CSMD Assignment-9

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A/ In order to apply non-symmetric load on our formulation, we can discretize our model along the angle around the axis of symmetry.

This way the angle gets discretized element by element and then integration is done on all elements along the circumference.

B/ Neglecting shear strains for an element with straight configuration, such that $R \to \infty$, the strains can be represented as–

$$\varepsilon = \begin{Bmatrix} \varepsilon_x \\ \varepsilon_\varphi \\ \cancel{\gamma_{xz}} \end{Bmatrix}_0 = \begin{Bmatrix} \frac{\partial u_0'}{\partial s} + \frac{\partial u'}{\partial s} \\ \frac{u_0 \cos\phi}{r} + \frac{u' \cos\phi}{r} - \frac{w_0' \sin\phi}{r} \end{Bmatrix} = \begin{bmatrix} \frac{\partial u_0'}{\partial s} - z\frac{\partial^2 w_0}{\partial s^2} \\ \frac{u_0' \cos\phi}{r} - z\frac{\partial w_0}{\partial s}\frac{\cos\phi}{r} - \frac{w_0 \sin\phi}{r} \end{bmatrix}$$

Considering $u_0 \simeq u_0^h = \sum_{i=1}^{n} N_i u_i$

& $w_0 \simeq w_0^h = \sum_{i=1}^{n} N_i w_i + \bar{N}_i \left(\frac{\partial w}{\partial s}\right)_i$

we have,

$$\varepsilon = \varepsilon_m + \varepsilon_b = \begin{bmatrix} \frac{\partial u_0'}{\partial s} \\ \frac{u_0' \cos\phi}{r} - \frac{w_0 \sin\phi}{r} \end{bmatrix} + \begin{bmatrix} -z\frac{\partial^2 w_0'}{\partial s^2} \\ -z\frac{\partial w_0}{\partial s}\frac{\cos\phi}{r} \end{bmatrix} = \begin{Bmatrix} \varepsilon_m \\ 0 \end{Bmatrix} + \begin{Bmatrix} z\varepsilon_b \\ \varepsilon_s \end{Bmatrix}_0$$

where, $\varepsilon_m \to$ Membrane strain

$\varepsilon_b \to$ Bending strain

$\varepsilon_s \to 0$ (Shear strain assumed to be zero)

$$\therefore \varepsilon = \begin{bmatrix} \varepsilon_m \\ z\varepsilon_b \end{bmatrix}$$

Now, expanding we get, $$\hat{\varepsilon}_{mi} = \begin{bmatrix} \frac{\partial \hat{N}_i}{\partial s} & 0 & 0 \\ \frac{\hat{N}_i \cos\phi}{r} & -\frac{N_i \sin\phi}{r} & -\frac{\bar{N}_i \sin\phi}{r} \end{bmatrix} \begin{bmatrix} u'^{h}_{0i} \\ w'^{h}_{0i} \\ \left(\frac{\partial w'_0}{\partial s}\right)^{h} \end{bmatrix}$$

$$\& \ \hat{\varepsilon}_b = \begin{bmatrix} 0 & -\frac{\partial^2 N_i}{\partial s^2} & -\frac{\partial^2 \bar{N}_i}{\partial s^2} \\ 0 & -\frac{\partial N_i}{\partial s}\frac{\cos\phi}{r} & -\frac{\partial \bar{N}_i}{\partial s}\frac{\cos\phi}{r} \end{bmatrix} \begin{bmatrix} u'^{h}_{0i} \\ w'^{h}_{0} \\ \left(\frac{\partial w_0}{\partial s}\right)^{h} \end{bmatrix}$$

$$\therefore B = \begin{bmatrix} \frac{\partial \hat{N}_i}{\partial s} & 0 & 0 \\ \frac{\hat{N}_i \cos\phi}{r} & -\frac{N_i \sin\phi}{r} & -\frac{\bar{N}_i \sin\phi}{r} \\ 0 & -\frac{\partial^2 N_i}{\partial s^2} & -\frac{\partial^2 \bar{N}_i}{\partial s^2} \\ 0 & -\frac{\partial N_i}{\partial s}\frac{\cos\phi}{r} & -\frac{\partial \bar{N}_i}{\partial s}\frac{\cos\phi}{r} \end{bmatrix}$$

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