UNIVERSITAT POLITÈCNICA DE CATALUNYA

MASTER IN COMPUTATION MECHANICS AND NUMERICAL METHODS IN ENGINEERING

COMPUTATIONAL STRUCTURAL MECHANICS AND DYNAMICS

Assignment 7

by

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1- Introduction

The goal of the assignment is to apply the concepts of the plates' theories. Among the plate theories, the Kirchoff theory for thin plates and the Reissner-Mindlin theory for thick plates are considered. The MCZ plate element and the RM plate element with full were considered in the present study.

2 – Assignment A

Considering the problem data presented in the assignment [1], the following geometry and mesh were considered:

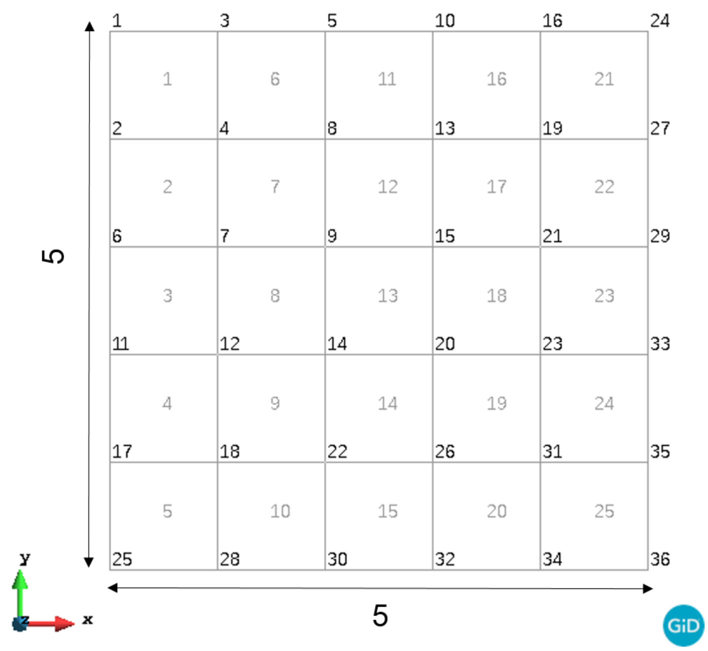

Figure 1. Geometry and mesh considered for Assignment A.

For the mesh presented in Figure 1, two element formulations for plates were considered: the MZC element and the RM element fully integrated [1]. To analyze the behavior of both formulations, five values of thickness were considered for the same load case. Maximum vertical displacements (w), maximum x and y rotations ($\theta_{x,\ y)}$ and maximum bending moments in x and y $M_{x,y}$ were the parameters evaluated for both elements. As boundary conditions, the plate was considered simply supported in a weak form. Table 1 presents the values of such parameters applying the MZC element formulation. Table 2 presents the values of such parameters applying the RM element formulation.

Table 1. Values of parameters evaluated considering MCZ element formulation.

Thickness (m)	**Maximum displacement (w) [mm]**	**Maximum rotation ($\theta_{x,y}$)**	**Maximum bending moment ($M_{x,y}$) [Nmm]**
0.001	-24.357	16.94000000000	1183.9000
0.01	-0.024357	0.01697400000	1183.9000
0.02	-0.0030447	0.00212170000	1183.9000
0.1	-0.000024357	0.00001697400	1183.9000
0.4	-3.8056E-07	0.00000026521	1183.9000

Table 2. Values of parameters evaluated considering RM element formulation.

Thickness (m)	**Maximum displacement (w) [mm]**	**Maximum rotation ($\theta_{x,y}$)**	**Maximum bending moment ($M_{x,y}$) [Nmm]**
0.001	-22.971	16.17300000000	1104.5000
0.01	-0.022973	0.01617500000	1104.6000
0.02	-0.0028724	0.00202660000	1104.9000
0.1	-0.000023271	0.00001634600	1113.2000
0.4	-3.9662E-07	0.00000027884	1193.2000

According to the values presented in Tables 1 and 2, the maximum displacement w does not change significantly from one formulation to the other, even with different values of thickness. The values of maximum displacement and rotations presented by the RM element formulation are smaller than the ones presented by the MZC element formulation when the thickness varies from 0.001 to 0.1 In such range, the plate is considered thin and the RM element formulation when fully integrated is not suitable for such scenario. Therefore, when applied to it, the RM element formulation presents a transverse shear locking effect. At such condition, the components of the shear stiffness matrix increase greatly, neglecting the components of the bending stiffness matrix, and diminishing the values of displacement and rotations. Nevertheless, a greater difference in the values of maximum displacement and rotations between the two element formulations was expected. A finer mesh could be applied to the geometry verify such behavior. Regarding the bending moments, the values are very similar comparing both element formulations. Such outcome is due to the small difference between the displacement and rotations presented by both formulations.

3 – Assignment B

To perform the Patch Test for the MZC element formulation, the following plate with thickness of 0.001 and mesh were considered:

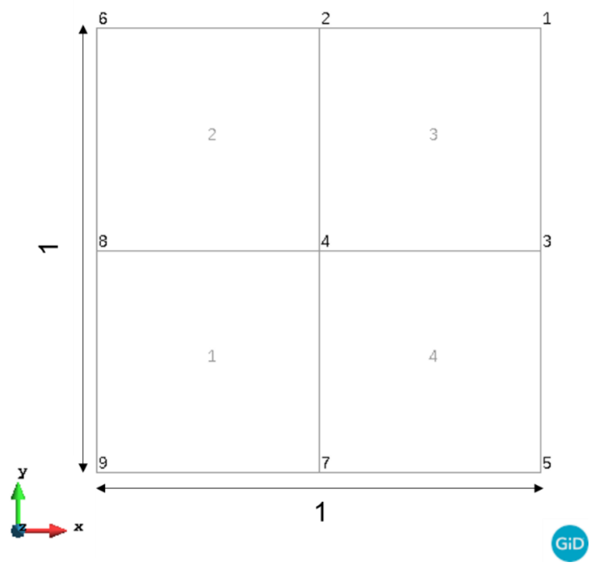

Figure 2. Geometry and mesh considered for the Patch Test of MCZ element formulation.

The material properties applied in the present assignment are the same as the Assignment A [1]. The assumed linear displacement and rotation fields are the following:

$$w(x, y) = -0.005y \quad (1)$$

$$\theta_x(x, y) = 0.0003x \quad (2)$$

$$\theta_y(x, y) = 0.0001y \quad (3)$$

The displacements and rotations were calculated for all nodes in the mesh and are presented in Table 3. All the displacements and rotations calculated were applied to the all nodes. In such manner, the following system of equations must be satisfied for the mid node (Figure 2) [2] so that the MZC element formulation verifies the Patch Test:

$$\boldsymbol{K}_{ij}\boldsymbol{a}_j = \boldsymbol{0} \quad (4)$$

where $\mathbf{a}_j$ is the displacement vector calculated from Equations 1-3. No body forces were considered.

Table 3. Displacement and rotation values for each node according to Equations 1-3.

	Coordinates [mm]		**Calculated displacements (w) and rotations (θ_x and θ_y)**		
Node	**x**	**y**	**w [mm]**	$\boldsymbol{\theta_x}$	$\boldsymbol{\theta_y}$
1	1	1	-0.005	0.0003	0.0001
2	0.5	1	-0.005	0.00015	0.0001
3	1	0.5	-0.0025	0.0003	0.00005
4	0.5	0.5	-0.0025	0.00015	0.00005
5	1	0	0	0.0003	0
6	0	1	-0.005	0	0.0001
7	0.5	0	0	0.00015	0
8	0	0.5	-0.0025	0	0.00005
9	0	0	0	0	0

According to Equation 4, the equations related to the degrees of freedom of node 4 (rows 10, 11 and 12 of global stiffness matrix multiplied by the displacement vector) must equal zero, obeying the equilibrium. Computing the stiffness matrix using the MATLAB file provided for the CMZ element formulation and multiplying the respective rows by the displacement vector, the following residual vector **r** was obtained:

$$\boldsymbol{r} = [0.2371\,N, 0.211\,Nmm, 6.0045\,Nmm\,]^{\boldsymbol{T}}$$

The force component and the moment components of the residual vector are considered small. Such values can be considered in agreement with the Patch Test, validating the convergence of the MZC element. Also, more suitable displacement fields, such as a complete first order polynomials could be applied to obtain a residual vector with values closer to zero. Nevertheless, it is worth mentioning that the MZC element formulation for arbitrary quadrilateral elements fail the Patch Test. Such observation is an important drawback of the formulation, limiting its usage to suitable shaped structured meshes. Appendix A present the stiffness matrix coefficients for the rows referring to the degrees of freedom from node 4.

4 - Conclusions

In Assignment A, the RM element formulation, with full integration, presented a mild transverse shear locking effect. There was no significant difference between the maximum values of displacement and rotations considering the RM and MZC element formulations. Specially for the thinnest plates considered in such assignment, the transverse shear locking effect should have been more present. In such scenario, the components of the shear stiffness matrix would greatly overcome the components of the bending stiffness matrix, reducing the values of displacement and rotations. Since it was not case, the results show that the bending stiffness matrix dominated the problem for the RM element formulation. Finer meshes could be considered to analyze if the same behavior would be present. In Assignment B, the MZC element was considered in agreement with the Patch Test considering structured rectangular elements. Smaller values for the components of the residual vector could be obtained if a finer mesh was considered.

5 – References

[1] – Presentation “TEMA-08-HOME-WORK”, Computational Structural Mechanics and Dynamics, Master of Science in Computational Mechanics, 2020.

[2] – “Structural Analysis with the Finite Element Method – Linear Statics”, Oñate, E., Vol. 1, Springer, 2013.

Appendix A - Coefficients of the stiffness matrix related to degrees of freedom of node 4

Components of row 10 * 10000 [N/mm] – d.o.f w_4		**Components of row 12 * 10000 [N/mm] – d.o.f $\theta_{x,4}$**		**Components of row 13 *10000 [N/mm] – d.o.f $\theta y_{,4}$**	
K10,1	-0.079	**K11,1**	-0.02	**K12,1**	-0.02
K10,2	0.0198	**K11,2**	0.0045	**K12,2**	0
K10,3	0.0198	**K11,3**	0	**K12,3**	0.0045
K10,4	-0.317	**K11,4**	0	**K12,4**	-0.079
K10,5	0	**K11,5**	0.0109	**K12,5**	0
K10,6	0.0793	**K11,6**	0	**K12,6**	0.0109
K10,7	-0.317	**K11,7**	-0.079	**K12,7**	0
K10,8	0.0793	**K11,8**	0.0109	**K12,8**	0
K10,9	0	**K11,9**	0	**K12,9**	0.0109
K10,10	1.5853	**K11,10**	0	**K12,10**	0
K10,11	0	**K11,11**	0.0575	**K12,11**	0
K10,12	0	**K11,12**	0	**K12,12**	0.0575
K10,13	-0.079	**K11,13**	-0.02	**K12,13**	0.0198
K10,14	0.0198	**K11,14**	0.0045	**K12,14**	0
K10,15	-0.02	**K11,15**	0	**K12,15**	0.0045
K10,16	-0.079	**K11,16**	0.0198	**K12,16**	-0.02
K10,17	-0.02	**K11,17**	0.0045	**K12,17**	0
K10,18	0.0198	**K11,18**	0	**K12,18**	0.0045
K10,19	-0.317	**K11,19**	0	**K12,19**	0.0793

K10,20	0	**K11,20**	0.0109	**K12,20**	0
K10,21	-0.079	**K11,21**	0	**K12,21**	0.0109
K10,22	-0.317	**K11,22**	0.0793	**K12,22**	0
K10,23	-0.079	**K11,23**	0.0109	**K12,23**	0
K10,24	0	**K11,24**	0	**K12,24**	0.0109
K10,25	-0.079	**K11,25**	0.0198	**K12,25**	0.0198
K10,26	-0.02	**K11,26**	0.0045	**K12,26**	0
K10,27	-0.02	**K11,27**	0	**K12,27**	0.0045