Computational Structural Mechanics and Dynamics

Assignment – Beams

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a) Program Timoshenko 2-nodes beam with reduce integration for shear stiffness matrix

Timoshenko is one of the more common elements used to solve beam element problems, but it shows some inconvenient to obtain a good approach in some cases. Defining the bean slenderness ratio as $\lambda = L^2/h$, as the beam slenderness increases, the solution for Timoshenko element is progressivity stiffer than the exact one. So that, it is conclude that the element is not the good one for working with this kind of beams.

In order to deal with this problem, different procedures have been developed. One of the most typical is the "reduced integration" Timoshenko, that is able to reduce the influence of the transverse stiffness using a Gauss quadrature of one order less than is needed to integrate exactly the terms in ${K_s}^e$.

Reduce integration Timoshenko MATLAB code

Taking as reference Timoshenko traditional code (Figure 1), some modifications are going to be implemented in order to reproduce in MATLAB the reduced integration variations (Figure 2).

As it was said, the order of the Gauss quadrature implemented at the reduce version should be one order less, so that for 2-node beam elements just one Gauss point is going to be needed.

```
gaus1 =-1/sqrt(3);
gaus2 = 1/sqrt(3);

bmat_b=[ 0, -1/len, 0, 1/len];
bmat_s1=[-1/len,-(1-gaus1)/2, 1/len,-(1+gaus1)/2];
bmat_s2=[-1/len,-(1-gaus2)/2, 1/len,-(1+gaus2)/2];

Str(ielem,1) = D_matb*(bmat_b *transpose(u_elem));
Str(ielem,2) = D_mats*(bmat_s1*transpose(u_elem));
Str(ielem,3) = D_mats*(bmat_s2*transpose(u_elem));
```

Figure 1. Timoshenko 2-Gauss points MATLAB code

```
gaus0 =  0.0;

bmat_b = [ 0, -1/len, 0, 1/len];
bmat_s1 = [-1/len,-(1-gaus0)/2, 1/len,-(1+gaus0)/2];

Str1_g0 = D_matb*(bmat_b *transpose(u_elem));
Str2_g0 = D_mats*(bmat_s1*transpose(u_elem));
```

Figure 2. Reduced integration Timoshenko 1-Gauss point MATLAB code

Then, the ${K_s}^e$ should be modified too whereas the ${K_b}^e$ remains as in the traditional Timoshenko case. For a homogeneous material with a single integration point it turns to

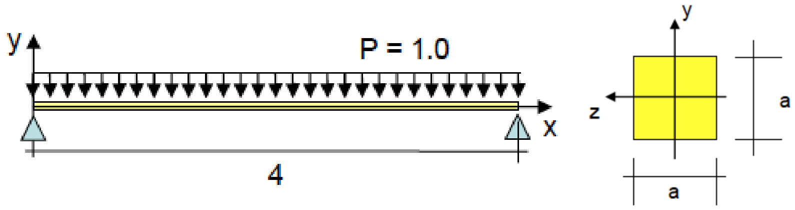
```
K_s = [   1    ,   len/2 ,    -1   ,   len/2 ;
        len/2  , len^2/4 , -len/2  , len^2/4 ;
         -1    ,  -len/2 ,     1   ,  -len/2 ;
        len/2  , len^2/4 , -len/2  , len^2/4 ];
```

Figure 3. Ks matrix for reduced Timoshenko MATLAB code

Applying this new code, Timoshenko will be able to solve as well as Euler element any kind of beam

b) Solve the following problem with 64 elements mesh with
2 nodes Euler Bernulli element
2 nodes Timoshenko Full Integrate element
2 nodes Timoshenko Reduce Integration element.
Compare the maximum displacements, moments and shear for the 3 elements against the a/L relationship.

$$\text{Beam data:} \begin{cases} E = 21000 \\ \nu = 0.25 \\ L = 4 \\ variable\ a\ and\ area \\ inertia = a^4/_{12} \end{cases}$$



The problem is going to be solved for each a value applying the three mentioned elements. Them, the results will be compared by representing in graphics the maximum y−displacement, moment and shear for each element type against the a/L and discussed.

Before calculating some conclusions can be made looking at the problem statement. As de problem is symmetric, acting at the beam a distributed constant load:

- The maximum y-displacement (absolute value) is going to be obtained at the beam middle point, $x = 2$.
- The moment graphics will have parabolic shape, achieving the maximum value at the middle point too.
- About the shear, at the middle point its value should be zero. The maximum values (positive and negative) equals in absolute value due to the symmetry, will be obtained at the two beam extreme points.

<u>Results and conclusions</u>

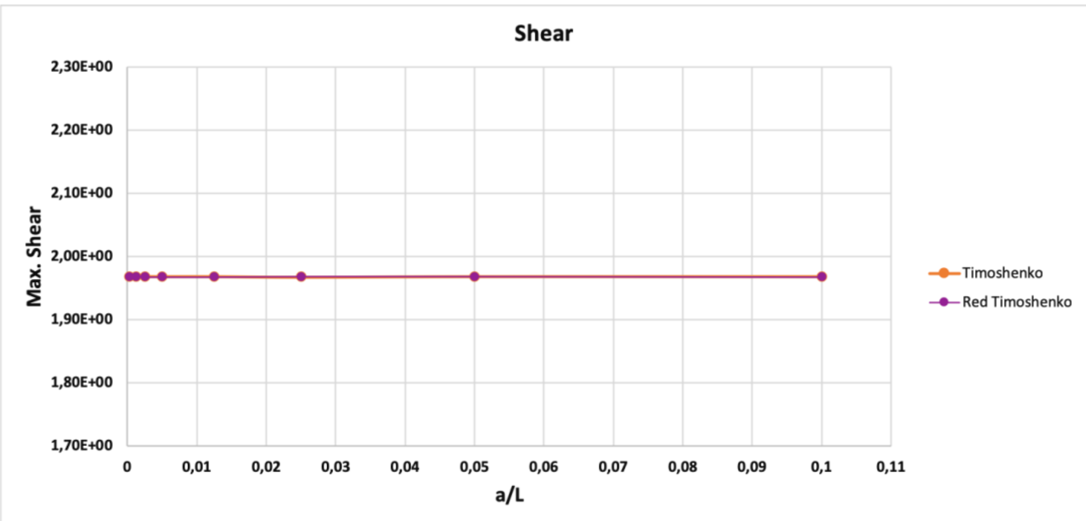


Figure 4. Max Shear graphic

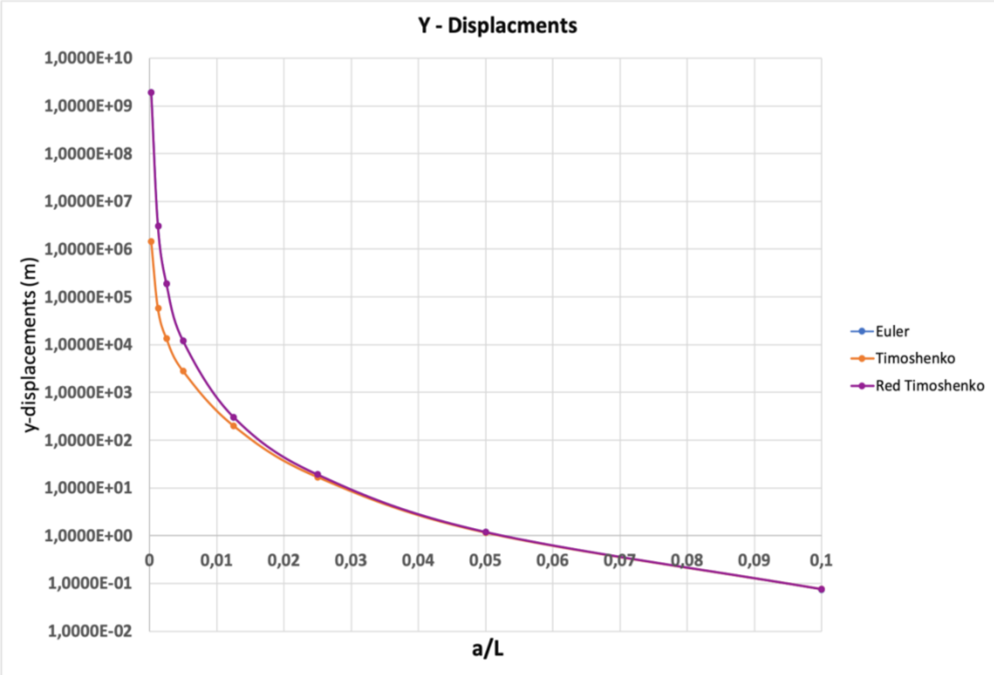


Figure 5. Max Y-displacement graphic

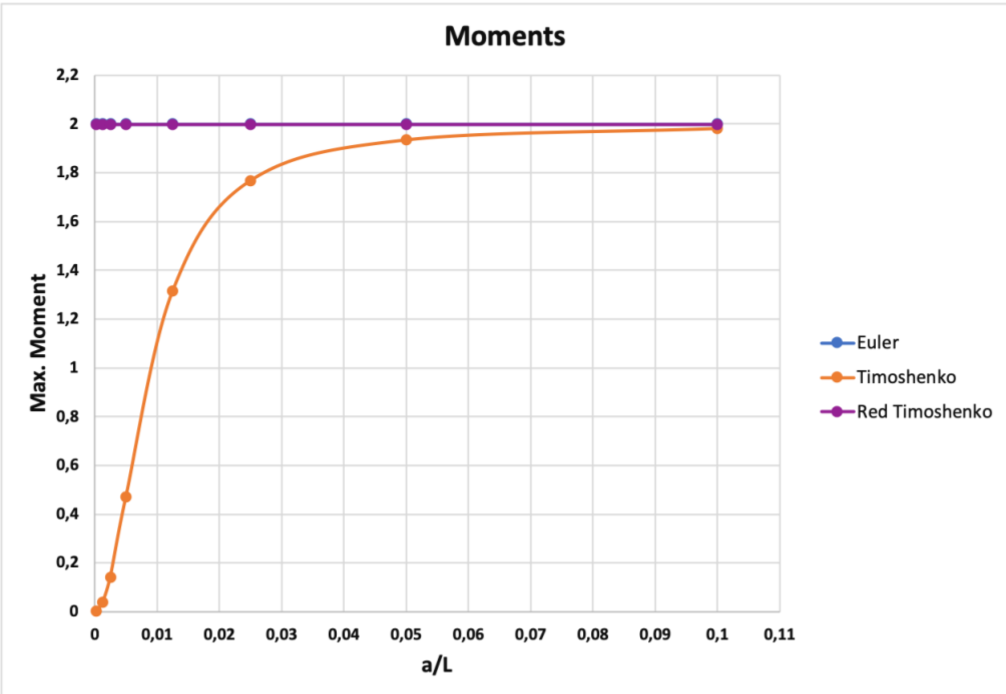


Figure 6. Max Moments graphic

In general, it can be said that the results for shear, displacement and moment fulfils with the expected. About the shear, small differences appear for the two presented Timoshenko types, and the value at the different points a/L is almost constant for each element type. This is due to the fact that a constant distributed load is applied along all the beam where the self-weight effects is almost negligible for such small areas. So that, the main force to taking into account in the shear calculation is going to be always the applied load, equal for all the studied cases.

At displacement and moment graphics, bigger differences between the element's types are observed, capturing the fact that basic Timoshenko element shows problems for that beams whose slenderness ratio is higher. Defining the slenderness ratio parameter as $\lambda = L^2/a$, check at the graphics how the difference between Euler and Timoshenko gets bigger as a/L goes to cero, whereas the solutions are almost the same as we move to the right side. The bad performance of Timoshenko is easy to observe at the moment graphics, where the values should be for any a/L almost constant and they are not.

Once the shear locking effect is removed from Timoshenko by introducing the "reduced integration", the results between Euler and this new element are so close that it is not possible to differentiating them at the graphics. It can be saw how at some point, as the a/L value increases, the solution between the two Timoshenko elements turns to be finally the same. This confirms that with some modifications, the Timoshenko elements achieve performing as well as Euler, allowing the user to choose between the most suitable option.