Polytechnic University of Catalonia

Msc. in Numerical Methods

Computational Structural Mechanics and Dynamics

Assignment 2.1:"FEM Modelling Introduction"

Assignment 2.2: "Variational Formulation of Bar Element"

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February 21, 2016

Assignment 2.1: FEM Modelling Introduction

1. Formulate three of your doubts about this subject.

a) It is known that the aspect ratio of the elements affect the quality of the results. The proportion between lengths and angles of the elements and the size compared to other elements can be used to quantify the mesh quality. What is the criteria to decide if an element is "good" or "bad" in a quantitative way? Are sometimes "bad" elements used in practice?

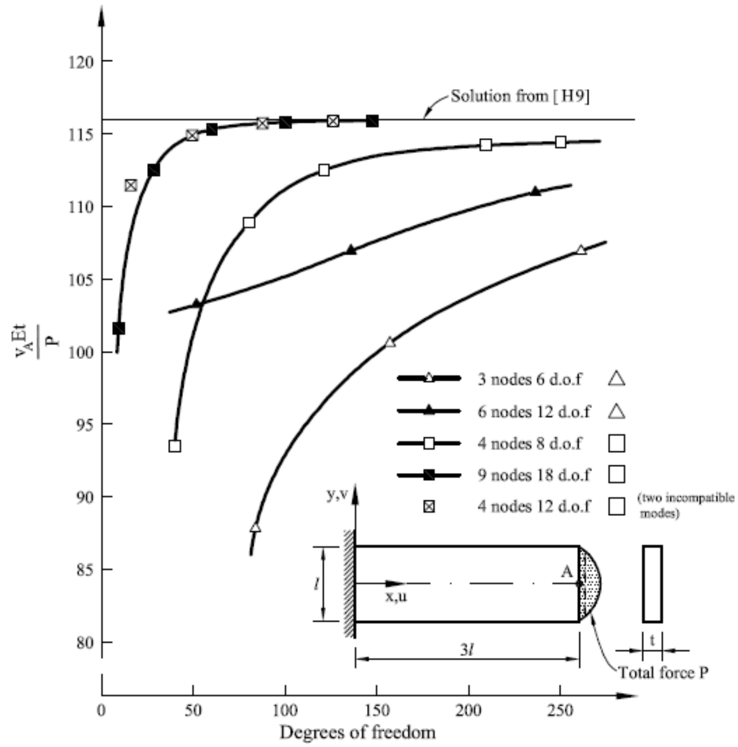
b) How are Macro Elements defined and why are they useful?

c) How are elastic boundary conditions applied in the stiffness equation system?

2. Formulate three questions that you would ask in an exam on this subject. Write down the answers that you would consider correct.

a) Why quadrilateral elements are more precise than triangles elements?

The interpolation function of the linear triangular element includes only linear terms (1,x,y) while the linear quadratic element includes and additional quadratic term (1,x,y,xy). The same concept can be extender to higher order or 3D elements. This leads to a higher convergence ratio, or a better approximation of the stress field and therefore the forces, as can be observed in the following figure:



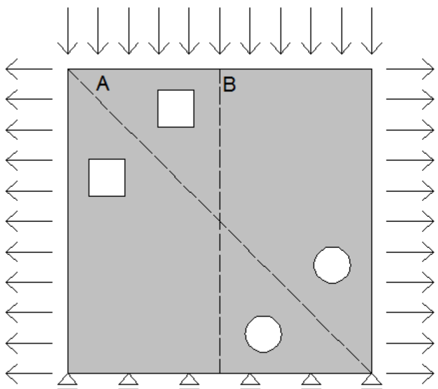
Cantilever deep beam under parabolic edge load (v = 0.2). Analysis with 3- and 6-noded triangles, 4- and 9-noded rectangles and the 4-noded rectangle with two incompatible modes.
Ref: Fig. 9.13 OÃśate, Diez, Zarate, Larese - Introduction to Finite Element Method - Oct. 2008

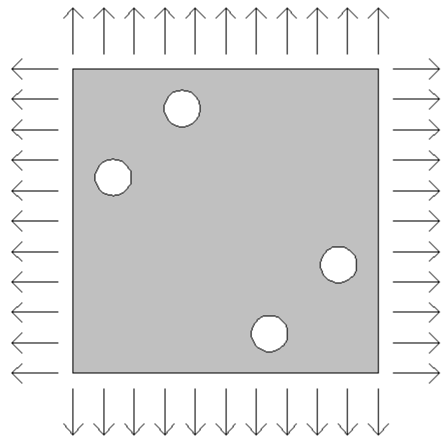
b) Modelling, discretization and solution errors are a consequence of the solution of a physical problem using discrete methods. How are they generated, which is the most dangerous one and why?

Modelling errors come from the process of converting a physical system in a mathematical model. To do this hypothesis are done, therefore physical reality is not represented exactly.
Discretization errors appear when the mathematical model (usually a PDE) that can not be solved analytically is decomposed in a simpler system of equations (using FEM or FDM). This transformation implies an approximation to the real functions that govern the mathematical model.
Finally, solutions errors are the product of solving the system of equations using computer algorithms and the limitations they imply.

Of the three of them, the most critical is the modelling error because the misunderstanding of the physical phenomena can lead to solving an unreal problem, and although the discretization and the solving of the problem are done right, the answers would be totally wrong.

c) Can the following domains be divided applying symmetry of BC?





1. No, it can not. It exist symmetry of both geometry (along line A) and boundary conditions (along line B), but they are not coincident. Meaning that if the domain was divided following geometrical symmetry the loads and boundary conditions would be different for the other domain, and the same the other way round.
2. Yes, it can. Due to symmetry along both diagonals (for geometry and boundary conditions), the domain can be divided in four triangles. Restrictions to displacements should be applied in the contact lines, allowing the displacement parallel to them.

Assignment 2.2: Variational Formulation of Bar Element

1. Explain the kinematic admissibility requirements stated in slide 6 in terms of physics, namely ruling out the possibility of gaps or interpenetration as the bar material deforms.

a) Continuity over the bar length: if the displacements were discontinuous, that would mean that the bar could have at the same time one physical part at two different positions.
b) Satisfaction of displacement at BC: boundary conditions are places in the mathematical (and discrete) model were displacements are known. This comes from the analysis of the physical model and the idealization of the structure. There would be no physical sense in results that would violate already known displacements.

2. Dr. Who proposes "improving" the result for the example truss of the 1st lesson by putting one extra node, 4 at the midpoint of member (3) 1-3, so that it is subdivided in two different members: (3) 1-4 and (4) 3-4. His "reasoning" is that more is better. Try Dr. Who's suggestion by hand computations and verify that the solution "blows up" because the modified master stiffness is singular. Explain physically.

From the example solved in the example, we know the stiffness system for elements 1 and 2. The new elements 3 y 4 have analogue equations than the original element 3, but with half the length, therefore the stiffness doubles.

Globalized Element Stiffness Equations

Element 1:

$$\begin{bmatrix} f_{x1}^1 \\ f_{y1}^1 \\ f_{x1}^1 \\ f_{y1}^1 \end{bmatrix} = 10 \begin{bmatrix} 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 \\ -1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} u_{x1}^1 \\ u_{y1}^1 \\ u_{x1}^1 \\ u_{y1}^1 \end{bmatrix}$$

Element 2:

$$\begin{bmatrix} f_{x2}^2 \\ f_{y2}^2 \\ f_{x3}^2 \\ f_{y3}^2 \end{bmatrix} = 10 \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0 & -0.5 \\ 0 & 0 & 0 & 0 \\ 0 & -0.5 & 0 & 0.5 \end{bmatrix} \begin{bmatrix} u_{x2}^2 \\ u_{y2}^2 \\ u_{x3}^2 \\ u_{y3}^2 \end{bmatrix}$$

Element 3:

$$\begin{bmatrix} f_{x1}^3 \\ f_{y1}^3 \\ f_{x4}^3 \\ f_{y4}^3 \end{bmatrix} = 10 \begin{bmatrix} 2 & 2 & -2 & -2 \\ 2 & 2 & -2 & -2 \\ -2 & -2 & 2 & 2 \\ -2 & -2 & 2 & 2 \end{bmatrix} \begin{bmatrix} u_{x1}^3 \\ u_{y1}^3 \\ u_{x4}^3 \\ u_{y4}^3 \end{bmatrix}$$

Element 4:

$$\begin{bmatrix} f_{x4}^4 \\ f_{y4}^4 \\ f_{x3}^4 \\ f_{y3}^4 \end{bmatrix} = 10 \begin{bmatrix} 2 & 2 & -2 & -2 \\ 2 & 2 & -2 & -2 \\ -2 & -2 & 2 & 2 \\ -2 & -2 & 2 & 2 \end{bmatrix} \begin{bmatrix} u_{x4}^4 \\ u_{y4}^4 \\ u_{x3}^4 \\ u_{y3}^4 \end{bmatrix}$$

After the assembly of the global system and reductions due to boundary conditions ($x1$,$y1$,$y2$), the global reduced equations take take the form of:

$$\begin{bmatrix} 0 \\ 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} = 10 \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ & 2 & 2 & -2 & -2 \\ & & 2.5 & -2 & -2 \\ & & & 4 & 4 \\ symm & & & & 4 \end{bmatrix} \begin{bmatrix} u_{x2} \\ u_{x3} \\ u_{y3} \\ u_{x4} \\ u_{y4} \end{bmatrix}$$

The determinant of the stiffness matrix is zero, therefore the system is singular and can not be solved.

Physically, this phenomena can be explained due to the fact the structure is hipostatic. There are now 3 nodes aligned (1,4,3), which result in an unstable configuration, as the bar elements used can not bear and transmit bending moment.