MASTER'S DEGREE NUMERICAL METHODS IN ENGINEERING

UNIVERSITAT POLITÈCNICA DE CATALUNYA BARCELONATECH

COMPUTATIONAL STRUCTURAL MECHANICS AND DYNAMICS

Homework 7: Plates designment

Author:
Mariano Tomás Fernandez

Professor:
Miguel Cervera
Francisco Zárate

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Contents

Assignment 7.1

Analyze the shear blocking effect on the Reissner Mindlin element and compare with the MZC element. For the Simple Support Uniform Load square plate.
Use the 5x5 mesh.

- t=0.001;
- t=0.010;
- t=0.020;
- t=0.100;
- t=0.400;

Material properties: $E = 10.92$, $\nu = 0.3$, $Q = 1.0$

Assignment 7.2

Define and verify a patch test mesh for the MZC element.

1 Assignment 7.1

In this section the idea is to analyse the behaviour of two plate element formulation using two different MATLAB codes, one representing the MZC formulation (based on the Kirchoff classical theory) and the other one representing the Reissner-Mindlin formulation.

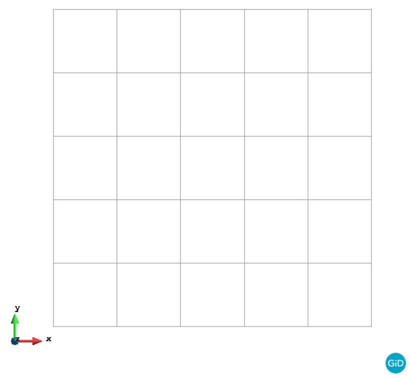
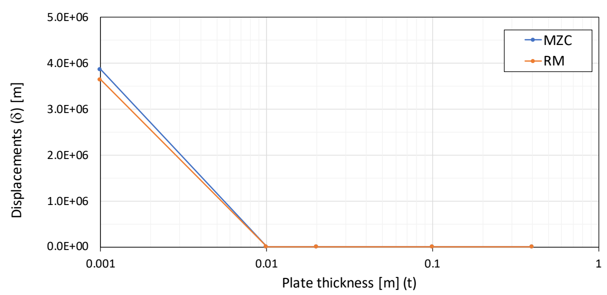


Figure 1: Analysed mesh over the 1x1 domain problem

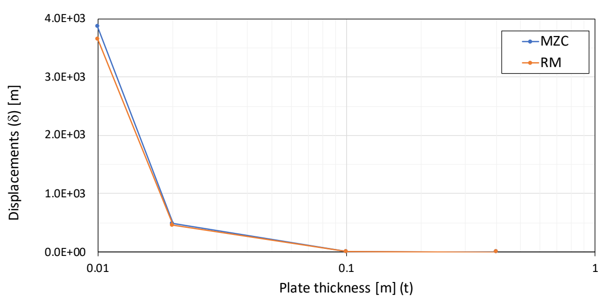
In Figure 1 the mesh used in the model is shown. The structure of the plate is 1.0 m x 1.0 m square-plate with 5 elements in each direction. In Table 1 the results obtained are expressed, also in Figure 2, Figure 3 and Figure 4 the displacements and bending moments are shown. As the mesh remain unchanged, these plots could have been done with the X-axis as a/L or simply a. As $L = 0.2\ m$, the X-axis is very similar in both cases (when plotting in logarithmic scale) therefore the a X-axis was adopted.

	MZC					RM				
a	Displ.	$\lvert Mx_{max}\rvert$	$\lvert My_{max}\rvert$	$\lvert Mx_{min}\rvert$	$\lvert My_{min}\rvert$	Displ.	$\lvert Mx_{max}\rvert$	$\lvert My_{max}\rvert$	$\lvert Mx_{min}\rvert$	$\lvert My_{min}\rvert$
[m]	[m]	[N m]	[N m]	[N m]	[N m]	[m]	[N m]	[N m]	[N m]	[N m]
0.001	$3.9e^{6}$	$4.7e^{-2}$	$4.7e^{-2}$	$6.2e^{-4}$	$6.2e^{-4}$	$3.6e^{6}$	$4.4e^{-2}$	$4.4e^{-2}$	$9.0e^{-7}$	$9.0e^{-7}$
0.010	$3.9e^{3}$	$4.7e^{-2}$	$4.7e^{-2}$	$6.2e^{-4}$	$6.2e^{-4}$	$3.6e^{3}$	$4.4e^{-2}$	$4.4e^{-2}$	$8.6e^{-5}$	$8.6e^{-5}$
0.020	$4.8e^{2}$	$4.7e^{-2}$	$4.7e^{-2}$	$6.2e^{-4}$	$6.2e^{-4}$	$4.6e^{2}$	$4.4e^{-2}$	$4.4e^{-2}$	$3.1e^{-4}$	$3.1e^{-4}$
0.100	$3.9e^{0}$	$4.7e^{-2}$	$4.7e^{-2}$	$6.2e^{-4}$	$6.2e^{-4}$	$4.2e^{0}$	$4.9e^{-2}$	$4.9e^{-2}$	$1.5e^{-3}$	$1.5e^{-3}$
0.400	$6.0e^{-2}$	$4.7e^{-2}$	$4.7e^{-2}$	$6.2e^{-4}$	$6.2e^{-4}$	$1.3e^{-1}$	$5.4e^{-2}$	$5.4e^{-2}$	$6.3e^{-7}$	$6.3e^{-3}$

Table 1: Results obtained in the models tested using MZC and RM elements in plates.

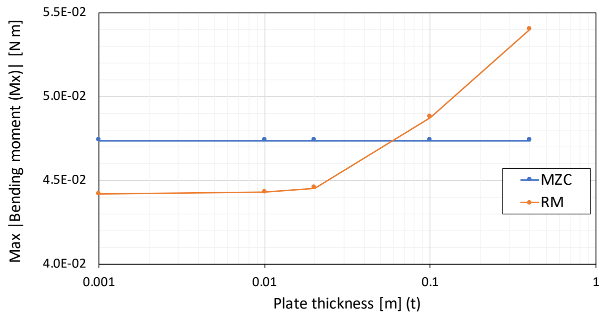


(a) Displacement registered

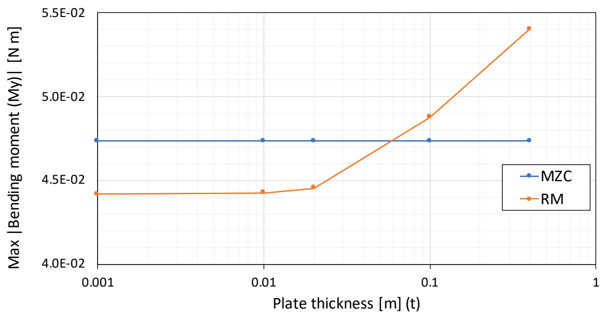


(b) Displacements for thicknesses bigger than $t = 0.001$.

Figure 2: Mesh of the problem and associated displacements for different plate thickness.

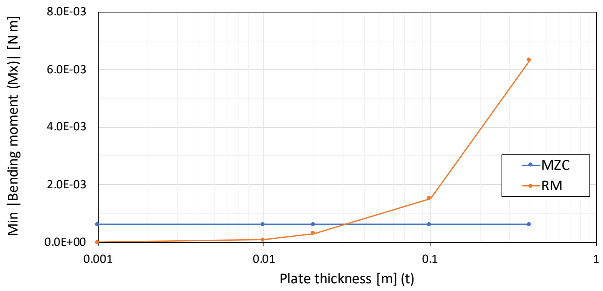


(a) X-direction bending moment

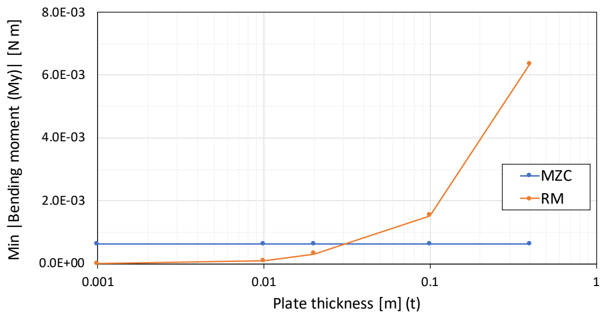


(b) Y-direction bending moment

Figure 3: Maximum absolute values of bending moments for different plate thickness.



(a) X-direction bending moment



(b) Y-direction bending moment

Figure 4: Minimum absolute values of bending moments for different plate thickness.

As seen in Figure 2a, the displacements registered for thickness of 0.001 m is far from being inside the *small displacements* framework as the displacement is more than one million times bigger than the plate width (this kind of plates should be described under different theories as *Fopp-von Kármán*), again in Figure 2b for thickness bigger of equal to $t = 0.01\ m$ the displacements are not enough small, from the Figure 2b it is see that for thicknesses smaller than 0.1 the displacements are too high ($\delta = 500$). From Table 1 it is seen that regarding to the displacements, the only plate that can be considered as small displacements is the one with the biggest thickness.
The shear blocking effect in the *Reissner Mindlin* element is observed in Figures 3a, Figure 3b, Figure 4a and Figure 4b as the RM method diverge from the result registered with the MZC method. For higher thicknesses both methods becomes closer, but it is not very comforting noticing that again the RM method diverge from the MZC.

2 Assignment 7.2

Define and verify a patch test mesh for the MZC element.

The patch test is based in imposing at the boundary of a patch element a displacement field which can be exactly reproduced by the shape functions.
This test is satisfied if the displacements and strains within the patch coincide with the exact values deduced from the prescribed displacement field.

To this end a constant displacement field will be required in the previously used plate model to check whether the displacements and strains generated match the theoretically expected ones, which will be constant displacement and no strains. In Figure 5 the patch test conditions are shown, where a *-1* displacement is imposed in the Z direction and no rotations are allowed in the inner nodes of the square plate.
The results are shown in Figure 6 where the conditions for the patch test are meet. Also in Figure 7a and Figure 7b the rotation conditions for the patch test are shown and are also in agreement with expected.

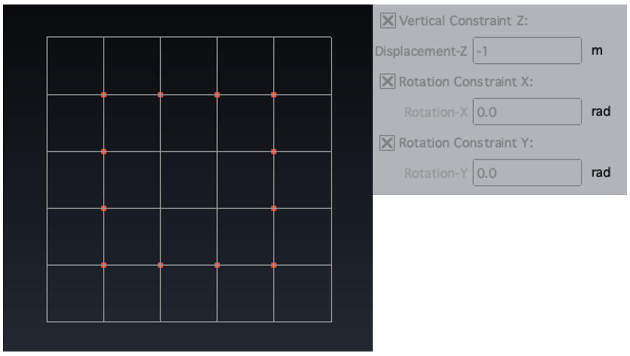


Figure 5: Boundary conditions for the Patch test.

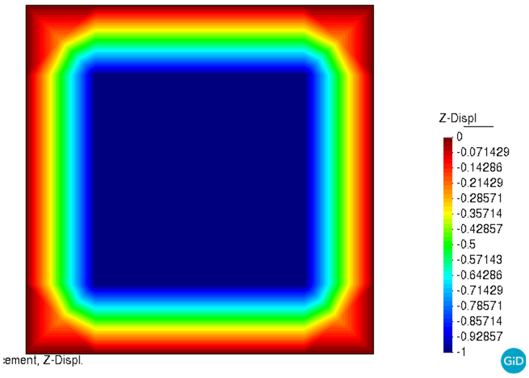
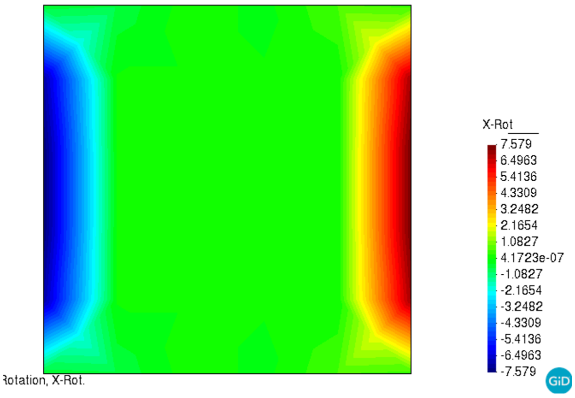
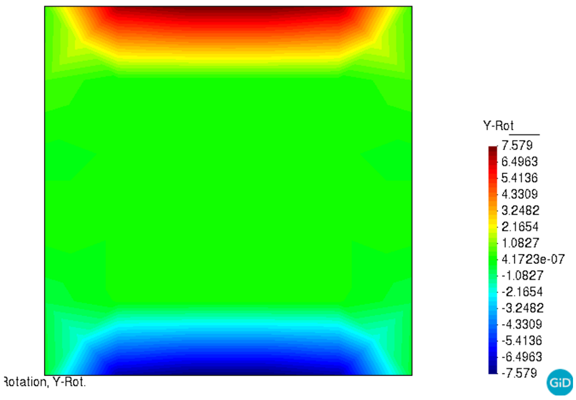


Figure 6: Displacement in the plate for the Patch test.



(a) X-direction rotation



(b) Y-direction rotation

Figure 7: Rotations on the plate for the Patch test.