
0,005	3,05E-01	5,74E-03	3,05E-01
0,010	1,90E-02	1,36E-03	1,90E-02
0,020	1,19E-03	2,80E-04	1,19E-04
0,050	3,05E-05	2,00E-05	3,05E-05
0,100	1,90E-06	1,69E-06	1,91E-06
0,200	1,19E-07	1,16E-07	1,20E-07
0,400	7,44E-09	7,56E-09	7,62E-09

Analytic solution	
Euler-Bernoulli	Timoshenko
1,90E+02	1,90E+02
3,0476E-01	3,05E-01
1,9048E-02	1,90E-02
1,1905E-03	1,19E-03
3,0476E-05	3,0488E-05
1,9048E-06	1,90762E-06
1,1905E-07	1,19762E-07
7,4405E-09	7,61905E-09

Maximum moment			
a	Euler	Timo,full	Timo,red
0,001	2,003	0,002	1,990
0,005	2,003	0,038	1,990
0,010	2,003	0,143	1,990
0,020	2,003	0,470	1,990
0,050	2,003	1,314	1,990
0,100	2,003	1,769	1,990
0,200	2,003	1,936	1,990
0,400	2,003	1,983	1,990

Maximum shear			
a	Euler	Timo,full	Timo,red
0,001	2,000	1,969	1,969
0,005	2,000	1,969	1,969
0,010	2,000	1,969	1,969
0,020	2,000	1,969	1,969
0,050	2,000	1,969	1,969
0,100	2,000	1,969	1,969
0,200	2,000	1,969	1,969
0,400	2,000	1,969	1,969

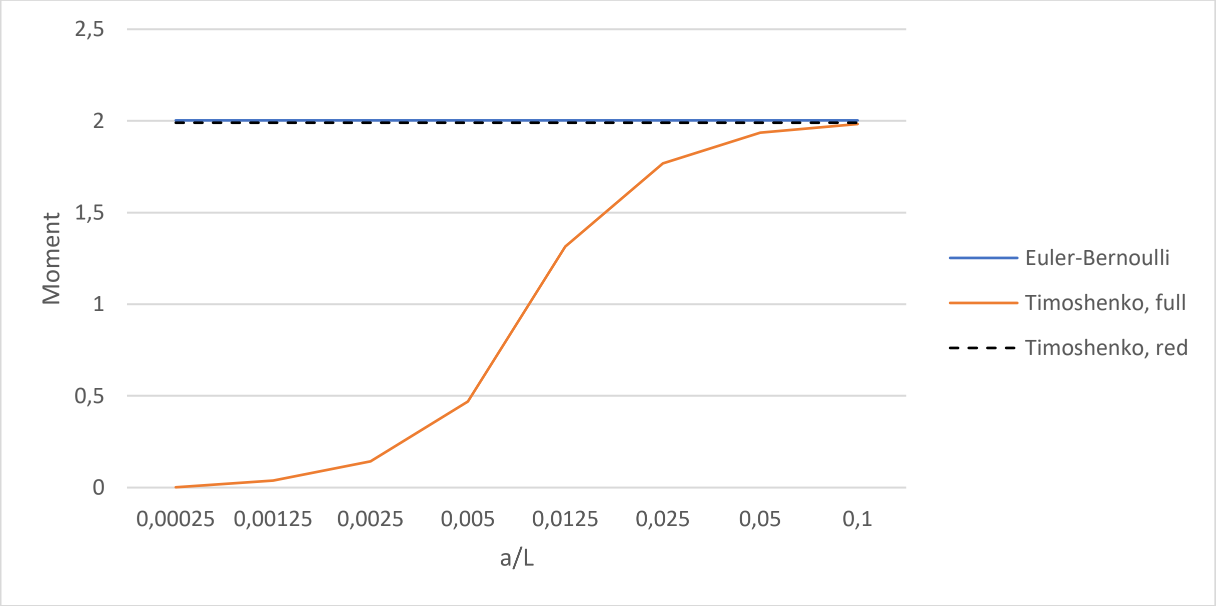

Figure 2 - Maximum moment in the beam for different a/L relationships for the three elements

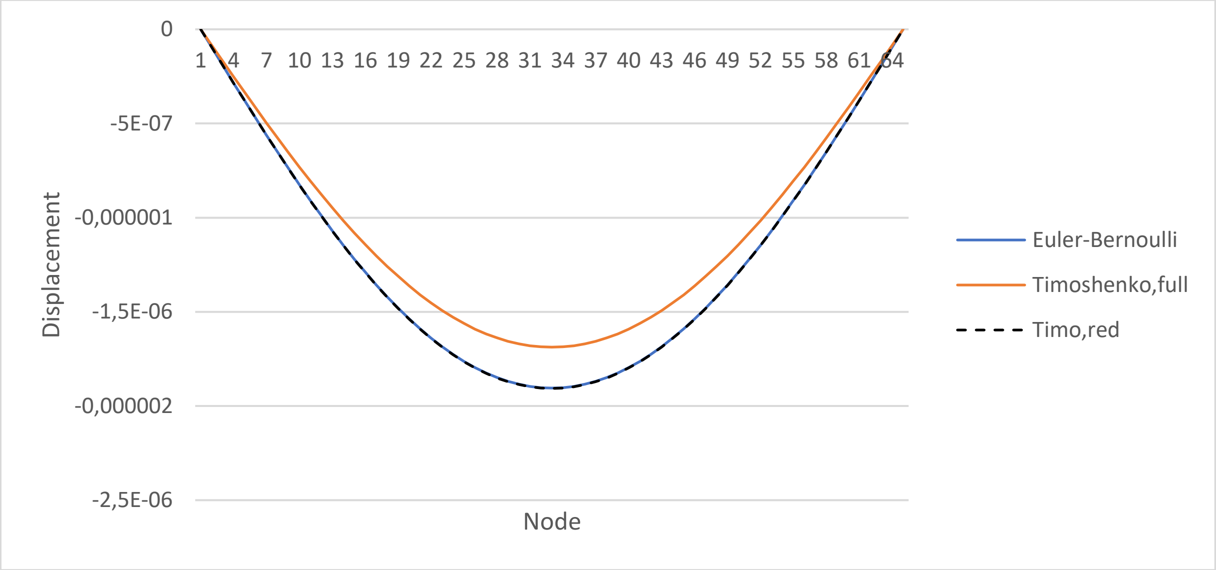

Figure 3 - Displacement at the nodes for a=0.1

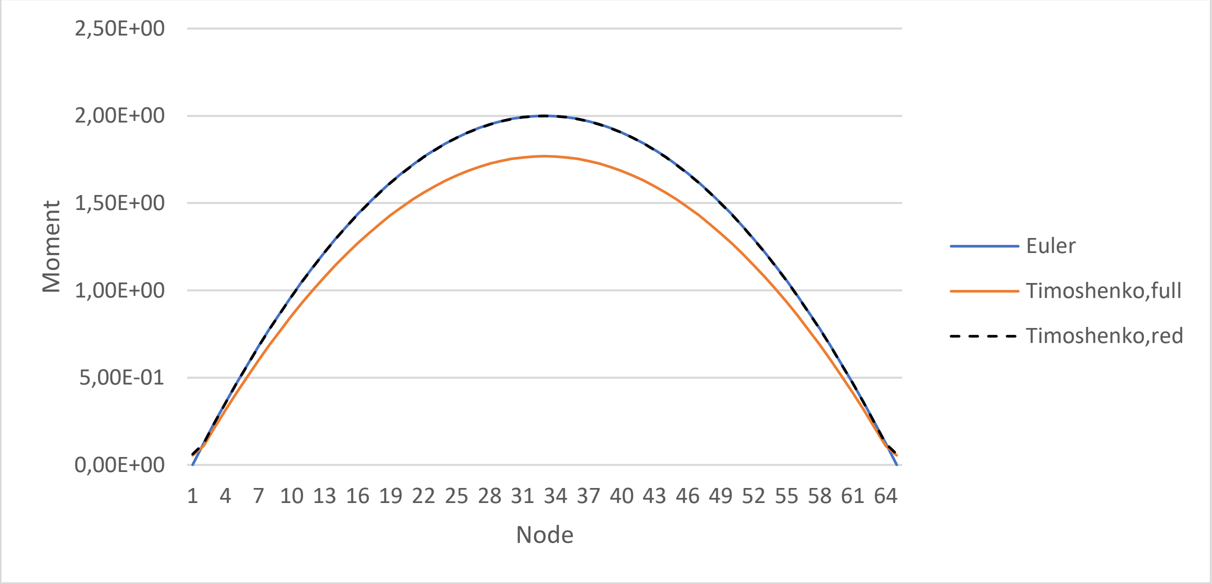

Figure 4 - Moment at the nodes for a=0.1

From the results obtained, is it possible to see that the 2-noded Timoshenko full integrated element underestimates the displacement and moment, but not the shear force. The interesting thing to see is how the moment value obtained from this method, gets closer to the exact solutions as the relationship a/L rises. This is shown in Figure 2.

The analytical solutions are attached in Table XX, and we can see that the values obtained from the Euler-Bernoulli element and the Timoshenko Reduced Integrated element corresponds with these.