CSMD Assignment 2

Jaume Betran 2017-Feb-14

1 Try by hand computations adding node 4 between nodes 1 and 3. Verify that the solution "blows up" because the modified master stiffness matrix is singular

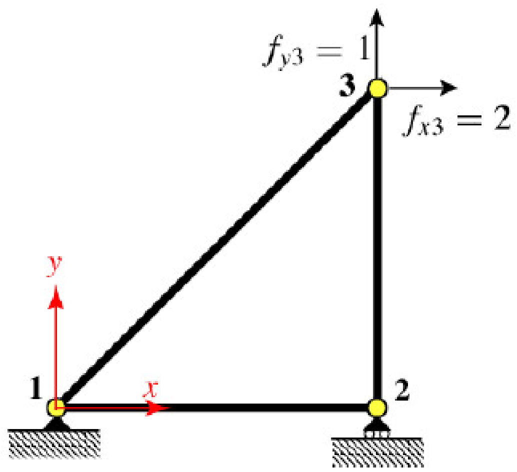

Figure 1: Original Truss

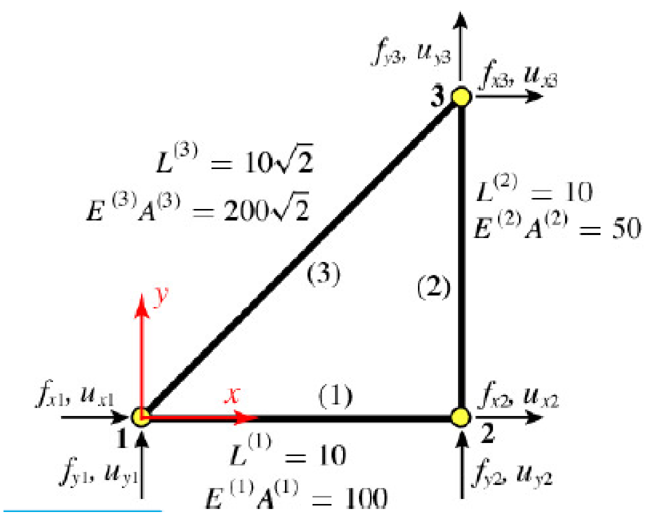

Figure 2: Original Truss with dimentions

It is assumed that the proposed solution consists of splitting bar 3 in two equal bars 3 and 4, with length L_4, and linking them by means of a joint of the same type as those present in nodes 1, 2 and 3.

The Hook law for a bar in the local reference system reads

$$\bar{\mathbf{f}}^e = \bar{\mathbf{K}}^e \bar{\mathbf{u}}^e$$

and, explicitly

$$\bar{\mathbf{f}}^e = \frac{EA}{L_4}\begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}\bar{\mathbf{u}}^e$$

For bars $e = 3, 4$, connecting node pairs 1-4 and 3-4 respectively, the transformation between local and global coordinates systems reads

$$\bar{\mathbf{u}}^e = \mathbf{T^e}\mathbf{u}^e$$

$$\bar{\mathbf{f}}^e = \mathbf{T^e}\mathbf{f}^e$$

with

$$\mathbf{T^e} = \begin{bmatrix} 1 & 1 & 0 & 0 \\ -1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \end{bmatrix}$$

And the elemental stiffness matrix in global coordinates system for bars $e = 3, 4$

$$\mathbf{K}^e = (\mathbf{T^e})^T\bar{\mathbf{K}}^e\mathbf{T^e} = \frac{EA}{L_4}\begin{bmatrix} 1 & 1 & -1 & -1 \\ 1 & 1 & -1 & -1 \\ -1 & -1 & 1 & 1 \\ -1 & -1 & 1 & 1 \end{bmatrix}$$

$$\mathbf{K^e} = \begin{bmatrix} \mathbf{K^e_d} & \mathbf{K^e_{offd}} \\ \mathbf{K^e_{offd}} & \mathbf{K^e_d} \end{bmatrix}$$

In view of the elemental matrix for bars 3 and 4 there is no need to completely build the global stiffness matrix. The global stiffness matrix has the following shape:

$$\mathbf{K} = \left[\begin{array}{ccc|c} & & & \mathbf{K^e_{offd}} \\ & \mathit{Symmetry} & & \mathbf{0} \\ & & & \mathbf{K^e_{offd}} \\ \hline \mathbf{K^e_{offd}} & \mathbf{0} & \mathbf{K^e_{offd}} & 2\mathbf{K^e_d} \end{array}\right]$$

Sub-matrices $\mathbf{K^e_{offd}}$ and $\mathbf{K^e_d}$, from the elemental stiffness matrix, are singular. For the present particular geometry, with two truss bars at $45\deg$ with a common node, the singularity is made evident before computing the complete global matrix and trying to compute its inverse. Rows 7 and 8 of are exactly equal and therefore the matrix is singular.

2 Explain physically

The reason why the global stiffness matrix is singular for this truss is because the system has at least one kinematic degree of freedom. The system is indeed a four-bar linkage which is known to have one degree of freedom.

The original truss consisted of 3 bars with three links and three fixations u_{x1}, u_{y1} and u_{y2}. Bars have 3 DOFs (x, y and θ), links cancel 2 DOFs (x and y displacements). The latte configuration is a static system. The modified truss consists of 4 bars with four links and three fixations which results in 1 kinematic DOF.

Table 1: Degrees if freedom

members	Original Truss	Modified Truss
bars	$3 \times 3 = 9$	$4 \times 3 = 12$
links	$3 \times 2 = 6$	$4 \times 2 = 8$
fixations	3	3
k-DOFs	0	1