COMPUTATIONAL STRUCTURAL MECHANICS AND DYNAMICS

MASTERS IN NUMERICAL METHODS

ASSIGNMENT 9

Axisymmetric Shells

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1 Problem A

Describe in extension how can be applied a non symmetric load on this formulation

In this case where non symmetric loads are applied to an axisymmetric structure, both symmetric and non symmetric displacements need to be accounted. According to Zienkiewicz and Taylor [1] , the problem of an axisymmetric solid subjected to non-symmetric loads may be solved by expressing displacements, as well as force components, as Fourier series expansions, splitting them into symmetric and antisymmetric components.

We can develop a Fourier series to split the force in two components and obtain two stiffness matrices. For the case of axisymmetric thin shells (Euler-Bernoulli formulation), the definition of strains is also modified to take into account the displacements and force in all three directions

$$\epsilon = \begin{bmatrix} \epsilon_a \\ \epsilon_\theta \\ \gamma_{a\theta} \\ \lambda_a \\ \lambda_\theta \\ \lambda_{a\theta} \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial a} \\ \frac{1}{r}\frac{\partial v}{\partial \theta} + (ucos\theta - \frac{w}{r}sin\theta) \\ \frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial a} - \frac{v}{r}cos\theta \\ -\frac{\partial^2 w}{\partial a^2} \\ -\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} - \frac{1}{r}\frac{\partial w}{\partial a}cos\theta + \frac{1}{r}\frac{\partial V}{\partial \theta}sin\theta \\ \frac{2}{r}(-\frac{\partial^2 w}{\partial a \partial \theta} + \frac{1}{r}\frac{\partial w}{\partial \theta} + \frac{\partial v}{\partial a} - \frac{v}{r}sin\theta cos\theta \end{bmatrix} \tag{1}$$

Now, three membrane and three bending effects are now present in the stress vector.

$$\begin{bmatrix} N_a & N_\theta & N_{a\theta} & M_a & M_\theta & M_{a\theta} \end{bmatrix}' \tag{2}$$

2 Problem B

Using thin beams formulation, describe the shape of the B(e) matrix and comment the integration rule.

Following the Krichoff formulation of thin beams, the B matrix will be composed only by the membrane strain and bending strain. This is because under Kirchoff assumption, shear stresses are negligible, so the general deformation matrix is defined as following:

$$B^e = \begin{bmatrix} B^e_m \\ B^e_b \end{bmatrix} = \begin{bmatrix} \frac{\partial N^e_a}{\partial a} & 0 & 0 \\ \frac{N^e_u cos\theta}{r} & -\frac{N^e_w sin\theta}{r} & -\frac{\bar{N}^e_w sin\theta}{r} \\ 0 & \frac{\partial^2 N^e_w}{\partial a^2} & \frac{\partial^2 \bar{N}^e_w}{\partial a^2} \\ 0 & \frac{cos\theta}{r}\frac{\partial^2 N^e_w}{\partial a^2} & \frac{cos\theta}{r}\frac{\partial^2 \bar{N}^e_w}{\partial a^2} \end{bmatrix} \tag{3}$$

Since a lot of the terms in the above equation contain r in the denominator, when r $\rightarrow$ 0 the terms will tend to infinity. This problem can be overcome if the integration is done at using Gauss quadrature, in this method the value is not evaluated at the node but at certain specific gauss points.

References

[1] Zienkiewicz, O.C.; Taylor, R.L., *The Finite Element Method Volume 2: Solid Mechanics, Fifth Edition.* Butterworth Heinemann, 2000