Computational Structural Mechanics and Dynamics - Practice 3

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1 Clamped plate with a uniform load

1.1 Introduction

The objective of this exercise is to analyze the differences between three type of elements in a problem modeled within the theory of thin plates. The problem is a concrete square fixed in all the borders under a normal load (Figure 1) with the properties of Table 1. Three different elements have to be compared: linear triangle (DKT element), quadratic Reissner-Midlin triangles (RM elements) and linear quadrilaterals (CLLL element). These numerical results have to be compared with the maximum displacement of the analytical solution at the central point.

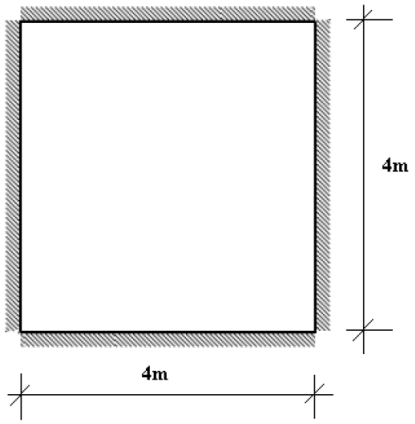


Figure 1: Problem Statement

E	ν	t	q
$3.0 \cdot 10^{10} \frac{N}{m^2}$	0.2	0.1m	$1 \cdot 10^{4} \frac{N}{m^2}$

Table 1: Problem properties

1.2 Preprocess

The models has been created in GID with the problem type Ramseries Educational 2D, plates extension. The geometry of the example is just a 4m square which is very easy to define with one rectangle and the thickness of 0.1m is introduced in the material properties. The boundary conditions are fixed vertical displacement in all the border and are imposed in the label of displacement constrains, impeding the external lines. The load is introduced as uniform normal load in all the surface.

The problem has been modeled with three different meshes using DKT, RM and CLLL elements but all of them have been meshed with a mesh size of 0.1m. The resulting meshes dimensions are summarized in Table 2.

Element type	Nº elements	Nº nodes
DKT	3200	1681
RM	3200	6561
CLLL	1600	1681

Table 2: Mesh properties summary.

1.3 Results

To be able to compare the numerical solution with an analytical one the following formula for the maximum displacement of a thin plate fixed in all the boundary with uniform load has been used (1):

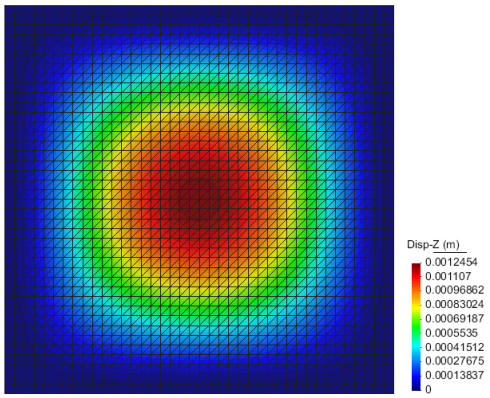
$$d_{max} = \frac{0.0284qb^4}{2.056Et^4} \tag{1}$$

The solution to this problem intuitively can be guessed that the maximum displacement will occur in the central point and that in the rest of the domain there will be a continuum distribution till 0 on the borders due to the boundary conditions. This is in agreement with the numerical results obtained and presented in .
The results of the analytical solution and of the three numerical solutions and its error are presented in Table 3.

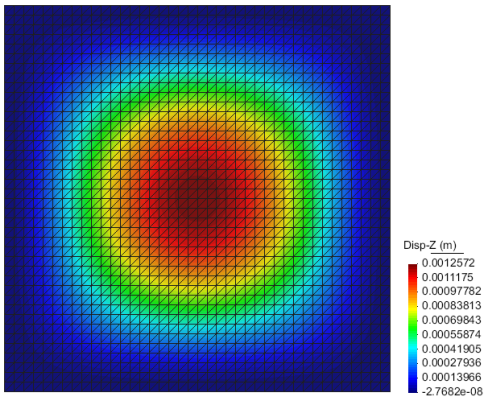
Method	d_{max}	Error (%)
Analytical	0.0011787	0.00
DKK	0.0012454	5.656184
RM	0.0012572	6.657262
CLLL	0.0012567	6.614844

Table 3: Results and relative errors for the three different meshes.

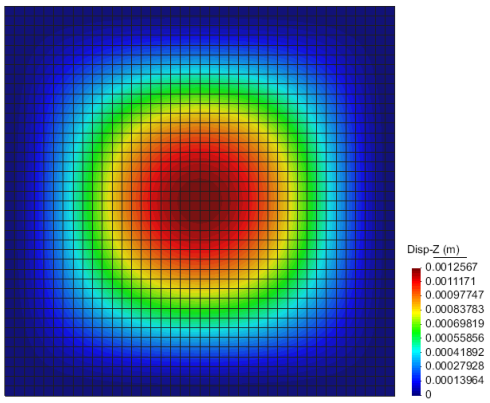
In this case looking at the results for the same element size the method with few error is the DKK linear triangle followed by the CLLL linear quadrilateral and the RM quadratic triangle.



(a) DKT mesh



(b) RM mesh



(c) CLLL mesh

Figure 2: Vertical displacement distribution of the Example 1 for three element types.

2 Thin Plate with Internal Hole

2.1 Introduction

In this exercise, a steel plate supported on four columns has to be structurally analyzed. The theory to use is the one of thin plates, with thickness $t = 0.05$ and a load applied to the full plate equal to $q = 1e4\frac{N}{m^2}$.

If we recall the theory of the subject, only Z-displacement should appear, while X and Y displacements remain unaltered.

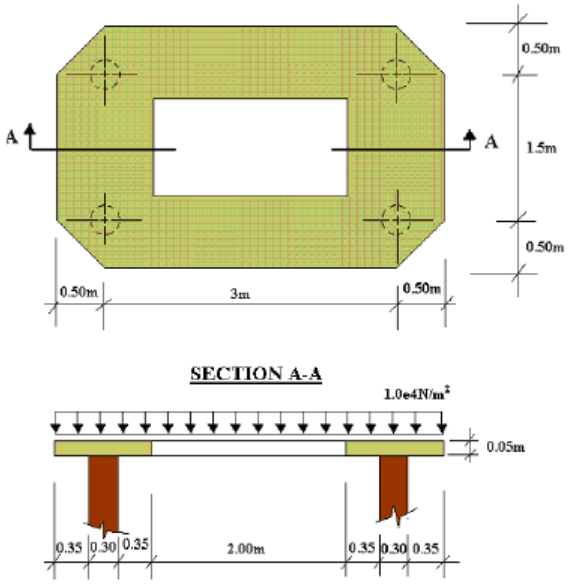


Figure 3: Problem Statement

2.2 Preprocess

To obtain a continuum mesh between the plate and the steel columns, different surfaces must be created. A plate with holes and 4 additional surfaces, one for each hole, are required. The mesh used has the following characteristics:

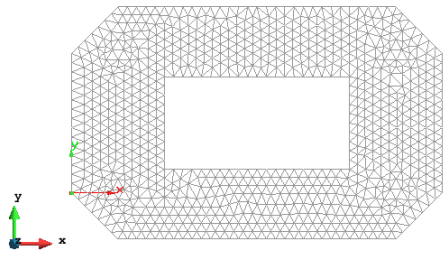


Figure 4: Mesh used. 896 nodes, 1614 elem.

2.3 Results

We obtain logical results, with bigger Z-displacements in the areas where the columns are not holding the structure. Alternatively, stresses are concentred in the column supports, where we get all the reactions to gravity and the load applied.

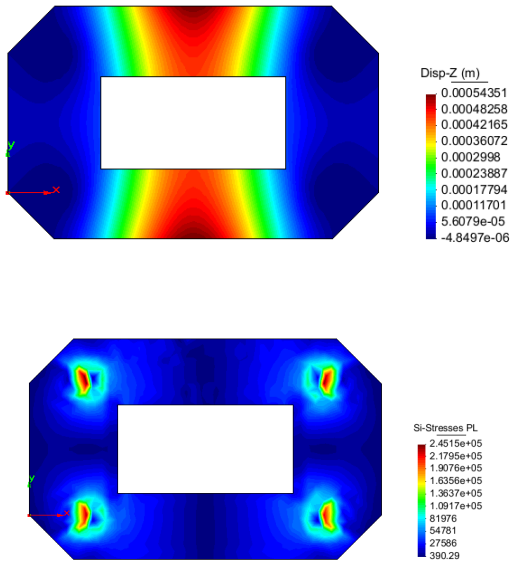


Figure 5: Z displacements and Principal Stresses S_1

3 Thick circular plate with internal hole

Type of Element	N° of Elements	N° of nodes
R-M 6 nodes	455	995

Table 4: Caption

3.1 Introduction

This exercise propose a circular reinforced concrete plate geometry, supported with four columns (Figure: 6). Now, the results will be analyse using thick plates Reissner-Mindlin theory.

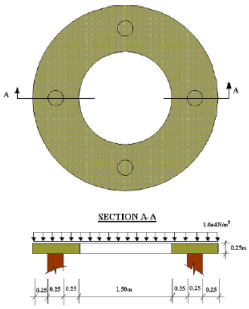


Figure 6: Problem Statement

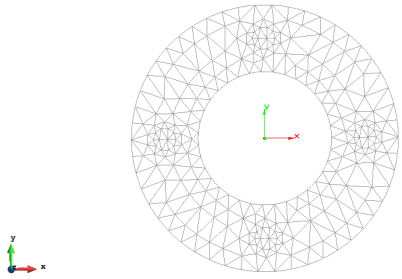


Figure 7: Mesh

3.2 Preprocess

Similarly to exerceise 2, to obtain a continuum mesh between the plate and the four columns, different surfaces had been created. A plate with holes and 4 additional surfaces, one for each hole, are required. The mesh used has the following characteristics:

3.3 Results

We obtain as before, the results expected: with the maximum Z-displacements in the areas where the columns are not holding the structure. Stresses are concentrate in supports, where the reaction to the loads applied appears. It is interesting to observed how the displacements are located in the exterior radius of the circular plate, in this area the distance between the supports is bigger an then large displacements occurs.

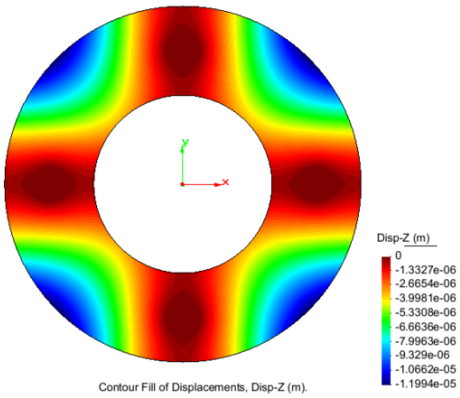


Figure 8: Z-Displacements

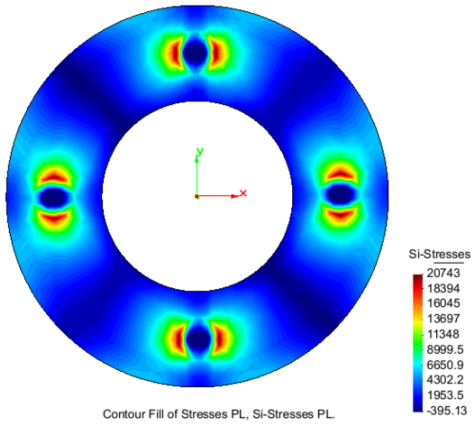


Figure 9: Si Stresses