

Plates Assignment

Hanna, John

1 Problem 1

A 5*5 plate simply supported plate with uniform load is studied. 5*5 quadrilateral elements are used: once using MZC element, and once using the RM element. Different thicknesses are used to study the shear locking phenomenon in the RM element.

Thickness	z-disp (MZC)	z-disp (RM)	% difference
0.001	2.41E+09	2.28E+09	5.67
0.01	2.41E+06	2.28E+06	5.67
0.02	3.02E+05	2.85E+05	5.66
0.1	2.41E+03	2.28E+03	5.45
0.4	3.77E+01	3.69E+01	2.18

Percentage difference in disp of the 2 elements
vs plate thickness

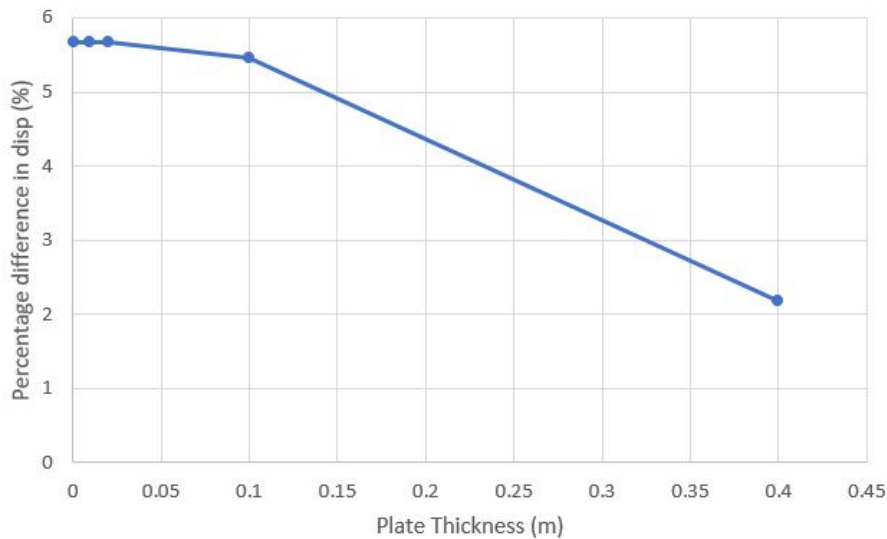


Figure 1: percentage difference in displacement vs thickness

The percentage of the difference of z-displacement between the 2 elements vs the thickness is plotted above. It's clear that the results of RM elements are lower than those of MZC elements. This difference vanishes as the thickness of the plate increases.

At low thickness values, the difference is about 5.7 %. That's due to the shear locking effect in the RM element (stiffness goes to infinity as the thickness approaches zero). As the thickness increase, the difference decrease since the shear locking effect vanishes at large thicknesses.

2 Problem 2

To do the patch test for MCZ elements, we study a patch of elements for a 3*3 plate, once using rectangles and then using random quadrilaterals as shown below. A linear boundary condition is applied ($u_z = 1 - x - y$), if the solution satisfies the formula given for the boundary conditions, then the elements converge, otherwise, they don't converge.



Figure 2: Rectangular structured mesh

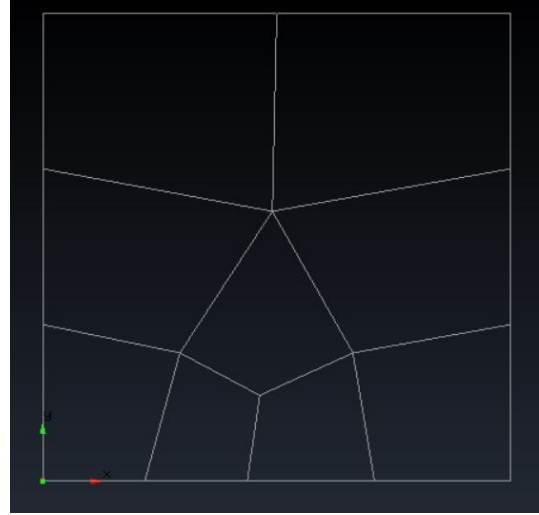


Figure 3: unstructured nonuniform mesh

Node coordinates	Exact sol	FEA sol
(1,1)	-1	-1
(2,1)	-2	-2
(1,2)	-2	-2
(2,2)	-3	-3

Results for uniform mesh

The results of the structured mesh follow the prescribed boundary conditions $u_z = 1 - x - y$ which shows that the elements pass the test and the solution will converge to the exact one using these kind of elements.

Node coordinates	Exact sol	FEA sol
(0.9533, 1.9755)	-1.928782475	-1.92879
(0.8088, 0.9824)	-0.791158566	-0.79116
(2.0044, 1.9196)	-2.923971334	-2.924
(1.6055, 0.9198)	-1.525279776	-1.5253
(2.4336, 1.3989)	-2.832486845	-2.8325
(2.2963, 0.7772)	-2.073489203	-2.0735

Results for the nonuniform mesh

Ramseries plate solver is used to obtain the results for the unstructured mesh since the code only works for rectangles. The results of the unstructured mesh don't follow exactly the prescribed boundary conditions which shows that the elements don't pass the test and the solution will not converge to the exact one.

To conclude, the MCZ elements pass the patch test (convergence is guaranteed) only when the elements are rectangles; random quadrilaterals are not guaranteed to converge.