Universitat Polytechnica de Catalunya
MSc Computational Mechanics
Spring 2018

Computational Structural Mechanics and Dynamics

Practice 2

Due 06/04/2018

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Exercise 1

For the first part of this assignment a circular tank under hydro-static load, as depicted in Figure (1), will be analyzed.

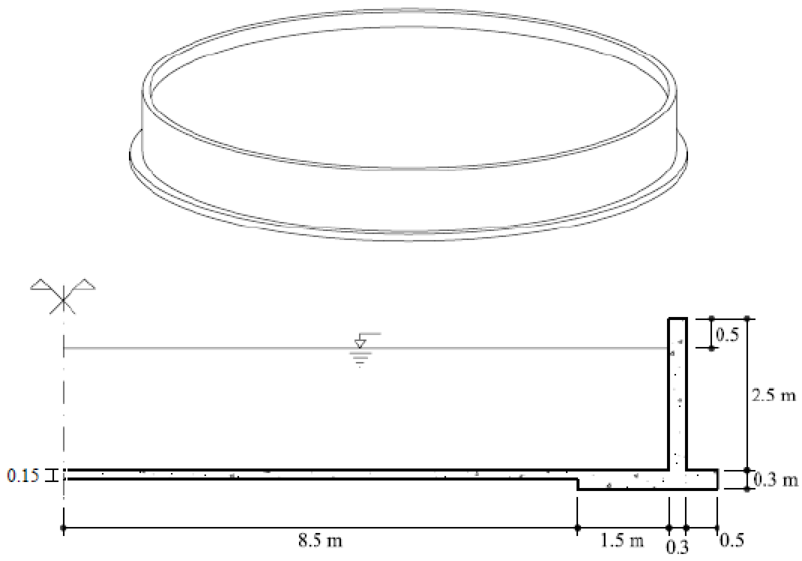


Figure 1: Circular Tank under hydrostatic load

The boundary conditions and assumptions for the problem are as follows:

- Linear z-constraint along the leftmost vertical edge as this is a structure of revolution
- Load in the negative y direction along the tank floor to simulate the water hydrostatic pressure
- Linear load along the right inner wall of the tank to simulate the water hydrostatic pressure on the wall linearly varying with depth
- Vertical elastic constraints along the two horizontal surfaces on the bottom of the tank
- Horizontally elastic constraint along the right-most edge

These elastic constraints will simulate soil. All boundary conditions can be seen in Figure (2).

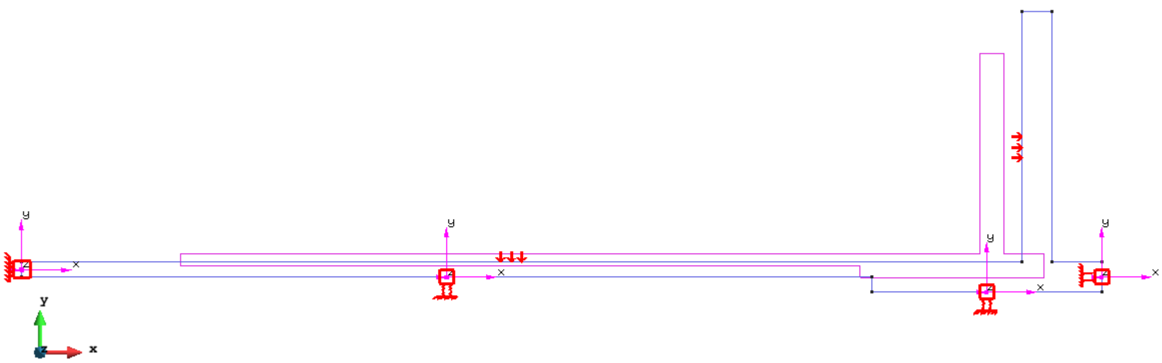


Figure 2: All conditions applied to tank

The material property and problem data are summarized as follows:

Material	**E [MPa]**	ν [-]
Concrete	3.0E10	0.2

Table 1: Material properties and problem data

In order to analyze the structure stress/strain state, a mesh convergence test is carried out for the 4-nodes bi-linear quadrilateral element. Figure (3) shows the convergence of the maximum displacement norm, which is located at the tank center (left endmost geometry side), with respect to mesh nodes density.

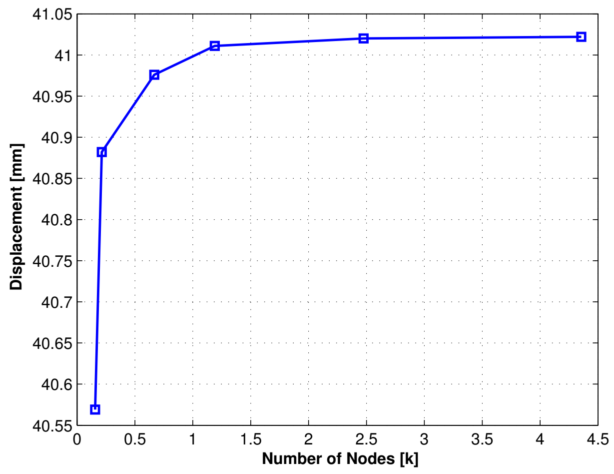


Figure 3: Mesh convergence test: Displacement norm

As observed in Figure (3), the displacement norm exhibits convergence for a mesh density of around 1100 nodes. We will use this mesh, shown in Figure (4), for the following analysis.

Figure 4: 1.1K nodes converged mesh

The vertical displacement u_y alongwith the structure deformed shape is shown in Figure (5).

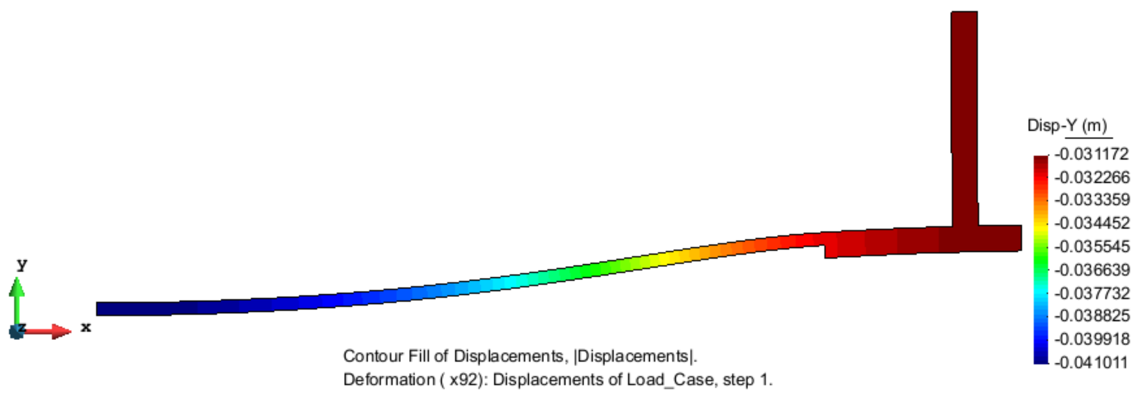


Figure 5: Tank u_y displacement

We can see from the above result that the center of the structure undergoes the higher vertical displacement. There is also some bending observed along the vertical walls because of the presence of linearly distributed horizontal load. This can be better understood from the following load diagram.

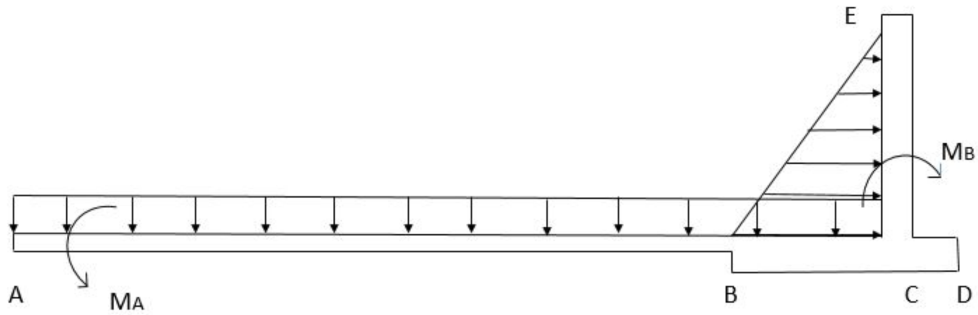


Figure 6: Simplified loads diagram

We can see from the above figure that there is practically no load acting on in the region C-D and a uniformly distributed load is acting on the region A-C. A horizontal linear load is acting against the walls of the tank E-C. We can see that there is least vertical displacement along the region E-C-D which makes sense because of the absence of any direct vertical load in that region. Considering this is a structure of revolution the maximum vertical displacement will naturally occur at point A, which can be verified from the results of post process. We also have moments resulting from these two loads. Analyzing the structure wrt point C, we find the moment MB tends to rotate the structure in clockwise direction while the moment MA tends to rotate the structure in counter-clockwise direction. This results in the displacement profile shown previously.

As the structure is made of **Concrete**, which is a **Brittle** material, for failure point of view analysis, Von-Mises stress can not be taken into account. In this case, principal stresses ((σ_1, σ_2 and σ_3) are the appropriate ones which are usually employed in failure theory criteria of Brittle materials (e.g.,Maximum Normal Stress, Coulomb-Mohr or Christensen) and compared with maximum tensile (σ_t) and compressive (σ_c) material strength.

As it can be seen in Figure (7), the maximum principal stress σ_1, which is tensile one, is at the junction of the vertical wall with the tank floor (stress conccentration point) and also at the upper floor surface, near the region the tank base changes its thickness. Those high stress are mostly due to the bending the hydrostatic water pressure causes in the tank vertical wall.

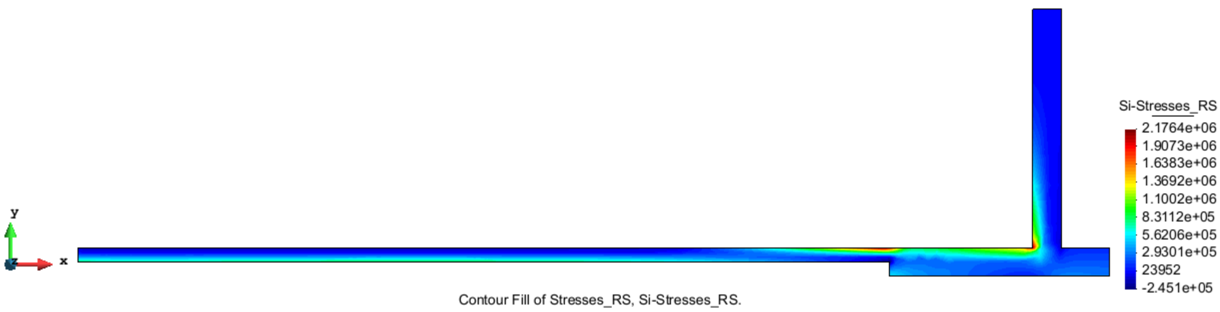


Figure 7: First principal stress σ_1

The higher principal stress $sigma_3$, which is a compressive one, seen in Figure (8) is found at the lower floor surface, just in the junction where the floor changes its thickness, which is a stress concentration point.

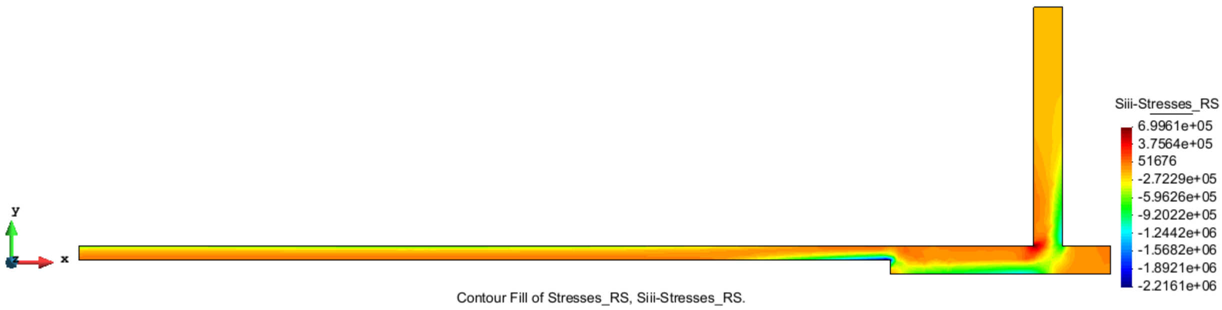


Figure 8: Third principal stress σ_3

Exercise 2

In the second part of this assignment a cantilever beam under a force couple at its far end, as seen in Figure (9), will be analyzed.

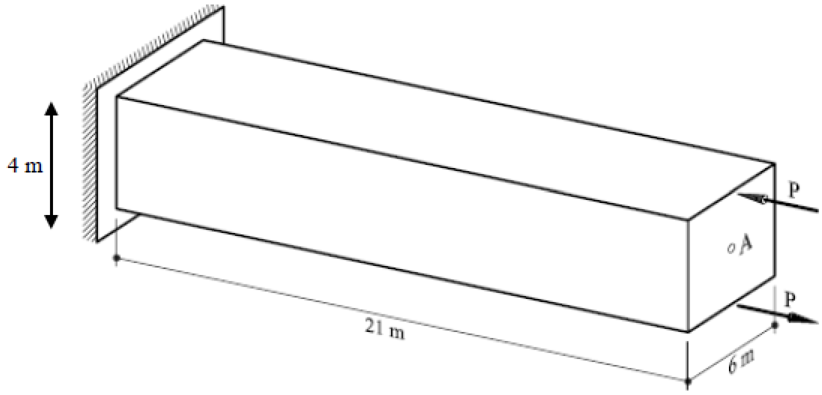


Figure 9: Cantilever Beam

The boundary conditions and assumptions for the problem are as follows-

- (x,y,z)-constraint on the left end of the beam (fixed).
- A compressive point load of 10000N applied at the middle of the top right end of the beam.
- A tensile point load of 10000N applied at the middle of the bottom right end of the beam.
- The right most end is free and without any constraints.

All boundary conditions can be seen in Figure (10).

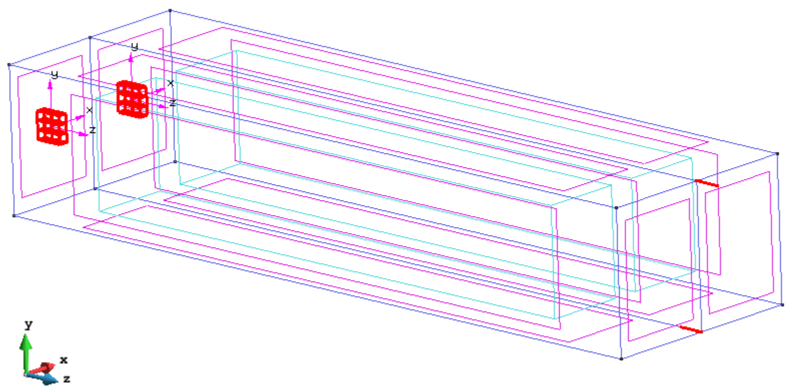


Figure 10: Cantilever beam Boundary conditions

The material property and problem data are summarized as follows:

Material	**E [MPa]**	ν [-]
Steel	2.1E11	0.2

Table 2: Material properties and problem data

Before starting the analysis, a mesh convergence test using hexahedral elements was conducted in order to define the *"ideal"* mesh for the problem. This was done by considering the maximum vertical displacement of the free end for different number of nodes.

The theoretical result for the maximum vertical displacement in the bem is given by:

$$u_{y_{max}} = -\frac{ML^2}{2EI} \tag{1}$$

Applying the problem data we can find the expected theoretical value:

M [N.m]	**L [m]**	**E [m]**	**I** $[m^4]$	u_{max} [m]
40×10^3	21	2.1×10^{11}	32	1.3125×10^{-6}

Table 3: Theoretical maximum displacement calculation data.

As can be seen from the result displacement convergence was not achieved for mesh densities up to 5k nodes. In order to avoid a higher computational cost a mesh with 5k nodes was chosen as the one to used in the next analysis. Also, the FEM results seems to converge for a different value than the exact one.

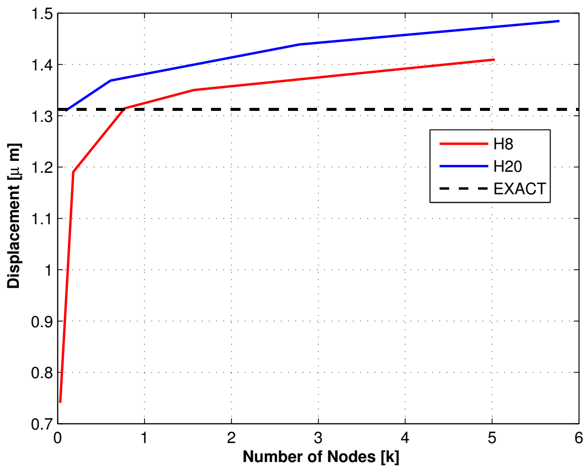


Figure 11: Mesh convergence test: Maximum Displacement

In order to keep a feasible computational cost, the mesh with 5k nodes, shown in Figure (12) was chosen as the *"ideal"* one.

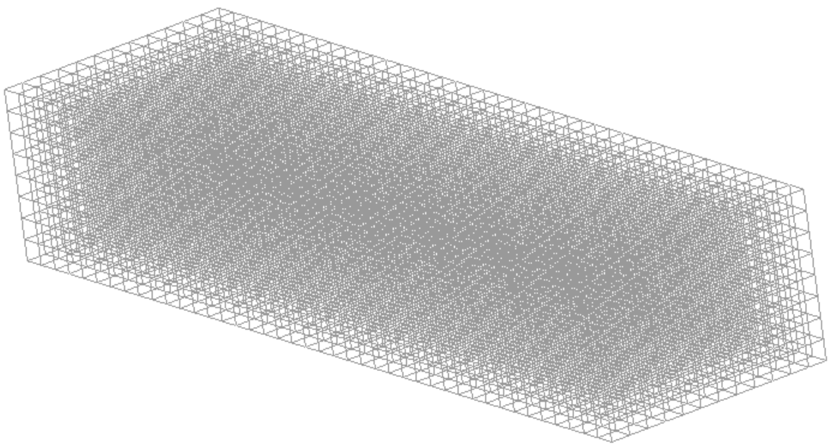


Figure 12: Hexahedral Mesh with 5k nodes

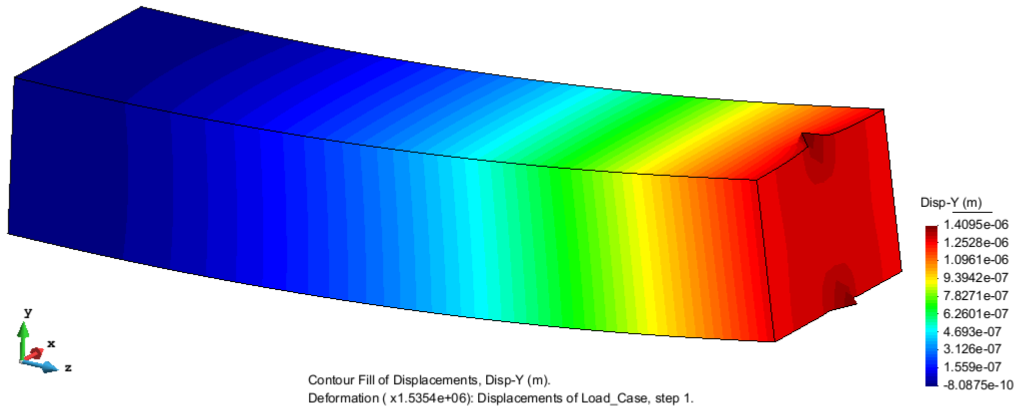
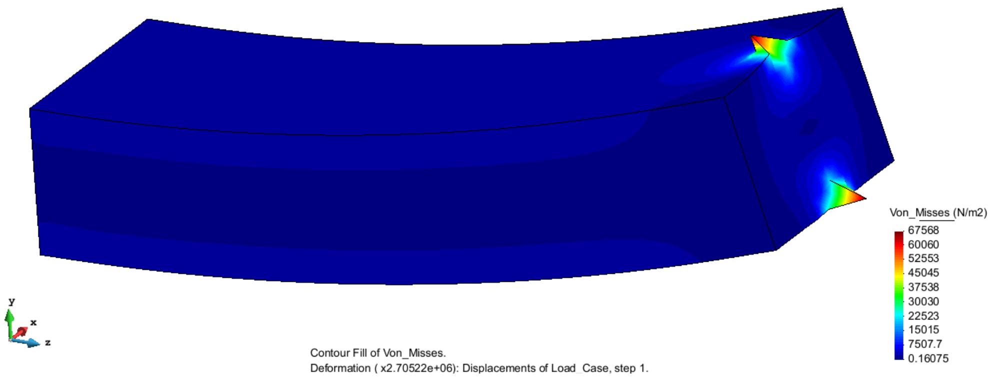


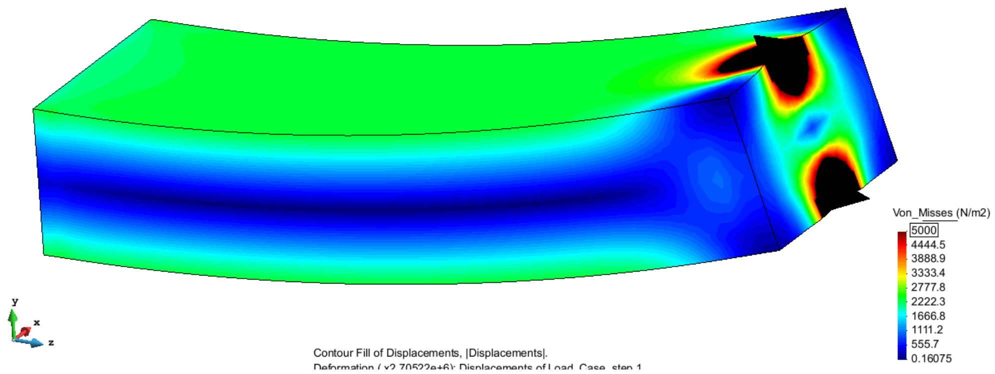
Figure 13: Vertical displacement in the cantilever beam

Since we have a force couple acting on the cantilever beam as shown in the problem, we have deformations as expected. The anti-clockwise couple generates a bending effect in the beam as can be seen in the vertical displacement profile of the cantilever beam, resulting from the compression of the top-most surface under the effect of a compressive force and the tensile effect of the tensile force on the lowermost surface. We also have tensile and compressive deformations on the points of applications of the respective forces.

Below we have the results of a Von-Mises stress analysis with shown deformation.



(a)



(b)

Figure 14: Von-Mises stresses generated in the cantilever beam: (a) Original;(b) Adjusted scale

The high intensity of stresses at the point of application of the loads was making it difficult to analyze the stress intensity at other regions. In order to visualize the results better, the maximum Von-Mises stress in the color bar was set to 5000 N/m^2 in Figure (14). As expected, we see the highest stresses are where the point loads are applied, due to the high stress concentration. Also, as expected by any beam theory, higher Von-Mises stress are found in top and bottom surface of the beam at the fixed supports. Also as we move in the direction of the beam mean line, the stresses are reduced near to zero.

Exercise 3

In the last part of this assignment, a foundation of a corner column, see in Figure (15), under eccentric point load is analyzed.

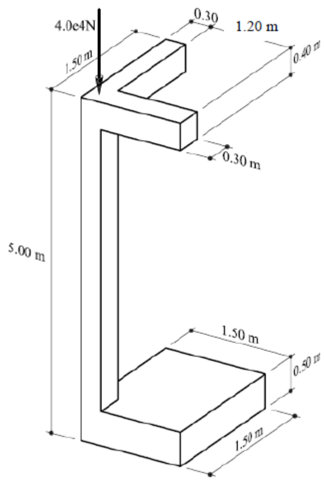


Figure 15: Foundation of a corner column.

The boundary conditions and assumptions for the problem are as follows-

- A point load of 40000N applied at the top of the concrete structure.
- Elastic soil constraints placed on the bottom of the structure and the two outer sides adjacent to the corner of the structure up to 1.5 meters.
- A surface load equivalent to the weight of 1 meter-deep soil applied to the top of the lower slab.
- (x,y)-directional constraints applied to the surfaces of the lower slab in normal directions as this structure only represents a quarter of the original structure.

The geometry of this problem can be seen in Figure (16).

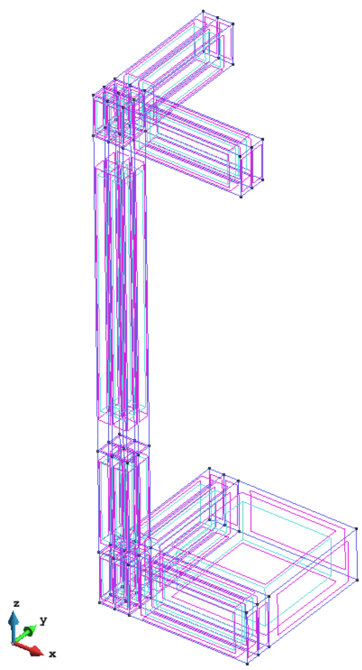


Figure 16: Problem geometry specialized for hexahedral elements

The ground was modelled by taking the Ballast Coefficient $K_x = 50E6N/m^3$. The material property and problem data are summarized as follows:

Material	**E [Pa]**	ν [-]
Concrete	3.0E10	0.2

Table 4: Material properties and problem data

As it was done for previous problems, before drawing conclusions towards the stress state of the structure, a mesh convergence test was carried out. Vertical displacement u_y was tracked for the top-left corner adjacent to the point of application of the compressive load, where the vertical displacement is maximum (17).

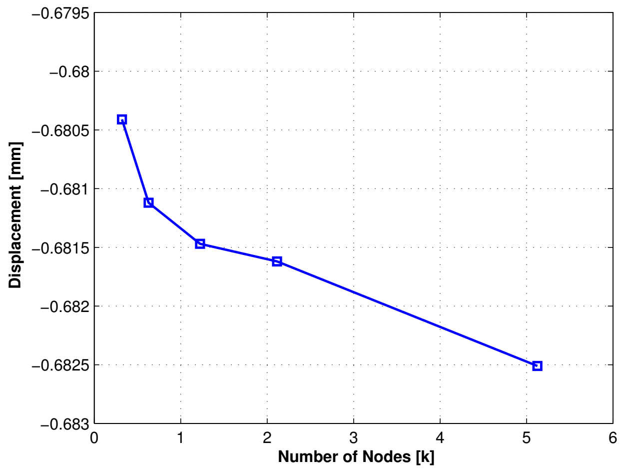


Figure 17: Mesh convergence test: Maximum Displacement.

As it can be seen, the displacement did not have full convergence due to limitations to run the problem with more dense meshes and the resulting high computational cost. Thus, results will be analyzed in a qualitative way using the most dense mesh obtained (5k nodes).

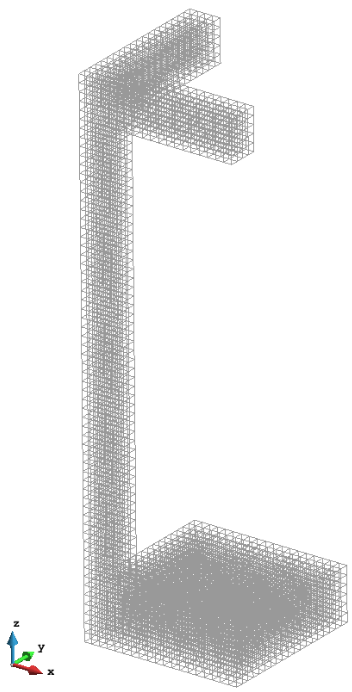
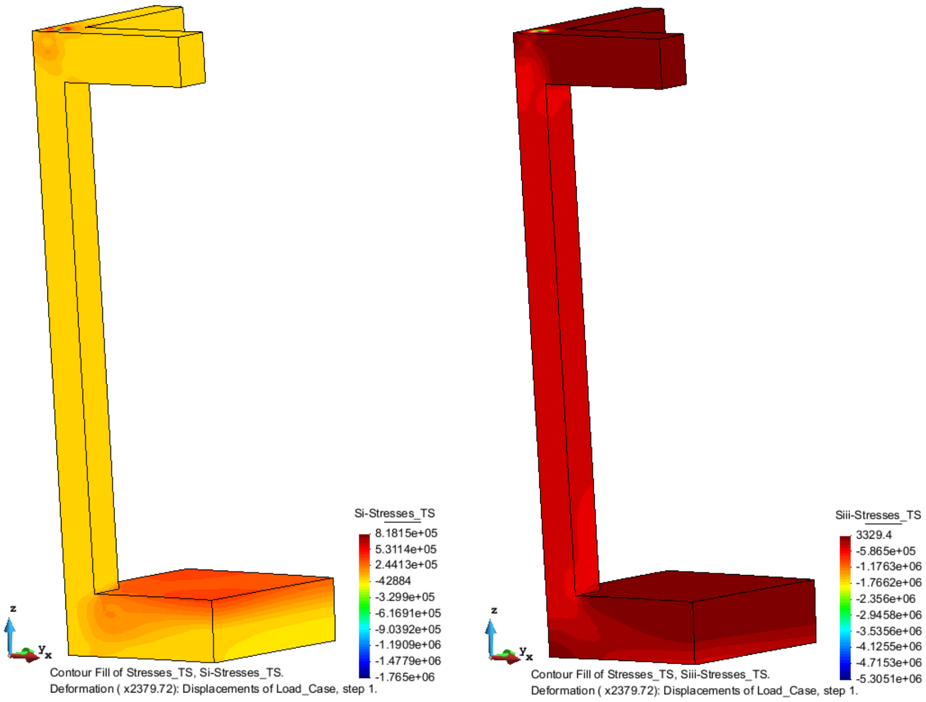


Figure 18: Hexahedral Mesh with 5k nodes

From Figure (19) it is observed the maximum principal stresses (σ_1 and σ_3) in the corner column/foundation structure.



(a) (b)

Figure 19: Principal stresses: (a) σ_1;(b) σ_3.

From this results we can see that the higher principal stresses σ_1 and σ_3 are located in the point where the load is applied. This expected given the high stress concentration implied by this load application.

Apart from that, the displacement variation along the bottom slab, as can be seen from the Figure(20), results in a stress generation along the surface of the bottom slab as can be perceived by the resultant principal stresses σ_1 and σ_3 in this region, shown in Figure (19).

Below we can see, in Figure (20), the resulting Z-direction displacement of the structure in question.

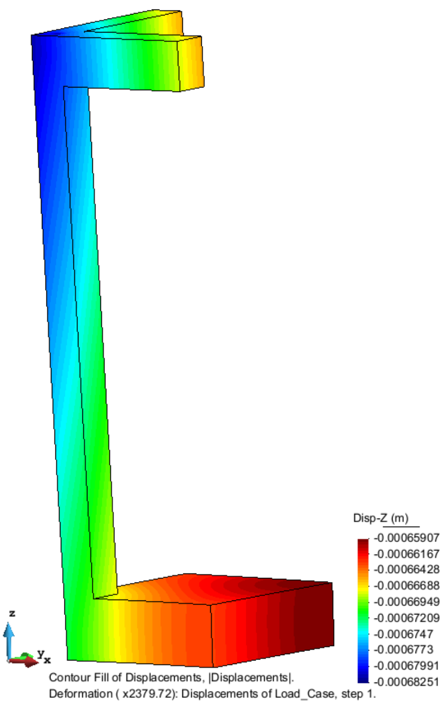


Figure 20: Displacement of structure in Z-direction

We can see that the region that experiences the most displacement is the top-left corner near where the point load is applied. The support reactions from the ground, resulting from the point load is eccentric and behaves in a non-uniform fashion. Since the reaction force on the bottom slab is non-uniform, the z-displacement is also non-uniform along the base of the slab. The part of the slab not connected to the structure undergoes a relatively low downward displacement compared to the rest of the base, resulting in a lifting effect.