Computational Structural Mechanics and Dynamics
MSc in Computational Mechanics
Universitat Politecnica de Catalunya

Assignment 9

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Contents

1. Abstract

In this report will be discussed axisymmetric shells.

2. Part a)

Describe in extension how can be applied a non-symmetric load on this formulation.

In case of a non-symmetric load applied on an axisymmetric shell is a non trivial case. This is because it generates both non symmetric and symmetric terms in the computation of the shell stress.

When this case happens, the Fourier series is needed to determine the formulation of the problem. It let us describe the harmonic behaviour of the structure with respect to the circumferential direction, allowing us to describe it in one dimension. The zeroth components of the series are the axisymmetrical loads as logic.

Developing the Fourier series will permit us to split the force in the symmetric and anti symmetric components and to obtain two stiffness matrices defined as following:

$$
\begin{aligned}
K_a &= \pi \int_A B_a^T D B_a dA \\
K_b &= \pi \int_A B_b^T D B_b dA
\end{aligned}
\tag{2.1}
$$

Being the loads and displacements described in polar coordinates and the loads from the Fourier series as:

$$
f(\theta) = \frac{a_0}{2} + \sum\left(\frac{cosn\theta}{\pi}\int_{-\infty}^{+\infty} f(\theta)cosn\theta d\theta) + \frac{sinn\theta}{\pi}\int_{-\infty}^{+\infty} f(\theta)sinn\theta d\theta\right. \tag{2.2}
$$

The strain vector can be described as following:

$$
\epsilon = \begin{bmatrix} \epsilon_a \\ \epsilon_\theta \\ \gamma_{a\theta} \\ \lambda_a \\ \lambda_\theta \\ \lambda_{a\theta} \end{bmatrix} = \begin{bmatrix} \frac{\partial u}{\partial a} \\ \frac{1}{r}\frac{\partial v}{\partial \theta} + (ucos\theta - \frac{1}{r}wsin\theta) \\ \frac{1}{r}\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial a} - \frac{1}{r}vcos\theta \\ -\frac{\partial^2 w}{\partial a^2} \\ -\frac{1}{r^2}\frac{\partial^2 w}{\partial \theta^2} - \frac{1}{r}\frac{\partial w}{\partial a}cos\theta + \frac{1}{r}\frac{\partial v}{\partial \theta}sin\theta \\ \frac{2}{r}(-\frac{\partial^2 w}{\partial a \partial \theta} + \frac{1}{r}\frac{\partial w}{\partial \theta} + \frac{\partial v}{\partial a} - \frac{1}{r}vsin\theta cos\theta) \end{bmatrix} \tag{2.3}
$$

3. Part b)

Using thin beams formulation, describe the shape of the B^e matrix and comment the integration rule.

Following the Krichoff formulation of thin beams, the B matrix will be composed only by the membrane strain matrix and the bending one. This is because under Kirchoff assumption, shear stresses are negligible, so the general deformation matrix is defined as following:

$$B^e = \begin{bmatrix} B^e_m \\ B^e_b \end{bmatrix} = \begin{bmatrix} \frac{\partial N^e_u}{\partial a} & 0 & 0 \\ \frac{N^e_u cos\theta}{r} & -\frac{N^e_w sin\theta}{r} & -\frac{\mathbf{N}^e_w sin\theta}{r} \\ 0 & \frac{\partial^2 N^e_w}{\partial a^2} & \frac{\partial^2 \mathbf{N}^e_w}{\partial a^2} \\ 0 & \frac{cos\theta}{r}\frac{\partial N^e_w}{\partial a} & \frac{cos\theta}{r}\frac{\partial \mathbf{N}^e_w}{\partial a} \end{bmatrix} \tag{3.1}$$

It can be noticed from the components of the deformation strain matrix, that for $r \to 0$ the integration method will present problems. This because some terms of B depends on $1/r$.

In order to face this issues, the integration Gauss quadratures are used. They are evaluated not in the node, but in points at a certain distance, depending on the order.