Computational Structural Mechanics and Dynamics

Masters in Numerical Methods

Assignment 6

Beam Theory

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1 Assignnment 6.1

1.1 Task

Program In MatLab the Timoshenko 2 Nodes Beam element with reduce integration for the shear stiffness matrix

1.2 Solution

The shear Stiffness matrix was changed to the follows in the code:

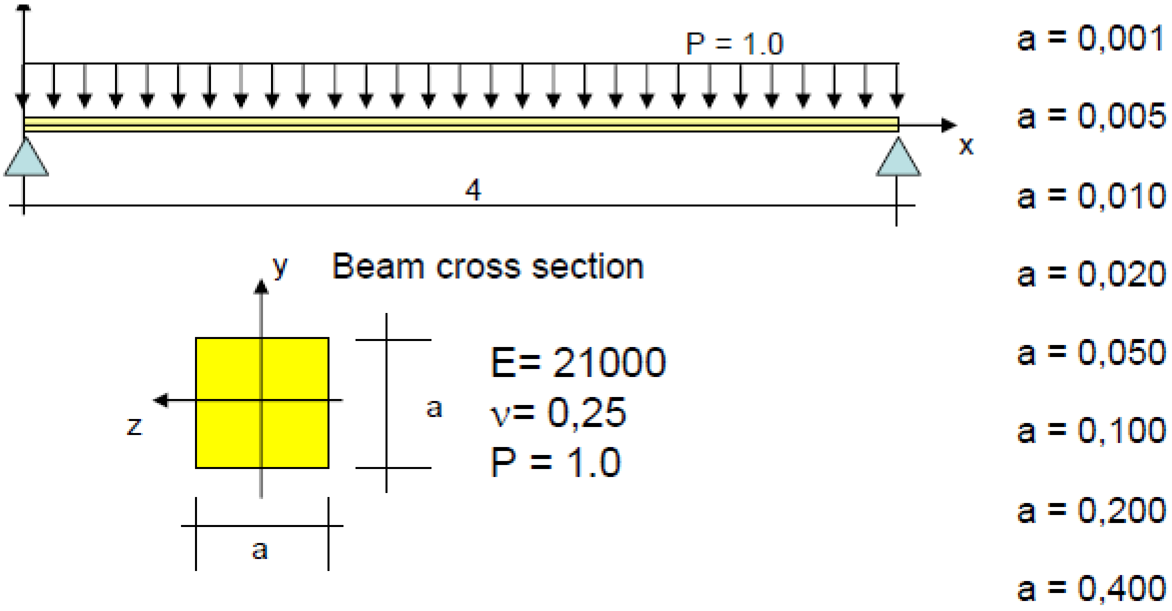
$$K_s = \begin{bmatrix} 1 & \frac{l}{2} & -1 & \frac{l}{2} \\ \frac{l}{2} & \frac{l^2}{4} & -\frac{l}{2} & \frac{l^2}{4} \\ -1 & -\frac{l}{2} & 1 & -\frac{l}{2} \\ \frac{l}{2} & \frac{l^2}{4} & -\frac{l}{2} & \frac{l^2}{4} \end{bmatrix} \tag{1}$$

2 Assignment 6.2

2.1 Task

Solve the following problem with

- Two noded Euler - Bernoulli Element
- Two noded Timoshenko Full Integrate Element
- Two noded Timoshenko Reduce Integrate Element



The MATFEM problem type for Euler Bernoulli and Timoshenko beams was used. The pre-processing such as defining the geometry and the mesh was performed once in GiD to obtain a matlab input data file for the solver programs, after this the variables of the data input file were modified for different values of a. The matlab programs for the Euler-Bernoulli,Timoshenko Full Integration and Timoshenko Reduce Integration cases generate a result file which was analysed for obtaining the maximum values of displacements, shear force and moments. A table for various values of a and the moment of inertia is presented in the fig 1. All the values were taken dimensionless, but the dimesions are consistent throughout the three cases.

L	a	Area	Moment of Inertia
4	0.001	1.00E-06	8.33E-14
4	0.005	2.50E-05	5.21E-11
4	0.01	1.00E-04	8.33E-10
4	0.02	4.00E-04	1.33E-08
4	0.05	0.0025	5.21E-07
4	0.1	0.01	8.33E-06
4	0.2	0.04	1.33E-04
4	0.4	0.16	2.13E-03

Table 1: Variation of length and Moment of Inertia

The results for the maximum displacement, Moment and Shear Force are tabulated below for each of the three cases.

a	a/L	Displacement	Mz	Shear Force
0.001	2.50E-04	1.90E+09	1.9999	2
0.005	1.25E-03	3.05E+06	1.9999	2
0.01	2.50E-03	1.90E+05	1.9999	2
0.02	5.00E-03	1.19E+04	1.9999	2
0.05	1.25E-02	3.05E+02	1.9999	2
0.1	2.50E-02	1.91E+01	1.9999	2
0.2	5.00E-02	1.4047	1.9999	2
0.4	1.00E-01	7.45E-02	1.9999	2

Table 2: Euler-Bernoulli

The variation of max displacement fig 1 and Mz moment fig 2 was plotted on a log-log scale for varied aspect ratio. It is clear that the Shear Force remains a constant as we change the aspect ratio in all three cases.

a	a/L	Displacement	Mz	Shear Force
0.001	2.50E-04	1.46E+06	0.0015	1.9687
0.005	1.25E-03	5.74E+04	0.0377	1.9687
0.01	2.50E-03	1.36E+04	0.1425	1.9687
0.02	5.00E-03	2.80E+03	0.47	1.9687
0.05	1.25E-02	2.00E+02	1.314	1.9687
0.1	2.50E-02	1.69E+01	1.7682	1.9687
0.2	5.00E-02	1.16E+00	1.98	1.9687
0.4	1.00E-01	7.56E-02	1.9829	1.9687

Table 3: Timoshenko

a	a/L	Displacement	Mz	Shear Force
0.001	2.50E-04	1.90E+09	1.999	1.9688
0.005	1.25E-03	3.05E+06	1.999	1.9688
0.01	2.50E-03	1.90E+05	1.999	1.9688
0.02	5.00E-03	1.19E+04	1.999	1.9688
0.05	1.25E-02	3.05E+02	1.999	1.9688
0.1	2.50E-02	1.91E+01	1.999	1.9688
0.2	5.00E-02	1.20E+00	1.999	1.9688
0.4	1.00E-01	7.62E-02	1.999	1.9688

Table 4: Timoshenko Reduced Integration

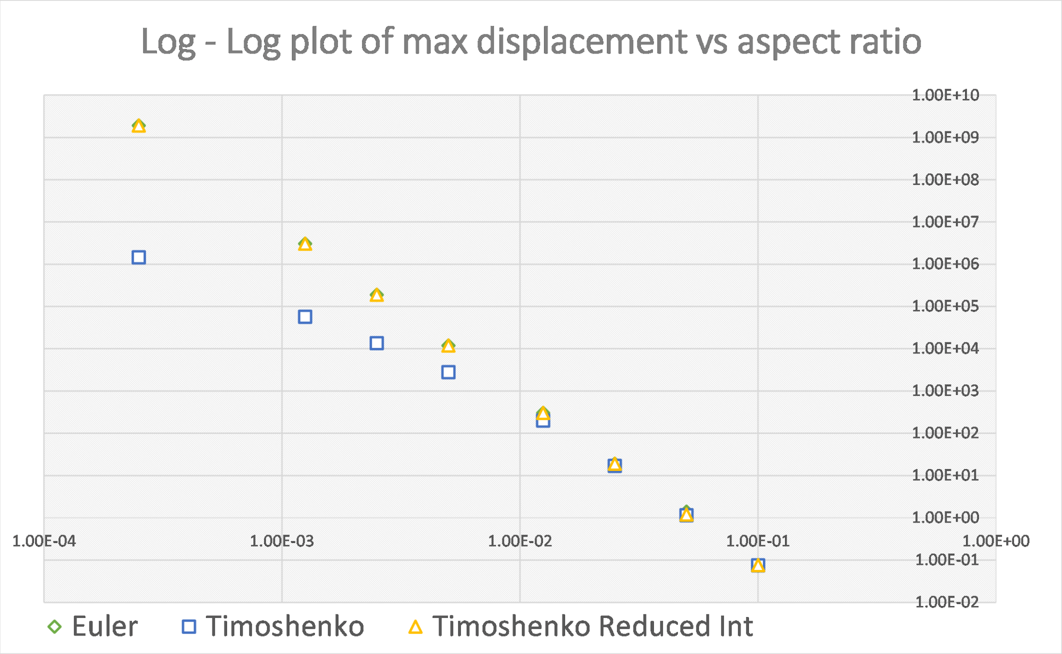


Figure 1: Max Displacement vs Aspect Ratio

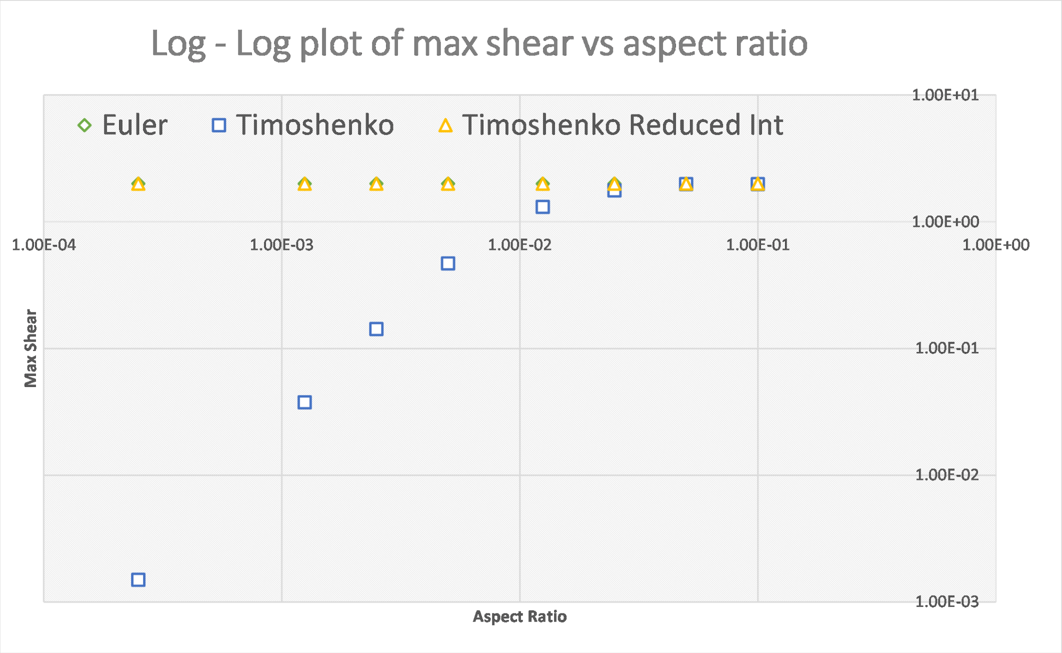


Figure 2: Max Shear Stress vs Aspect Ratio

2.2 Discussion

1. From the displacement plot, it is clear that as the aspect ratio (a/L) increases the maximum displacements decrease. Euler Bernoulli and Timoshenko reduced integration models follow almost the same trend on the log-log plot. The Timoshenko full integration model shows a deviation from this convergence as a/L reduces. This can be mathematically predicted before it has the length term to the third power in it's shear stiffness matrix whereas the other two methods have the length term to the second power. The displacements for full Timoshenko model are therefore more precise. When the shear locking occurs at small aspects ratios the element appears to be stiffer than it is actually is, thus under-estimating the bending displacements.

2. From the Mz moment plot we see the similar trend that Euler Bernoulli and Timoshenko reduced integration models converge to a value which is closer to the analytical value. While the Timoshenko Full Integration presents very different results when the a/L ratio is very small. This is again because of the concept of shear locking. Which causes the results to deviate.