CSMD: Assignment 7: Beams

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a)

The Reduced integration 2-node Timoshenko beam element has been implemented in the Matlab code provided. The changes made in the code are:

```
%Reduced integration
K_shear = [   1    ,    len/2 ,    -1    ,    len/2 ;
            len/2 , len^2/4 , -len/2 , len^2/4 ;
             -1    ,   -len/2 ,     1    ,   -len/2 ;
            len/2 , len^2/4 , -len/2 ,   len^2/4 ];
%
```

Figure 1: Stiffness matrix

```
%reduced integration
gaus2=0;
gaus1 = gaus2;
%
```

Figure 2: Gauss points for computation of stresses

b)

The problem depicted in figure 3 has been solved using the three beam elements studied:

1. 2 nodes Euler Bernoulli element
2. 2 nodes Timoshenko element
3. 2 nodes Timoshenko Reduced Integration element

From the analyticial solution of thin beams, it is known that:

$$M(x) = \frac{Px}{2}(L-x)$$

$$y(x) = \frac{Px}{24EI}\left(x^3 - 2Lx^2 + L^3\right)$$

The maximum displacement and moment take place at $\frac{L}{2}$:

$$M_{max} = \frac{PL^2}{8}$$

$$y_{max} = \frac{5PL^4}{384EI}$$

The shear force at x=0 will be:

$$V = \frac{PL}{2}$$

The problem has been solved using a mesh of 64 elements for different values of a and keeping b = 0.02 constant. The errors obtained for the maximum displacement, bending moments and shear force are depicted in figures 4, 5 and 6. As can be seen in the pictures, the error obtained with Euler Bernouilli beam is much lower for both bending moment and displacement since it uses a higher order polynomial to describe the unknowns. If comparing the results obtained for both Timoshenko elements, it is clear that for low values of $\frac{a}{L}$, the reduced integration element provides better results since shear locking effects are avoided. However, for large values of $\frac{a}{L}$ the behavior tends to be the opposite. Regarding shear force, the results obtained with Euler-Bernouilli and Timoshenko with reduced integration elements coincide, and full-integration Timoshenko element shows the same behavior than for bending moment.

It can be concluded that Euler-Bernouilli element provides the best accuracy for thin beams, but it has larger computational cost. Timoshenko elements is computationally cheaper, but shear-locking can take place in thin beams. This problem can be solved using reduced integration (one Gauss point), providing an element that shows a good balance between accuracy and computational cost.

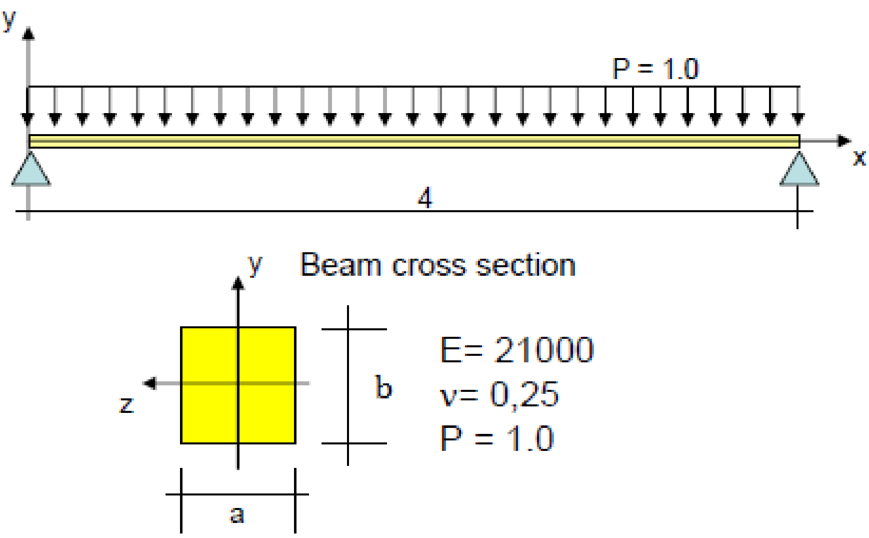

Figure 3

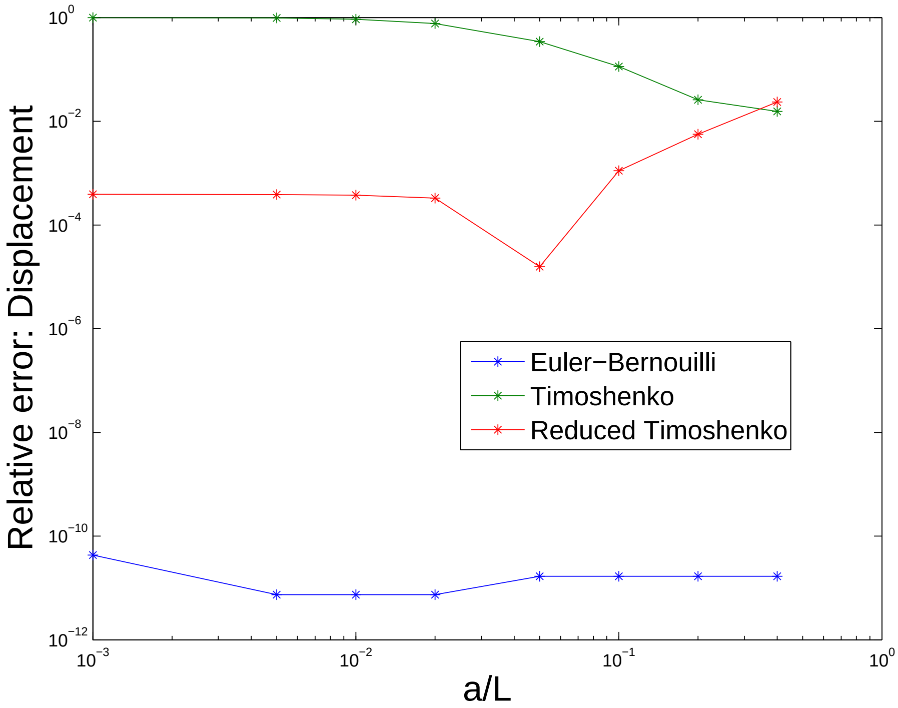

Figure 4: Error for displacement

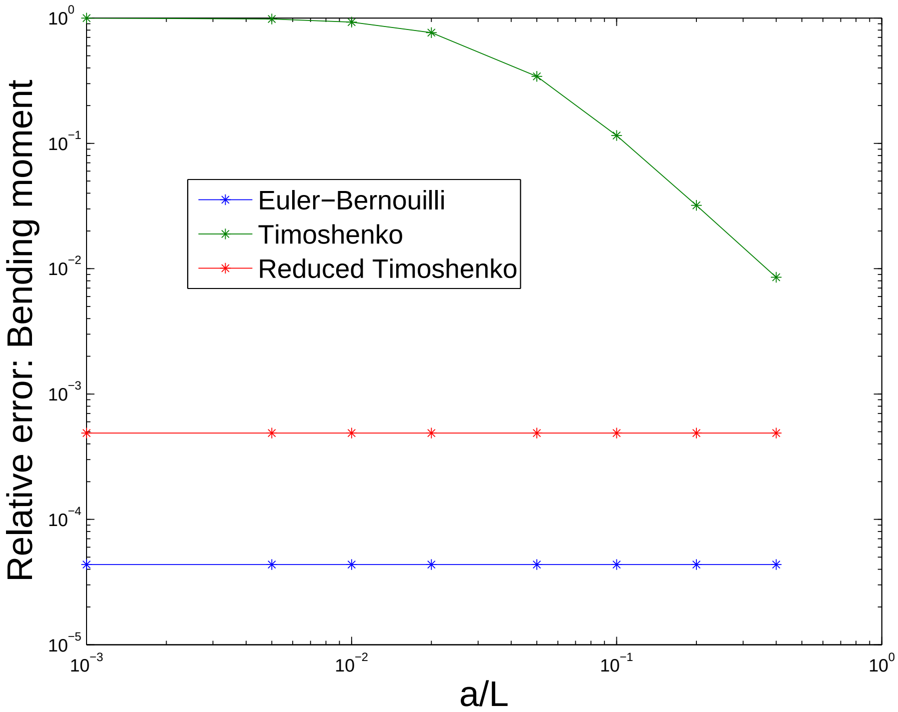

Figure 5: Error for bending moment

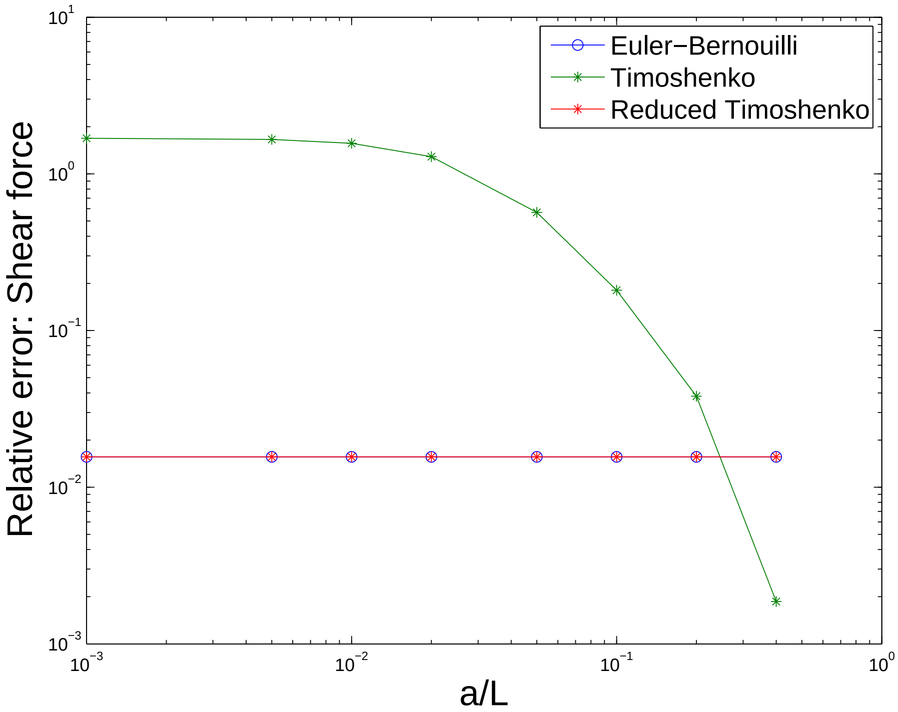

Figure 6: Error for shear force