Computational Structural Mechanics and Dynamics, Assignment 7

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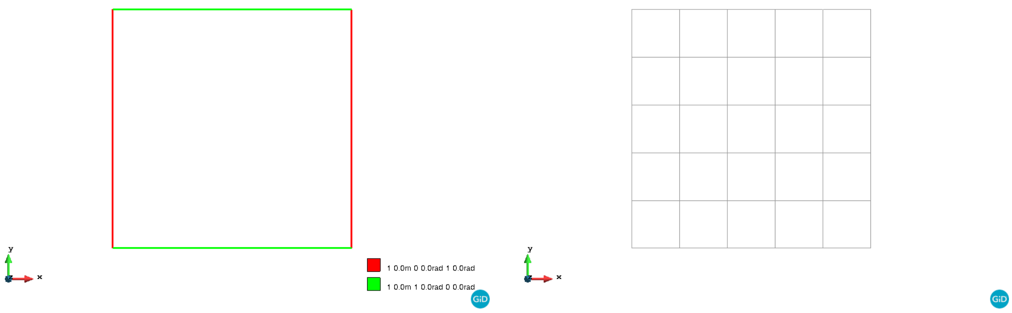
Assignment 7
On "Plate theory":

a) Analyze the shear blocking effect on the Reissner Mindlin element and compare with the MZC element. For the Simple Support Uniform Load square plate.

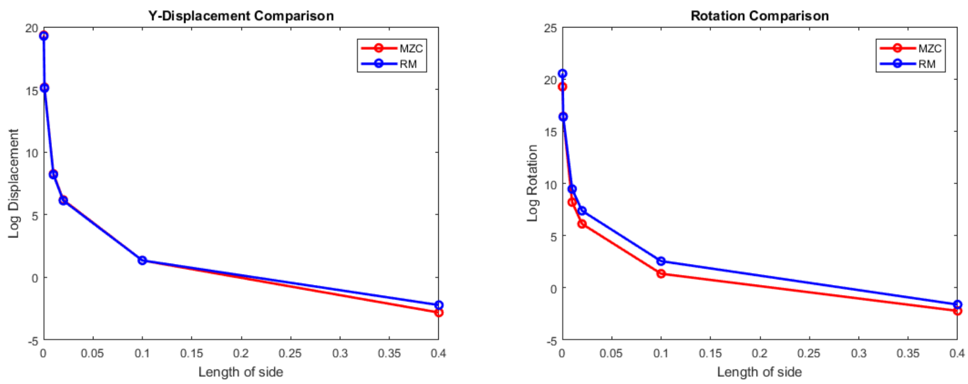
For this point, a 5×5 mesh is created in GiD. The length of the side of the square is set to 1m, and the given material properties are set. On all the edges, strong simple supported boundary conditions are imposed. The modelling is shown in figure 1.



(a) Strong Simple Support (b) Mesh to be used

Figure 1: Setting of the problem in GiD

After running the examples for the different thicknesses. One can get the plots shown in figure 2. Notice that the scale does not allow to observe much difference.



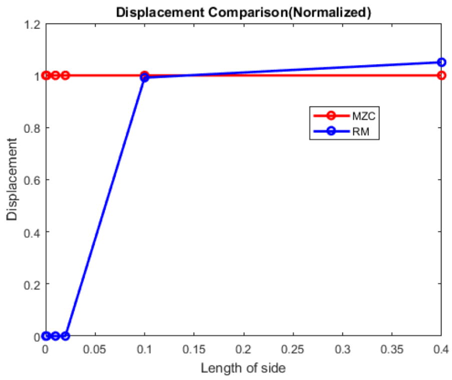
(a) Displacement (b) Rotation

Figure 2: Maximum value for each thickness

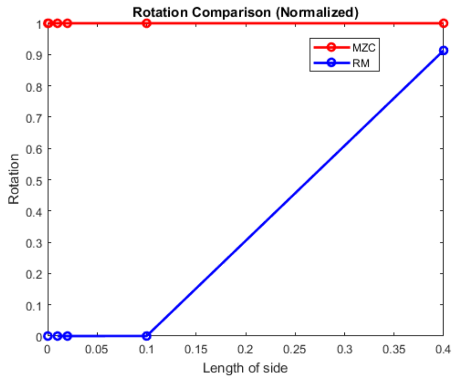
A way to observe the variation of the methods is to use a normalized measurement. That is, taking as a reference the MZC, and checking how the variation w.r.t. RM, using the expression:

$$ValuePlotted = e^{(MZC_{result} - RM_{result})}$$

The normalized results are shown in figure 3.



(a) Displacement



(b) Rotation

Figure 3: Maximum value for each thickness

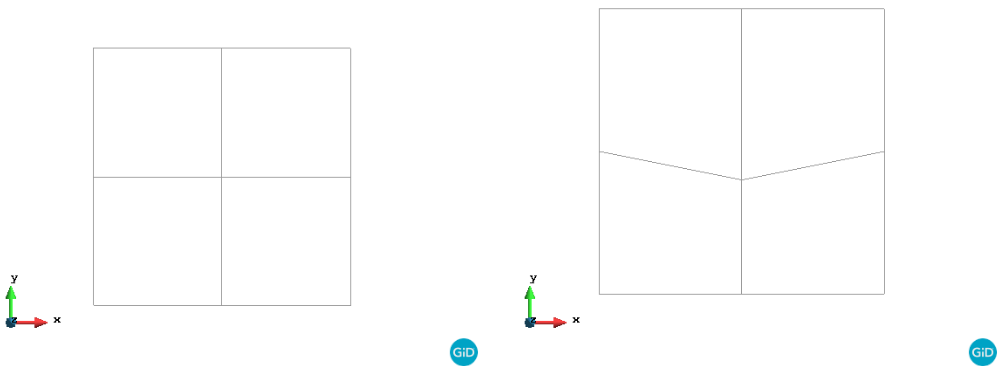
Conclusions, part a: Figure 3 shows how the RM element is giving extra stiff results for the case of thin plates. RM improves as the thickness increases and it approaches the results obtained using MZC element. The RM element counts with an easier formulation and it also provides information on the shear forces as a direct output and not as a postprocess, but in the case of thin plates, measurements should be taken to avoid the locking effects, for example using a DK (Discrete Kirchoff) element, or use a reduced integration for the shear matrix.

b) Define and verify a patch test mesh for the MCZ element.

For this exercise, the Zienkiewicz element is be tested using the following linear expression:

$$u = 1 + 3x - 2y \quad in[0,1]^2$$

Two meshes are used, one with homogeneous, perfectly squared elements, and another one with a slight distortion on the elements. Both meshes are shown in figure 4.



(a) Mesh 1. Squared Elements

(b) Mesh 2. Distorted Elements

Figure 4: Results of the simulation

The conditions on the edge nodes are imposed on both, displacement and rotations, considering the linear displacement field and the rotations, which are the derivatives of the field with respect to x and y. Tables and show the imposed values and expected values for the central node in both meshes.

IMPOSED VALUES FOR THE MESHES								
Node	(0,0)	(0.5,0)	(1,0)	(0,0.5)	(1,0.5)	(0,1)	(0.5,1)	(1,1)
w	1	2.5	4	0	3	-1	0.5	2
$\frac{dw}{dx}$	3	3	3	3	3	3	3	3
$\frac{dw}{dy}$	-2	-2	-2	-2	-2	-2	-2	-2

Table 1: Imposed Values

EXPECTED VALUES FOR THE CENTRAL NODE IN BOTH MESHES		
Central Node of	MESH 1 (0.5, 0.5)	MESH 2 (0.5,0.4)
w	1.5	1.7
$\frac{dw}{dx}$	3	3
$\frac{dw}{dy}$	-2	-2

Table 2: Expected Values

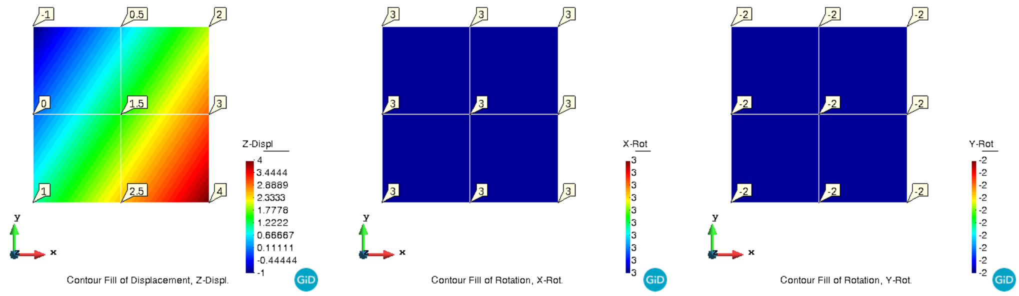
After running the simulations. The obtained results for are shown for the three variables (w, $\frac{dw}{dx}$, $\frac{dw}{dy}$) in figures 5, and 6.

It could be seen that distorted elements are not behaving as expected. But will rectangular elements also fail (not perfectly squared)? In order to test that, another mesh is attempted using non-distorted, yet rectangular elements. The results are shown in figure 7.

Now, it is also interesting to see whether the Zienkiewicz element is able to pass a second order patch test. For this case, the following equation is imposed for the displacement field.

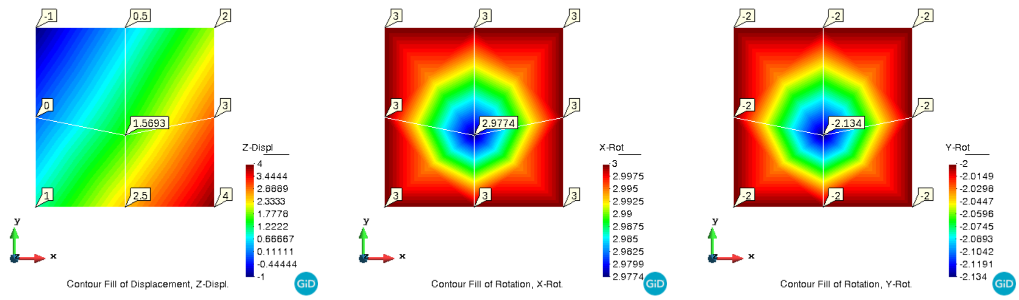
$$w = x^2 + y^2$$

The corresponding values are imposed to a regular mesh (mesh 1), and they are shown in table 3. The plots are shown in figure 8.



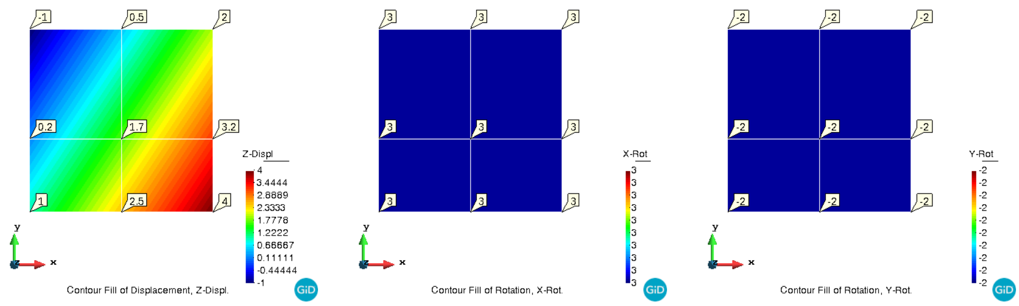
(a) w (b) dw/dx (c) dw/dy

Figure 5: Results for central node. Mesh 1



(a) w (b) dw/dx (c) dw/dy

Figure 6: Results for central node. Mesh 2

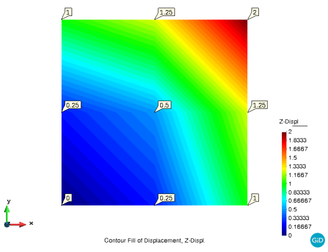


(a) w (b) dw/dx (c) dw/dy

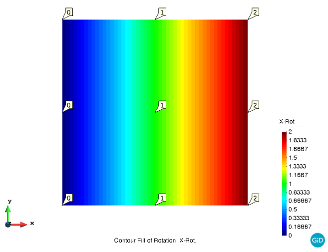
Figure 7: Results for central node. Rectangular Mesh (just for comparison, not much data is provided, but it behaves correctly)

IMPOSED VALUES FOR THE QUADRATIC MESH								
Node	(0,0)	(0.5,0)	(1,0)	(0,0.5)	(1,0.5)	(0,1)	(0.5,1)	(1,1)
w	0	0.25	1	0.25	1.25	1	1.25	2
$\frac{dw}{dx}$	0	1	2	0	2	0	1	2
$\frac{dw}{dy}$	0	0	0	1	1	2	2	2

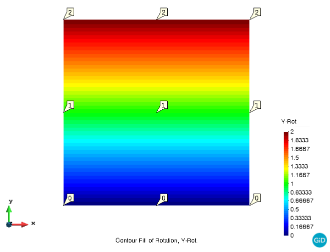
Table 3: Imposed Values quadratic field patch



(a) w



(b) dw/dx



(c) dw/dy

Figure 8: Quadratic Field Patch

Conclusions, part b: The patch test is key for proving that an element will converge to the exact solution as the the mesh is refined. It can clearly be seen that the Zienkiewicz element passes the patch test only when the elements are not distorted. However, even a slight distortion in the element caused deviations from the expected results. This element is therefore only suitable for non distorted meshes, orthogonal meshes. Either squared or rectangular.
Moreover, the quadratic field is also reproduced by the Zienkiewicz element. This means that a mesh using this type of elements will not need too small elements to capture the behavior of the solution, since relatively larger elements could capture second order solutions.