# Computational Structural Mechanics and Dynamics

Assignment 7

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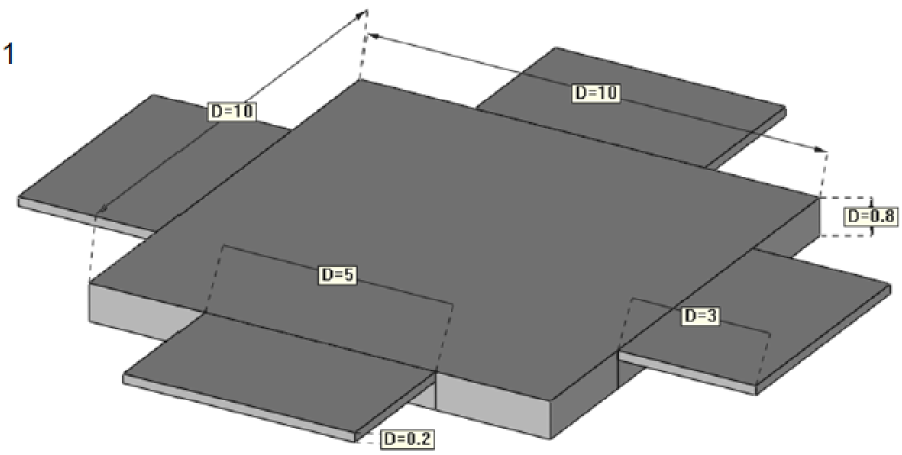
**a)** What kind of strategy (theory, elements, integration rule, boundary conditions, etc) will you use for solving the following problems:

a.1



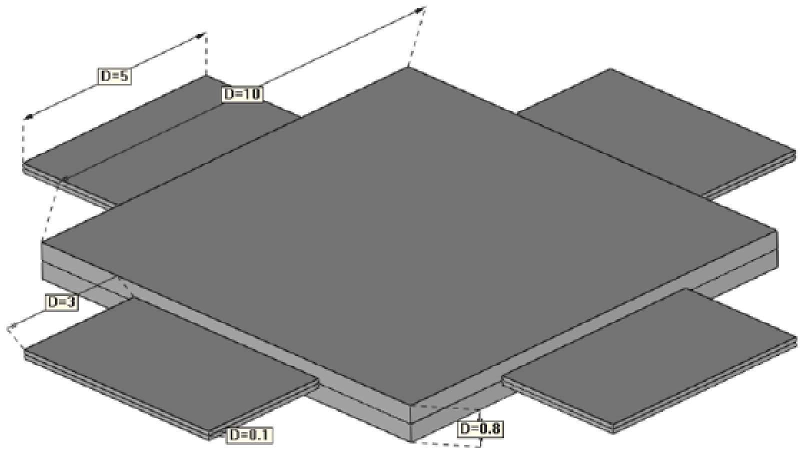

The geometry shows 5 plates four of which have the same dimensions. These plates don't share the same mid-plane with the middle plate. In order to reduce the computation costs, we can divide the structure into a quarter and apply the suitable boundary conditions regarding the axis of symmetry. Afterwards, we can do the FE analysis for each plate separately since they don't have same mid-planes, which means to consider local reference systems to define the stiffness matrix for each plate and then assemble the matrices taking into account the assembly rules for different local coordinates.

The boundary conditions that should be applied in this case are the clamped boundaries between each plate and zero rotation around x and y axis for the axis of symmetry which restricts the displacement in both x and y direction.

In order to analyze this structure we first have to evaluate the ratio of thickness/width of the plates to see if they are considered as thin or thick plates. Considering the dimensions provided by the problem, we can see that the lowest and the highest ratio are 0.04 and 0.08 which correspond to the side plates and the middle plate respectively. Since both ratios are less than 0.1, the plates are considered to be thin. So according to the fact that if the effect of shear stress distribution is considered to be important for us or not, we can use the Kirchhoff or Mindlin/Reissner's theory. In this problem, since the side plates are not sharing the same mid-plane, it seems that the shear stress distribution of the middle plate is also important for the side plates. Therefore, it might be better to use the Mindlin theory for the middle plate and Kirchhoff for the side plates since Mindlin also gives importance to shear stress but Kirchhoff gives less importance to shear and pays more attention to bending. In addition, if we look at the ratios of both plates, it is true that both are less than 0.1 but the ratio of the middle plate is twice compared to the side plates. So, when it comes to comparison, we can use the Mindlin theory for the analysis of this plate or at least use this theory for the parts of the geometry which are experiencing thickness change and use Kirchhoff for other parts. And in this case we also have to give special attention to shear-lock effects and how we are doing the assembly.

**b)** What kind of strategy (theory, elements, integration rule, boundary conditions, etc) will you use for solving the following problems:

a.2




This problem consists of 5 plates four of which have the same dimensions and the middle plate is different. The whole structure has one mid-plane. In order to reduce the computation costs we can divide the whole structure into one quarter same as the previous problem.

The boundary conditions on the axis of symmetry are defined as the previous problem which is zero rotation around x and y direction.

In this problem, in contrast to the previous one, we can take all the plates as one structure and we don't have to separate them to do the FE analysis since all of them share the same mid-plane. Therefore, regarding the calculation of the ratio of thickness/width we can say it is equal to the smallest thickness existing in the structure over the largest width, which means it would be equal to 0.1/16 that is 0.006. Since this ratio is less than 0.1 it is considered as thin plate and according to the previous explanations the Kirchhoff theory can be used for the whole structure. However, if we also want to give importance to shear distribution we have to use the Mindlin theory for the parts that thickness changes.

There is also another way of analyzing the plates separately like the previous problem but here we are not undergoing the problems related to different mid-planes. So if we take into account the ratio of thickness/width for each plate, it would be 0.02 and 0.08 respectively for the side and middle plates which again are less than 0.1. Now, taking into account the previous assumptions and explanations, Kirchhoff and Mindlin theory can both be used in this case keeping in mind the shear-lock effects and applying the appropriate reduced-integration techniques to solve this issues.

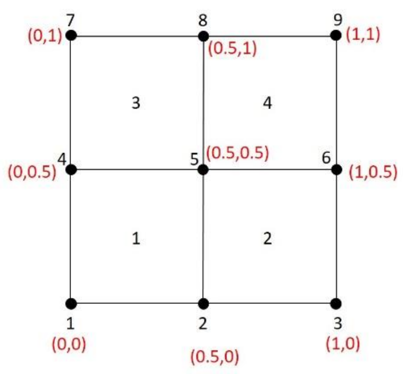
**c)** Define and verify a patch test mesh for the MCZ element.

The patch test is done by setting the interior nodal forces equal to zero and prescribing a specific value for x and y displacement for every node but not the one node in the middle. And then using this displacements, we shall compute the displacement and rotation functions and compare the results with that obtained by the code.

The displacement and rotation functions are chosen to be:

$$w = \frac{1}{2}x^2 + \frac{1}{2}y^2 + \frac{1}{2}xy$$
$$\theta_x = x + \frac{1}{2}y$$
$$\theta_y = y + \frac{1}{2}x$$

The mesh for the patch test is chosen as:




| node | x | y | w | theta_x | theta_y |
|---|---|---|---|---|---|
| **1** | 0 | 0 | 0 | 0 | 0 |
| **2** | 0.5 | 0 | 0.125 | 0.5 | 0.25 |
| **3** | 1 | 0 | 0.5 | 1 | 0.5 |
| **4** | 0 | 0.5 | 0.125 | 0.25 | 0.5 |
| **5** | **0.5** | **0.5** | **0.25** | **0.75** | **0.75** |
| **6** | 1 | 0.5 | 0.625 | 1.25 | 1 |
| **7** | 0 | 1 | 0.5 | 0.5 | 1 |
| **8** | 0.5 | 1 | 0.625 | 1 | 1.25 |
| **9** | 1 | 1 | 1 | 1.5 | 1.5 |

Now that we have calculated the w, theta_x and theta_y for all the nodes, we shall use the code for MCZ plates provided in the virtual center to find the result of patch test:

| node 5 | FEM | Analytical |
|---|---|---|
| w | 0.3751 | 0.25 |
| theta_x | 0.7493 | 0.75 |
| theta_y | 0.75 | 0.75 |
| eps_x | -1.0036 | -1 |
| eps_y | -0.9976 | -1 |
| gamma_xy | -1 | -1 |

According to the results, the epsilon remains the same which means the patch test is satisfied.