Assignment 2.1:

1)

I have attended to many courses related to the Finite Element Method, so I have no doubts about the main ideas presented in "FEM Modelling Introduction". However, after reading it I had some doubts about some specific points. My doubts were mainly related to the applications and functions of the two special elements presented, Crack and Infinite elements. In order to solve this doubts, I looked for examples of the applications of these elements. Regarding the Infinite elements I found that they can be used, amongst other applications, in fluid-structure problems to account for the effects of the outer fluid in the inner region[1]. Regarding crack elements, I found them to be very complex and used in different applications, so I will have to look for more information about them in the future.

After reading this file I realized that I did not know what the Spectral Element Method is. Thus, I looked for more information about it and found that it is a formulation of the FEM that uses high degree piecewise polynomials as basis functions[2].

2)

Proposed questions:

1. Could we solve a simple mechanical problem only using natural boundary conditions and the Finite Element Method?
2. When does a structure have symmetry?
3. Could the problem showed in figure 1 be reduced using either symmetry or antisymmetry?

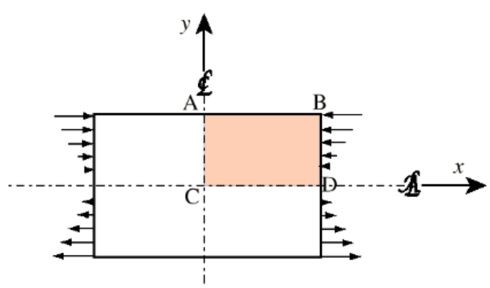

Figure 1

[1] Olson, L. and Bathe, K. "An infinite element for analysis of transient fluid-structure interactions", Eng. Comput., 1985, Vol. 2, December.

[2] https://en.wikipedia.org/wiki/Spectral_element_method

Solutions:

1. No. The stiffness matrix obtained would be singular. Moreover, the problem would have infinite solutions because we are only prescribing the derivative of the variable we are solving for.
2. A structure possesses symmetry if its components are arranged in a periodic or reflective manner.
3. Yes. The problem could be reduced to the problem depicted in figure 2.

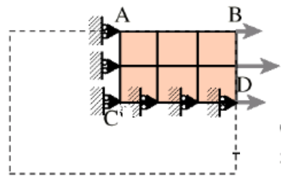

Figure 2

Assignment 2.2:

1)

δu will be kinematically admissible if both u(x) and u(x)+ $\delta u(x)$ are continuous over bar length and satisfy displacement boundary conditions. If the displacement field was not continuous, it would mean that there would be a gap in the point of the discontinuity or two different elements would interpenetrate each other (the material points initially placed at the left side of the discontinuity and the ones at the right side would have different displacements, yielding a gap or interpenetration).

2)

The local stiffness matrix obtained for elements 1 and 2 is the same as for the example of the 1^{st} lesson:

- Element 1:

$$K_1 = 10\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

- Element 2:

$$K_2 = 5\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 1 \end{bmatrix}$$

The local stiffness matrices of elements 3 and 4 are the same as for bar 3, but with half its length:

- Element 3:

$$K_3 = 2 \cdot 20 \begin{bmatrix} 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{bmatrix} = 40 \begin{bmatrix} 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{bmatrix}$$

- Element 4:

$$K_4 = 40 \begin{bmatrix} 0.5 & 0.5 & -0.5 & -0.5 \\ 0.5 & 0.5 & -0.5 & -0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \\ -0.5 & -0.5 & 0.5 & 0.5 \end{bmatrix}$$

Assembling:

$$K = \begin{bmatrix} 10+20 & 0+20 & -10 & 0 & 0 & 0 & -20 & -20 \\ & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\ & & 10 & 0 & 0 & 0 & 0 & 0 \\ & & & 5 & 0 & -5 & 0 & 0 \\ & & & & 20 & 20 & -20 & -20 \\ & & & & & 5+20 & -20 & -20 \\ & & & & & & 20+20 & 20+20 \\ \text{symm} & & & & & & & 20+20 \end{bmatrix}$$

The only external forces acting over the structure are $f_{x3} = 2$ and $f_{y3} = 1$. The system of equations obtained is:

$$\begin{bmatrix} 30 & 20 & -10 & 0 & 0 & 0 & -20 & -20 \\ & 20 & 0 & 0 & 0 & 0 & -20 & -20 \\ & & 10 & 0 & 0 & 0 & 0 & 0 \\ & & & 5 & 0 & -5 & 0 & 0 \\ & & & & 20 & 20 & -20 & -20 \\ & & & & & 25 & -20 & -20 \\ & & & & & & 40 & 40 \\ \text{symm} & & & & & & & 40 \end{bmatrix} \begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{bmatrix} = \begin{bmatrix} R_{x1} \\ R_{y1} \\ 0 \\ R_{y2} \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

The system has 5 unknowns. Applying Dirichlet boundary conditions ($u_{x1} = 0, u_{y1} = 0$ and $u_{y2} = 0$):

$$\begin{bmatrix} 10 & 0 & 0 & 0 & 0 \\ 0 & 20 & 20 & -20 & -20 \\ 0 & 20 & 25 & -20 & -20 \\ 0 & -20 & -20 & 40 & 40 \\ 0 & -20 & -20 & 40 & 40 \end{bmatrix} \begin{bmatrix} u_{x2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix}$$

The determinant of the reduced stiffness matrix is equal to 0. Thus, it is a singular matrix and the system has infinite solutions.

This infinite number of solutions is related to the fact that we have added a new node (4) in the middle of a bar which can only have axial forces and whose displacement should be in the same direction to that of node 3. However, in the system solved, the direction of the displacement of 4 was not prescribed and can take infinite solutions for a given set of external forces.

An alternative explanation is the fact that the system now has one degree of freedom and becomes a mechanism. Thus, we would need more information about the problem to solve it.