# Assignment Beams

### a) Program in Matlab the Timoshenko 2 Nodes Beam Element with reduced integration for the shear stifness matrix.

In File *Beam_Timoshenko*:

We define new $B_s$ shear strain matrix evaluating at Gauss point “0”.

```
 %Reduced integration. Gauss point = 0, Wq=2
   bmat_s=[-1/len,-1/2, 1/len,-1/2];
Stred= zeros  ( nelem , 3 );
Stred(ielem,1) = dmatf*(bmat_f *transpose(u_elem));
Stred(ielem,2) = dmats*(bmat_s*transpose(u_elem));
```

So curvature stress matrix remains the same.

Next we export the reduced stress matrix:

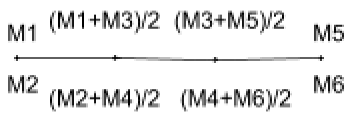
```
ToGiD_VigaD (file_name,u,reaction,Str,Stred);
```

Stred is matrix of as much rows as elements on mesh with two columns, first for bending stress and second for shear stress.

Following thing is two adapt file *ToGiD_Viga* adding new input:

```
function ToGiD_Viga (file_name,u,reaction,Str,Stred)
```

We smooth the  Shear Nodal Stresses defining new matrix for this, *“strnodred”.*We try here to give stress values in the nodes following this scheme:




Being the pair values the *shear* ones and the odd values the *Moments*, which will remain the same.

```
strnodred = zeros(npnod,3);
for ielem = 1 : nelem
  Inods_i = elements(ielem,1);
  Inods_j = elements(ielem,2);
  strnodred(Inods_i,1) = strnodred(Inods_i,1)+Stred(ielem,1);
  strnodred(Inods_j,1) = strnodred(Inods_j,1)+Stred(ielem,1);
  strnodred(Inods_i,2) = strnodred(Inods_i,2)+Stred(ielem,2);
  strnodred(Inods_j,2) = strnodred(Inods_j,2)+Stred(ielem,2);
  strnodred(Inods_i,3) = strnodred(Inods_i,3)+1;
  strnodred(Inods_j,3) = strnodred(Inods_j,3)+1;
end
for i = 1 : npnod
  strnodred(i,1:2) = strnodred(i,1:2)/strnodred(i,3);
end
```

```
fprintf(fid,['Result "Moment" "Load Analysis"  1  Vector OnNodes \n']);
fprintf(fid,['ComponentNames "Mx", "Sy", "Zero" \n']);
fprintf(fid,['Values \n']);
```

We end by choosing our method, complete or reduced. If method="red", complete integration is assumed, else, reduced integration is assumed:

```
method="red";
for i = 1 : npnod
    if red=="red"
   fprintf(fid,['%6.0f %12.5d  %12.5d  0.0 \n'],i,strnodred(i,1),strnodred(i,2));
    else
   fprintf(fid,['%6.0f %12.5d  %12.5d  0.0 \n'],i,strnod(i,1),strnod(i,2));
    end
end
```

Results change for the shear stress with the **two node/1 element** case and the default ***cant_P_1*** file delivered to students. Comparing them:

### Reduced integration:

```
Result "Moment" "Load Analysis"  1  Vector OnNodes
ComponentNames "Mx", "Sy", "Zero"
Values
    1 -4.95356e-02       -00001  0.0
    2 -4.95356e-02       -00001  0.0
```

### Fully integrate (2  Gauss Points):

```
Result "Moment" "Load Analysis"  1  Vector OnNodes
ComponentNames "Mx", "Sy", "Zero"
Values
    1 -4.95356e-02  -2.68915e+00  0.0
    2 -4.95356e-02   6.89152e-01  0.0
```

**b) Solve the following problem with a 64 element mesh with:**

- **2 nodes Euler-Bernouilli element**
- **2 nodes Timoshenko Full Integrate element**
- **2 nodes Timoshenko Reduce Integration**

**Compare max. displacements, moments and shear for the 3 elements against A/L relationship**

*E=21000*
*V=0,25*
*P=1*
*a=[0,001,0,005,0,010,0,020,0,050,0,1,0,2,0,4]*

*L=4*
*$I=a^4/12$*

Assuming Density=1, the Inertia moment is $1/12*a^2$ for square cross section.

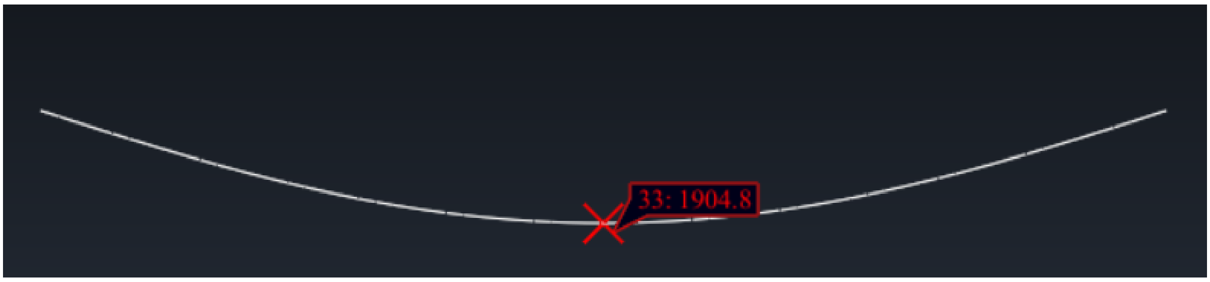
**Euler-Bernouilli**

With GID we can measure max. displacement and Mx. Both maximum values in the middle of the beam.

Moment does not change with A/L. Of course, when cross section reduced the max. displacement increases and vice versa.

Shear is 0, **at any point.**

e.g.




**Timoshenko Full Integrate element**

With Timoshenko, moment and displacement still are max.at middle point, and appears SHEAR stresses, which is zero at middle point and maximum at the edges, (but with different sign).

**Timoshenko Reduced Integrate element**

Moments and displacements still are the same but shears now are different (only one Gauss point).

Before comparison, we do calculate EXACT max. displacement, max Momentum and Max. Shear:

| real displ | real mom. | real shear |
|---|---|---|
| 1,91E+09 | 2 | 2 |
| 3,05E+06 | 2 | 2 |
| 1,91E+05 | 2 | 2 |
| 1,19E+04 | 2 | 2 |
| 3,05E+02 | 2 | 2 |
| 1,91E+01 | 2 | 2 |
| 1,19E+00 | 2 | 2 |
| 7,44E-02 | 2 | 2 |

Comparison on Displacements:

| i | a | a/L | MAX. DISPLACEMENT EULER | % error | MAX. DISPLACEMENT TIMOSHENKO | % error |
|---|---|---|---|---|---|---|
| 1 | 0,001 | 0,00025 | 1,90E+09 | 0,01% | 1,46E+06 | 130254,45% |
| 2 | 0,005 | 0,00125 | 3,05E+06 | 0,01% | 5,74E+04 | 5210,01% |
| 3 | 0,01 | 0,0025 | 1,90E+05 | 0,01% | 1,36E+04 | 1302,49% |
| 4 | 0,02 | 0,005 | 1,19E+04 | 0,04% | 2,80E+03 | 325,41% |
| 5 | 0,05 | 0,0125 | 3,05E+02 | 0,01% | 2,00E+02 | 52,08% |
| 6 | 0,1 | 0,025 | 1,90E+01 | 0,01% | 1,69E+01 | 12,89% |
| 7 | 0,2 | 0,05 | 1,19E+00 | 0,04% | 1,16E+00 | 2,62% |
| 8 | 0,4 | 0,1 | 7,44E-02 | 0,00% | 7,56E-02 | 1,59% |

So Euler method gives us  a wide range for a/L with very little error, while Timoshenko is only reliable for a/L>0,025.

Comparison on Moments:

| i | a | a/L | MAX. MOMENT EULER | % error | MAX. MOMENT TIMOSHENKO FULL&red. | % error |
|---|---|---|---|---|---|---|
| 1 | 0,001 | 0,00025 | 1,99991 | 0,0045% | 0,002 | 130272,15% |
| 2 | 0,005 | 0,00125 | 1,99991 | 0,0045% | 0,038 | 5210,93% |
| 3 | 0,01 | 0,0025 | 1,99991 | 0,0045% | 0,143 | 1302,77% |
| 4 | 0,02 | 0,005 | 1,99991 | 0,0045% | 0,470 | 325,73% |
| 5 | 0,05 | 0,0125 | 1,99991 | 0,0045% | 1,314 | 52,16% |
| 6 | 0,1 | 0,025 | 1,99991 | 0,0045% | 1,769 | 13,08% |
| 7 | 0,2 | 0,05 | 1,99991 | 0,0045% | 1,936 | 3,31% |
| 8 | 0,4 | 0,1 | 1,99991 | 0,0045% | 1,983 | 0,86% |

So maximum moments results are more reliable with Euler method for any a/L relation. Timoshenko will only be reliable for a/L > 0,0125. Even them. Euler methos is preferable on calculating moments.

Comparison on Shears

| i | a | a/L | MAX. SHEAR TIMOSHENKO FULL | % error | MAX. SHEAR TIMOSHENKO REDUCED | % error |
|---|---|---|---|---|---|---|
| 1 | 0,001 | 0,00025 | 1,439 | 39,02% | 1,96875 | 1,59% |
| 2 | 0,005 | 0,00125 | 1,377 | 45,24% | 1,96875 | 1,59% |
| 3 | 0,01 | 0,0025 | 1,198 | 66,94% | 1,96875 | 1,59% |
| 4 | 0,02 | 0,005 | 0,640 | 212,57% | 1,96875 | 1,59% |
| 5 | 0,05 | 0,0125 | 0,801 | 149,70% | 1,96875 | 1,59% |
| 6 | 0,1 | 0,025 | 1,576 | 26,91% | 1,96875 | 1,59% |
| 7 | 0,2 | 0,05 | 1,861 | 7,45% | 1,96875 | 1,59% |
| 8 | 0,4 | 0,1 | 1,941 | 3,03% | 1,96875 | 1,59% |

Shear is reliable for reduced integration at any a/L, while full integration is not-