Description of the problem

This report deals with the analysis of a hyperbolic shell. The shell is made out of concrete and it is subjected to self-weight only and fixed from all sides. Hyperbolic shells gain strength through their shape as opposed to deriving it from their mass as normal flat shells. The curvature of the shape reduces its tendency to buckle in compression (as a flat plane would) which means that they can achieve exceptional stiffness. By being braced in two directions they experience no bending and are able to withstand unequal loading.

The given structure has the following dimensions (Figure 1):

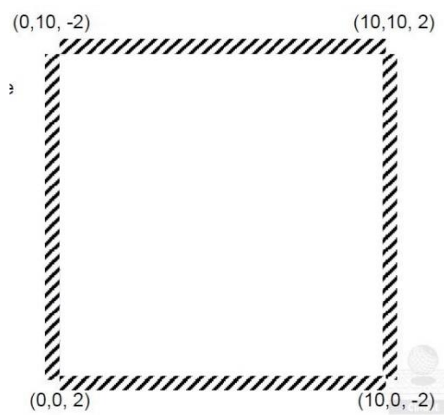

Figure 1 Shell dimensions

The structure was discretized using Reissner-Mindlin triangular shell element. In order to assure that the obtained solution would be of a sufficient accuracy, a fine mesh was employed. This resulted in the utilization of 1,800 elements with 961 nodes. The mesh is shown in Figure 2.

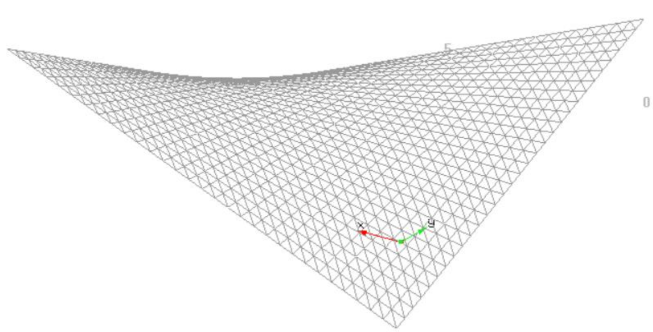

Figure 2 Hyperbolic shell mesh

Results and discussion

This analysis of the structure described in the above section generates the following results.

The deformation that occurs in the hyperbolic shape is shown in Figure 3.

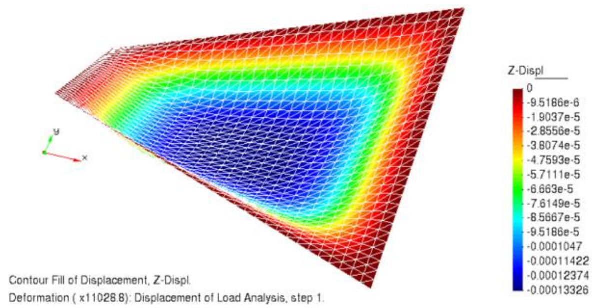

Figure 3 Deformed shape

In order to further analyze this deformation, the displacements in the x, y and z axis are shown in Figure 4.

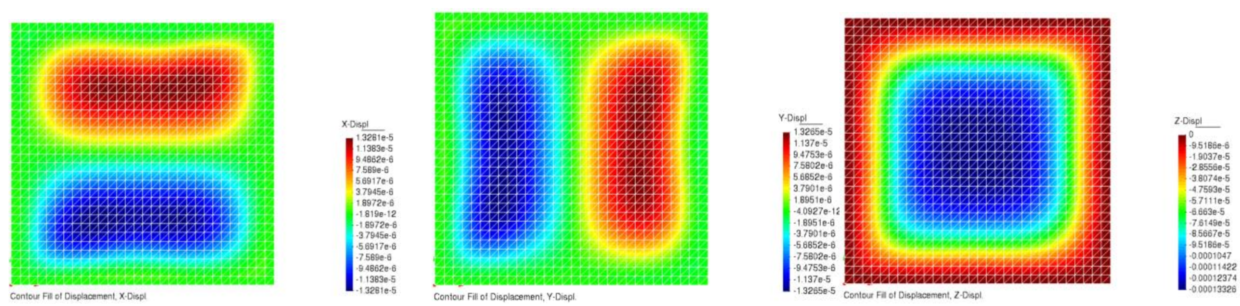

Figure 4 Results for the X, Y and Z displacements respectively

For the z displacement, it could be seen that, as expected, the maximum occurs at the middle of the shell structure. This is due to the effect of self-weight being a uniform load hence the maximum z displacement is expected to occur at the furthest point from the supports. As for the x and y displacement, it could be seen that they are aligned with the results shown with the results of the local rotations shown in Figure 5. The x rotation couple results in a displacement that is equal and opposite in the y direction. Similarly, the y rotation couple results in a displacement that is equal and opposite in the x direction.

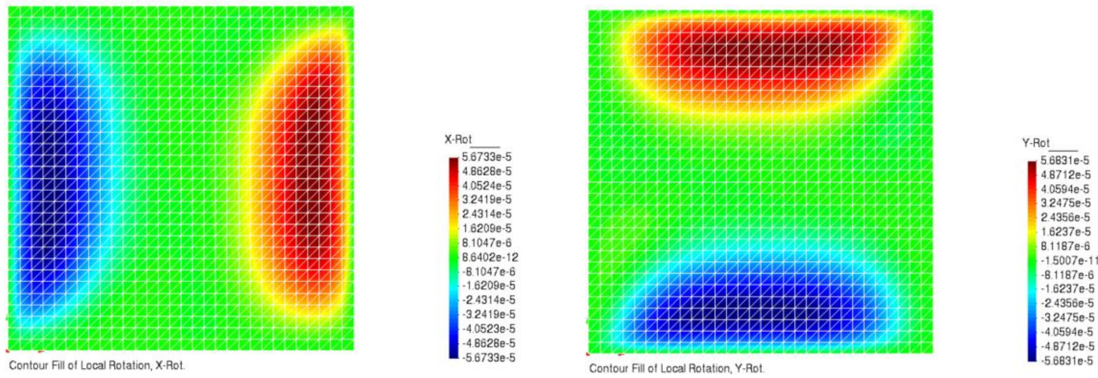

Figure 5 Results for the local rotations about the X and Y axis respectively

These displacements, or rather strains, result in stresses inside the structure. These stresses are categories as bending moment stresses, membrane stresses and shear stresses. There general configuration for an infinitesimal element is shown in Figure 6.

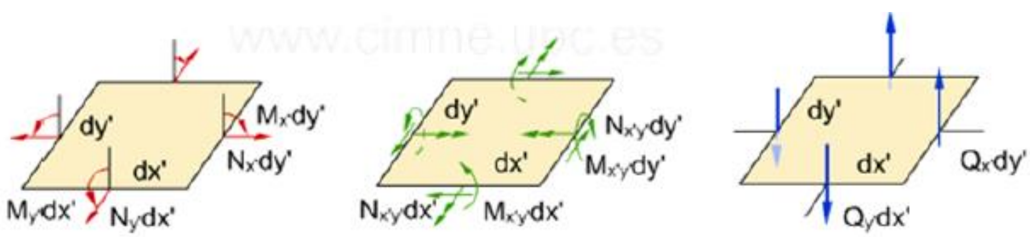

Figure 6 Stresses for an infinitesimal element

The results for the bending moment stress (Figure 7) are similar for the x and y directions. Depending on the normal to the direction, the maximum positive bending moment stress occurs near the supports. The minimum bending moment stress occurs in the regions with the highest magnitude of local rotations shown in Figure 5. It could be noted that the Mx and My bending moment stresses are minimal at the center of the shell. As for the twisting bending moment Mxy, it could be noted that its maximum values occur at the corners. In these positions the maximum and minimum points of the shell structure are present. The maximum positive twisting moment is correlated with the corners with the maximum points in the structure and vice versa.

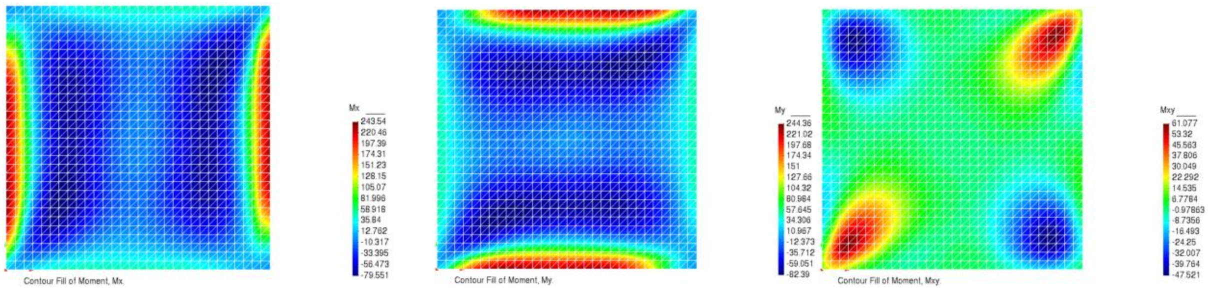

Figure 7 Results for the Moment stress Mx, My and Mxy respectively

The magnitude of the bending moment shown in Figure 8 could be used for a better interpretation of the results. From this plot, the previous observation that the middle of the plate is subjected to the minimal bending stresses is confirmed. It could be also noted that due to geometry of the hyperbolic shell, bending stress is distributed throughout the structure in a manner that reduces it. This is aligned with the statement mentioned in the introduction where the utilization of hyperbolic geometry would result in a stiffer structure as the stresses it is subjected to are reduced.

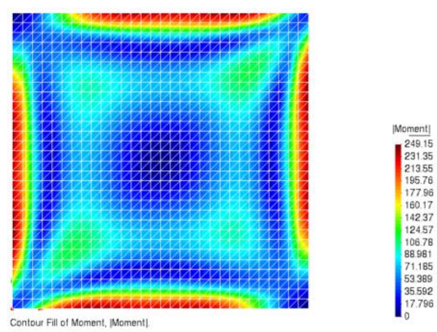

Figure 8 Magnitude of the bending moment

The results for the membrane stresses are shown in Figure 9. It could be noted that the membrane stresses occur in couples in the x and y directions. As for the Txy membrane stress it could be noted that the maximum value occurs at the center of the shell while the minimum occurs at the corners.

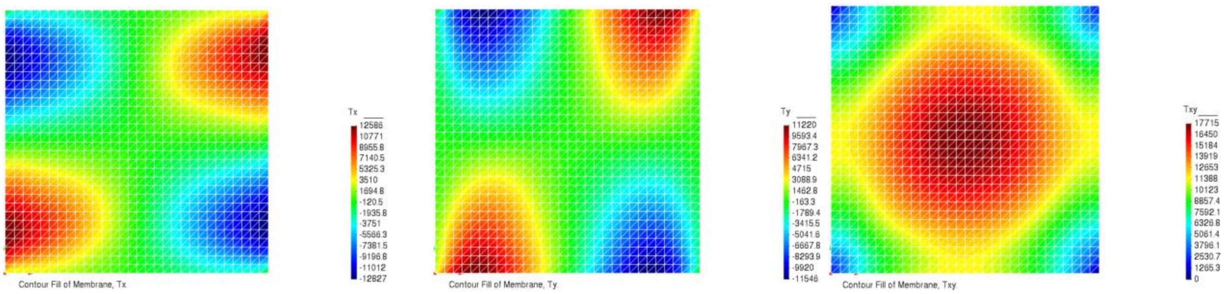

Figure 9 Results for the membrane stress Tx, Ty and Txy respectively

The magnitude of the membrane stress shown in Figure 10Figure 8 could be used for a better interpretation of the results. It could be seen that the maximum values occur at the middle of the shell while the higher values are concentrated around the diagonals.

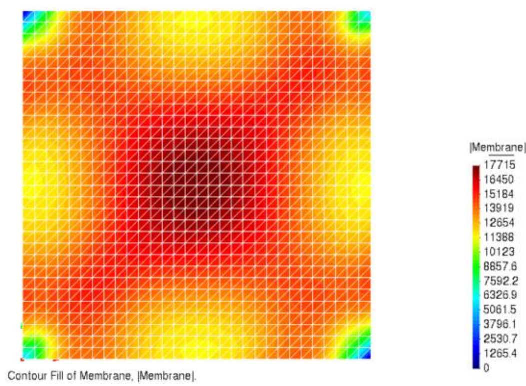

Figure 10 Magnitude of the membrane stress

The shear stresses are shown in Figure 11. It could be noted that the maximum values occur at the supports in a coupled manner which is an expected behavior when a uniform load is applied.

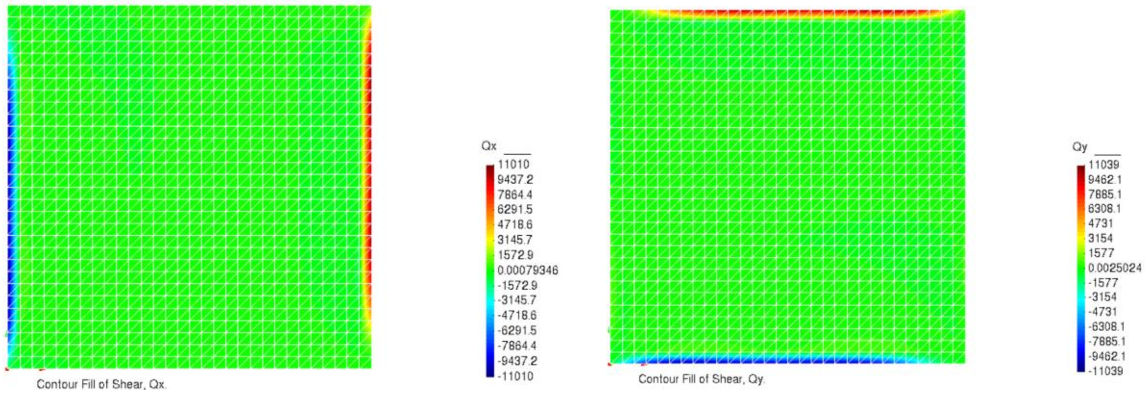

Figure 11 Results for the shear Qx and Qy respectively