Master of Science in Computational Mechanics 2017

Computational Structural Mechanics and Dynamics

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Assignment 1

On "The Direct Stiffness Method":

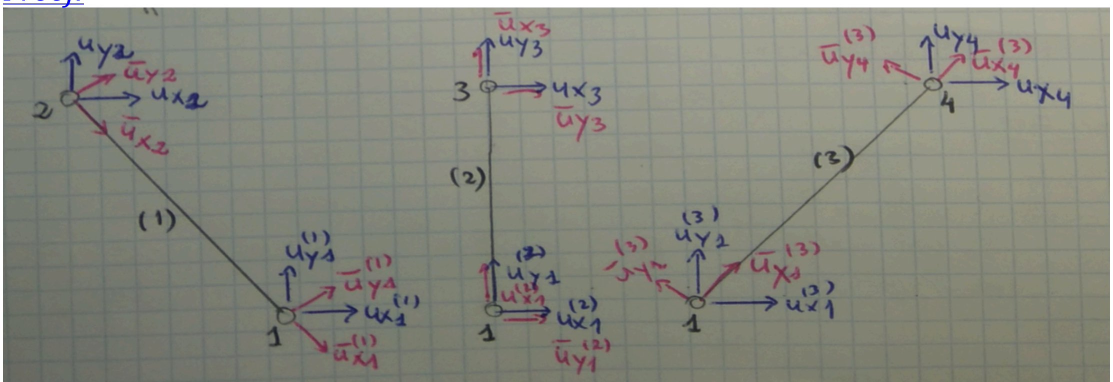
Consider the truss problem defined in the Figure. All geometric and material properties: *L, α, E* and *A*, as well as the applied forces *P* and *H*, are to be kept as variables. This truss has 8 degrees of freedom, with six of them removable by the fixed-displacement conditions at nodes 2, 3 and 4. This structure is statically indeterminate as long as $\alpha \neq 0$.

(a) Show that the master stiffness equations are

$$\frac{EA}{L}\begin{bmatrix} 2cs^2 & 0 & -cs^2 & c^2s & 0 & 0 & -cs^2 & -c^2s \\ & 1+2c^3 & c^2s & -c^3 & 0 & -1 & -c^2s & -c^3 \\ & & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\ & & & c^3 & 0 & 0 & 0 & 0 \\ & & & & 0 & 0 & 0 & 0 \\ & & & & & 1 & 0 & 0 \\ & & & & & & cs^2 & c^2s \\ \text{symm} & & & & & & & c^3 \end{bmatrix}\begin{bmatrix} u_{x1} \\ u_{y1} \\ u_{x2} \\ u_{y2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{bmatrix} = \begin{bmatrix} H \\ -P \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

in which $c = \cos\alpha$ and $s = \sin\alpha$. Explain from physics why the 5th row and column contain only zeros.

Proof:



$$K^e = (T^e)^T \bar{K}^e T^e$$

$$\bar{K}^e = \frac{EA}{L^e}\begin{pmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \; ; \; T^{(1)} = \begin{pmatrix} s & -c & 0 & 0 \\ c & s & 0 & 0 \\ 0 & 0 & s & -c \\ 0 & 0 & c & s \end{pmatrix} ;$$

$$T^{(2)} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix} \; ; \; T^{(3)} = \begin{pmatrix} s & c & 0 & 0 \\ -c & s & 0 & 0 \\ 0 & 0 & s & c \\ 0 & 0 & -c & s \end{pmatrix}$$

$$L^{(1)} = L^{(3)} = \frac{L}{c} \quad \& \quad L^{(2)} = L$$

The extended stiffness matrices:

$$K^{(1)} = \frac{EA}{L}\begin{pmatrix} cs^2 & -c^2s & -cs^2 & c^2s & 0 & 0 & 0 & 0 \\ -c^2s & c^3 & c^2s & -c^3 & 0 & 0 & 0 & 0 \\ -cs^2 & c^2s & cs^2 & -c^2s & 0 & 0 & 0 & 0 \\ c^2s & -c^3 & -c^2s & c^3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$K^{(2)} = \frac{EA}{L}\begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$K^{(3)} = \frac{EA}{L}\begin{pmatrix} cs^2 & c^2s & 0 & 0 & 0 & 0 & -cs^2 & -c^2s \\ c^2s & c^3 & 0 & 0 & 0 & 0 & -c^2s & -c^3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -cs^2 & -c^2s & 0 & 0 & 0 & 0 & cs^2 & c^2s \\ -c^2s & -c^3 & 0 & 0 & 0 & 0 & c^2s & c^3 \end{pmatrix}$$

The sum of which results in the global stiffness matrix shown earlier.

Due to symmetry of the structure and the equilibrium of the horizontal forces, the horizontal nodal force at node 3 is null, which is why the 5th row and column of the stiffness matrix contains only zeros.

(b) Apply the BCs and show the 2-equation modified stiffness system.

The BCs are $u_{x2} = u_{y2} = u_{x3} = u_{y3} = u_{x4} = u_{y4} = 0$ and the vertical (P) and horizontal (H) loads at node 1. Therefore, the reduce system looks as follows:

$$\frac{EA}{L}\begin{pmatrix} 2cs^2 & 0 \\ 0 & 1+2c^3 \end{pmatrix}\begin{pmatrix} u_{x1} \\ u_{y1} \end{pmatrix} = \begin{pmatrix} H \\ -P \end{pmatrix}$$

(c) Solve for the displacements u_{x1} and u_{y1}. Check that the solution makes physical sense for the limit cases $\alpha \to 0$ and $\alpha \to \pi/2$. Why does u_{x1} "blow up" if $H \neq 0$ and $\alpha \to 0$?

$$u_{x1} = \frac{H}{2cs^2} \cdot \frac{L}{EA}$$

$$u_{y1} = -\frac{P}{1+2c^3} \cdot \frac{L}{EA}$$

alpha tending to 0, means the three trusses will be vertical and parallel to each other leading to an unstable structure as there will be no horizontal restrains reason why u_{x1} blows up for non null values of H. Meanwhile for alpha tending to 45 degrees, the structure is restrained both vertically and horizontally.

At the same time we observe that alpha tending to zero leads to smaller vertical u_{y1} deflections (three trusses acting parallel to each other), exactly one third of what is obtained for a 45 degrees arrangement.

All the abovementioned conclusions make physical sense to me.

(d) Recover the axial forces in the three members. Partial answer: $F^{(3)} = -H/(2s) + Pc^2/(1+2c^3)$. Why do $F^{(1)}$ and $F^{(3)}$ "blow up" if $H \neq 0$ and $\alpha \to 0$?

$$\bar{u}^e = T^e u^e$$

$$\bar{u}^{(1)} = \begin{pmatrix} u_{x1} s - u_{y1} c \\ u_{x1} c + u_{y1} s \\ 0 \\ 0 \end{pmatrix}; \quad \bar{u}^{(2)} = \begin{pmatrix} u_{y1} \\ u_{x1} \\ 0 \\ 0 \end{pmatrix}; \quad \bar{u}^{(3)} = \begin{pmatrix} u_{x1} s + u_{y1} c \\ u_{y1} s - u_{x1} c \\ 0 \\ 0 \end{pmatrix}$$

$$d^{(1)} = u_{y1} c - u_{x1} s = \frac{L}{EA}\left(\frac{-Pc}{1+2c^3} - \frac{H}{2cs}\right)$$

$$F^{(1)} = \frac{EA \cdot c}{L} d^{(1)} = \frac{-Pc^2}{1+2c^3} - \frac{H}{2s};$$

$$d^{(2)} = -u_{y1} = \frac{P}{1+2c^3}\frac{L}{EA} \rightarrow F^{(2)} = \frac{EA}{L} d^{(2)} = \frac{P}{1+2c^3};$$

$$d^{(3)} = -u_{x1} s - u_{y1} c = \frac{L}{EA}\left(\frac{Pc}{1+2c^3} - \frac{H}{2cs}\right)$$

$$F^{(3)} = \frac{EA \cdot c}{L} d^{(3)} = \frac{Pc^2}{1+2c^3} - \frac{H}{2s}$$

These two axial forces blow up, for the same reasons that u_{x1} blows up (section c). As the angle alpha tends to zero, the axial forces F_1 and F_3 required to achieve equilibrium of the horizontal forces tend to infinite.

Assignment 2

Dr. Who proposes "improving" the result for the example truss of the 1st lesson by putting one extra node, 4 at the midpoint of member (3) 1-3, so that it is subdivided in two different members: (3) 1-4 and (4) 3-4. His "reasoning" is that more is better. Try Dr. Who's suggestion by hand computations and verify that the solution "blows up" because the modified master stiffness is singular. Explain physically.

Answer:

The new arrangement: segmenting truss (3) into two members, leads to another unstable structure, where the new member can move without restrains as it can rotate freely with regards to the new node. This translates on more than one possible solution, as the solution is not unique we end up with a singular stiffness matrix.

An additional horizontal member (normal to truss (2)) connecting with node 1 would do a better job.