Computational Structural Mechanics and Dynamics - Assignment Beams

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1 Introduction

This report describes the solution of the assignment concerning Beam theory in the subject Computational Structural Mechanics and Dynamics. First it will be described how the Timoshenko 2 Nodes Beam element with reduce integration was integrated. Then the results for the different method will be presented together with some discussion.

2 Assignment

The Timoshenko 2 nodes Beam with reduced integration was programmed with the use of Matlab. From the given matlab code for a full Timoshenko beam downloaded from CIMNE, the code could be altered so it worked with reduced integration instead. Firstly, the shear stiffness matrix was changed in the way given in the slides. Secondly, the number of Gauss points to integrate for the reduced method is only one point, as this is the minimum minimum order of integration required for exact evaluation of the strain energy.

All three method used the same input file describing the supported beam with a uniform load working in the y-direction. To solve the problem it was used 64 elements for the 4m long beam.

The objective of this assignment is to compare how the displacement, shear and moment changes for the different beam elements.

- 2 nodes Euler Bernoulli Element.
- 2 nodes Timoshenko Full integrate element.
- 2 nodes Timoshenko Reduce Integration element.

Below is tables showing the maximum displacement, shear and moments for the different elements with different heights of the cross section as explained in the assignment. The results will be discussed after the tables.

Table 1: Table showing the maximum displacement for the different methods for different sizes of the cross section.

Height	Euler Bernoulli	Timoshenko	Timoshenko Reduced
0.001	-190.4760	-0.1461	-190.4760
0.005	-0.3048	-0.0057	-0.3046
0.01	-0.0190	-0.0014	-0.0190
0.02	-0.0012	-2.7975E-04	-0.0012
0.05	-3.0467E-05	-2.0043E-05	-3.047E-05
0.1	-1.9046E-06	-1.6875E-06	-1.9069E-06
0.2	-1.1905E-07	-1.1596E-07	-1.1972E-06
0.4	-7.4405E-09	-7.5561E-09	-7.6161E-09

Table 2: Table showing the maximum shear forces. for the different methods for different sizes of the cross section.

Height	Euler Bernoulli	Timoshenko	Timoshenko Reduced
0.001	2.0	1.9688	1.9688
0.005	2.0	1.9688	1.9688
0.01	2.0	1.9688	1.9688
0.02	2.0	1.9688	1.9688
0.05	2.0	1.9688	1.9688
0.1	2.0	1.9688	1.9688
0.2	2.0	1.9688	1.9688
0.4	2.0	1.9688	1.9688

Table 3: Table showing the maximum moment for the different methods for different sizes of the cross section.

Height	Euler Bernoulli	Timoshenko	Timoshenko Reduced
0.001	2.0003	0.0015	1.9990
0.005	2.0003	0.0377	1.9990
0.01	2.0003	0.1426	1.9990
0.02	2.0003	0.4698	1.9990
0.05	2.0003	1.3144	1.9990
0.1	2.0003	1.7687	1.9990
0.2	2.0003	1.9360	1.9990
0.4	2.0003	1.9829	1.9990

As we could observe from the tables, the maximum displacement and moment with the use of the Timoshenko full integrate element, varies a lot from the maximum values obtained with Euler Bernoulli and Timoshenko reduced. At the same time we observe that the Timoshenko reduced and Euler Bernoulli conclude very well for the different heights.

The Euler Bernoulli element is best for "thin" beams since it does not take into account the shear deformations in the theory, while Timoshenko do. In general, it is often considered to use the Euler Bernoulli theory when the ratio between the length and the height is less than 10 and the displacements are small in comparison with the height of the beam. In this assignment we solve the problem for the beam up to a height of 0.4 which gives a ratio of $\frac{L}{H} = 10$. This can also be seen from our tables. When the height of the beam approaches this ratio, the maximum values with the Timoshenko element has approximately the same values as the Euler Bernoulli element. If we had solved the beam for even higher height of the cross section, the Timoshenko theory would have been preferred.

The Timoshenko reduced element was created to optimize the solution when using Timoshenko theory, and gives extraordinary good results. Even for thin beams the Timoshenko reduced element works very well and gives good results in comparison with the Euler Bernoulli element. It is also worth noticing that when the ratio of length and height is at 10 (height equal to 0.4) the Timoshenko reduce element results conclude more with the results for the Timoshenko element than for the Euler Bernoulli element, showing the strength of this method.

Below it is added plots for the displacement, shear distribution and moment distribution for three of the heights, showing how the solutions evolve based on the height of the cross section. As we could see from both the tables and the plots the values of the displacement gets very small when we increase the height of the cross section, since this increases the inertia with proportion of a^4. For many of the plots it is not possible to see the distributions with the Euler Bernoulli element. This is because the results for the Timoshenko reduced element where plotted on after the Euler Bernoulli, and since both elements match that well only the Timoshenko reduced element shows.

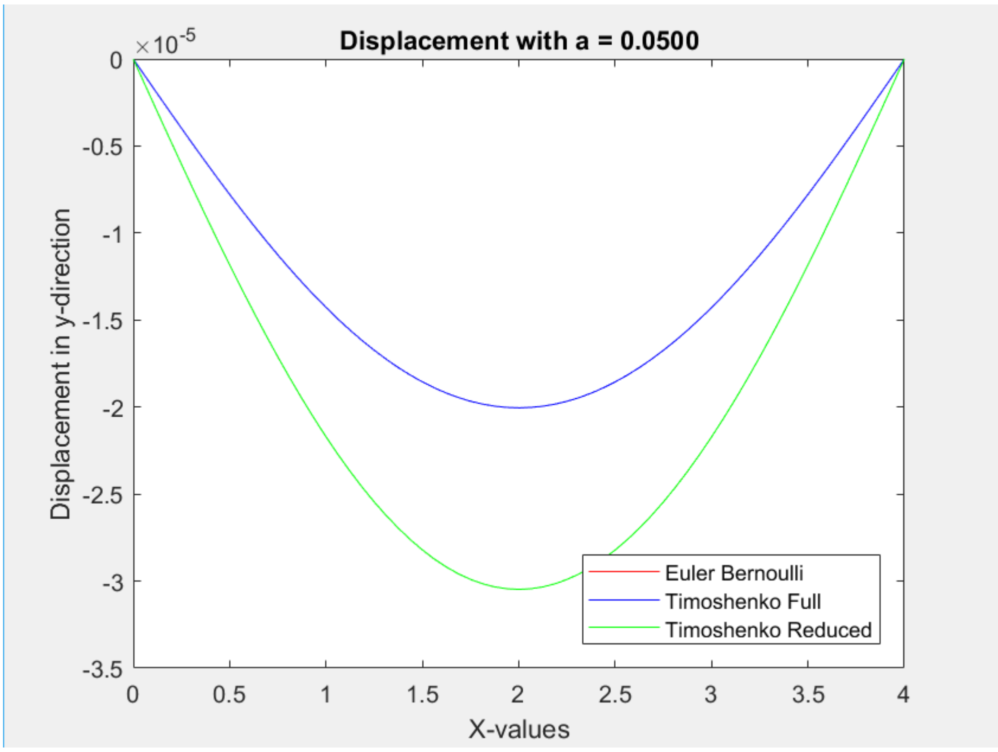

Figure 1: The displacement for the three different methods with a = 0.05.

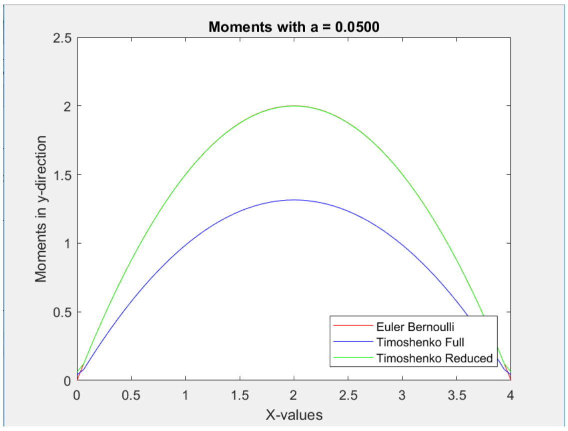

Figure 2: Moment distribution.

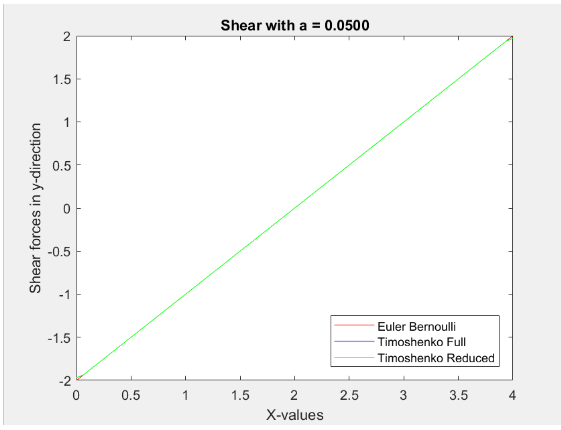

Figure 3: Shear distribution.

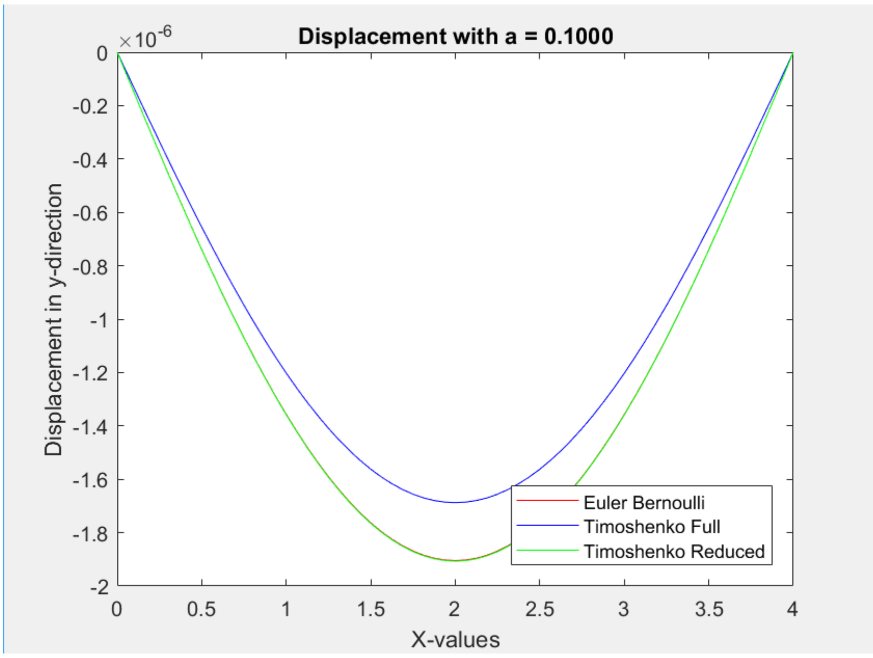

Figure 4: The displacement for the three different methods with a = 0.1.

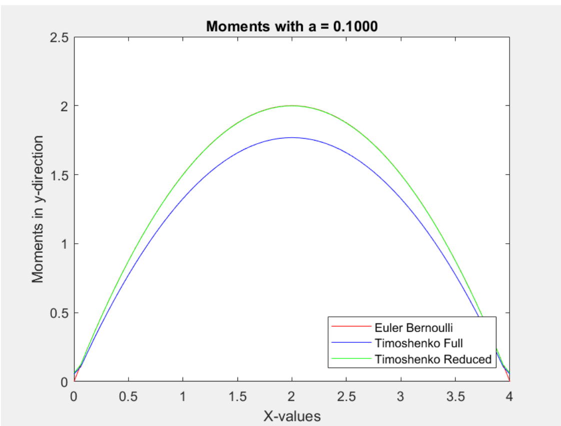

Figure 5: Moment distribution.

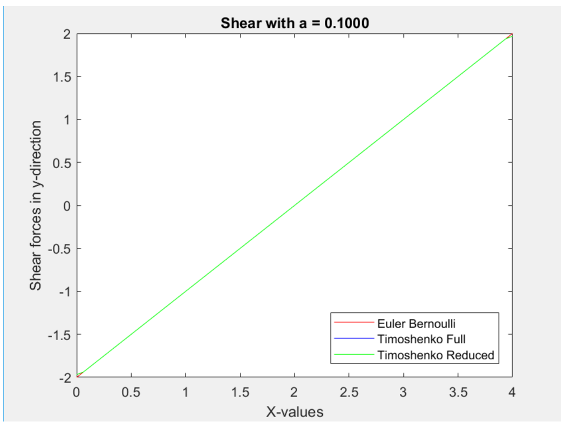

Figure 6: Shear distribution.

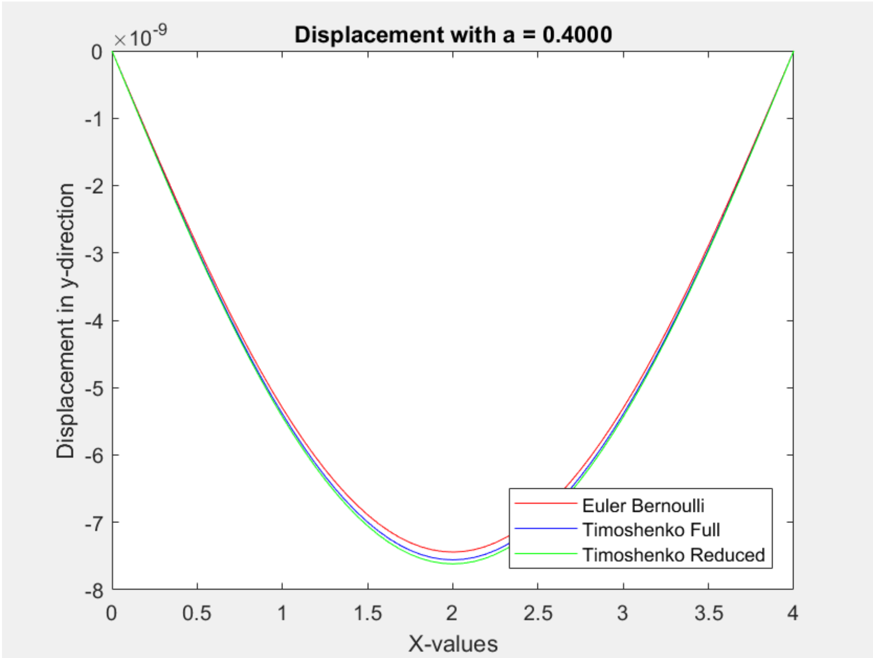

Figure 7: The displacement for the three different methods with a = 0.4.

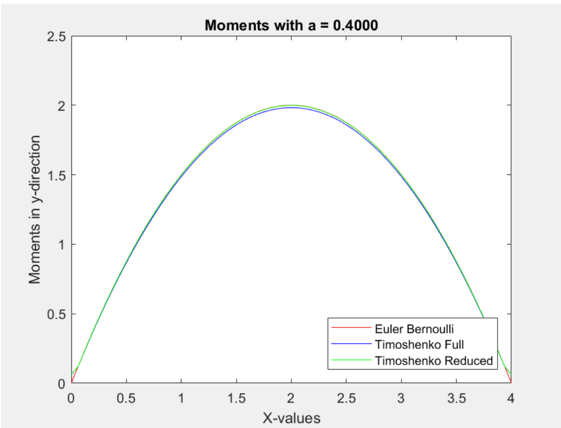

Figure 8: Moment distribution.

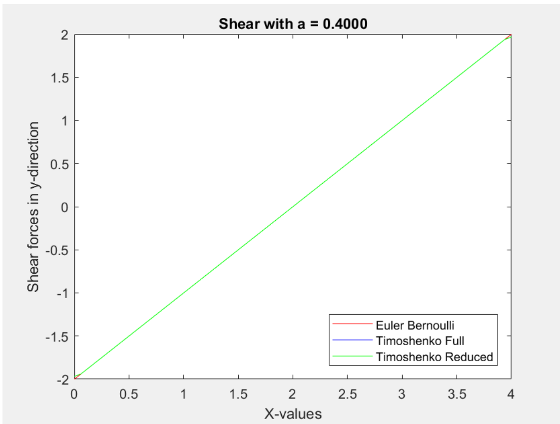

Figure 9: Shear distribution.