HOMEWORK 1: 1D EXAMPLE

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1 1D CONVECTION-DIFFUSION

In this assignment we solve the following equation and boundary conditions

$$au_x - \nu u_{xx} = s \qquad x \in [0, 1] \tag{1}$$

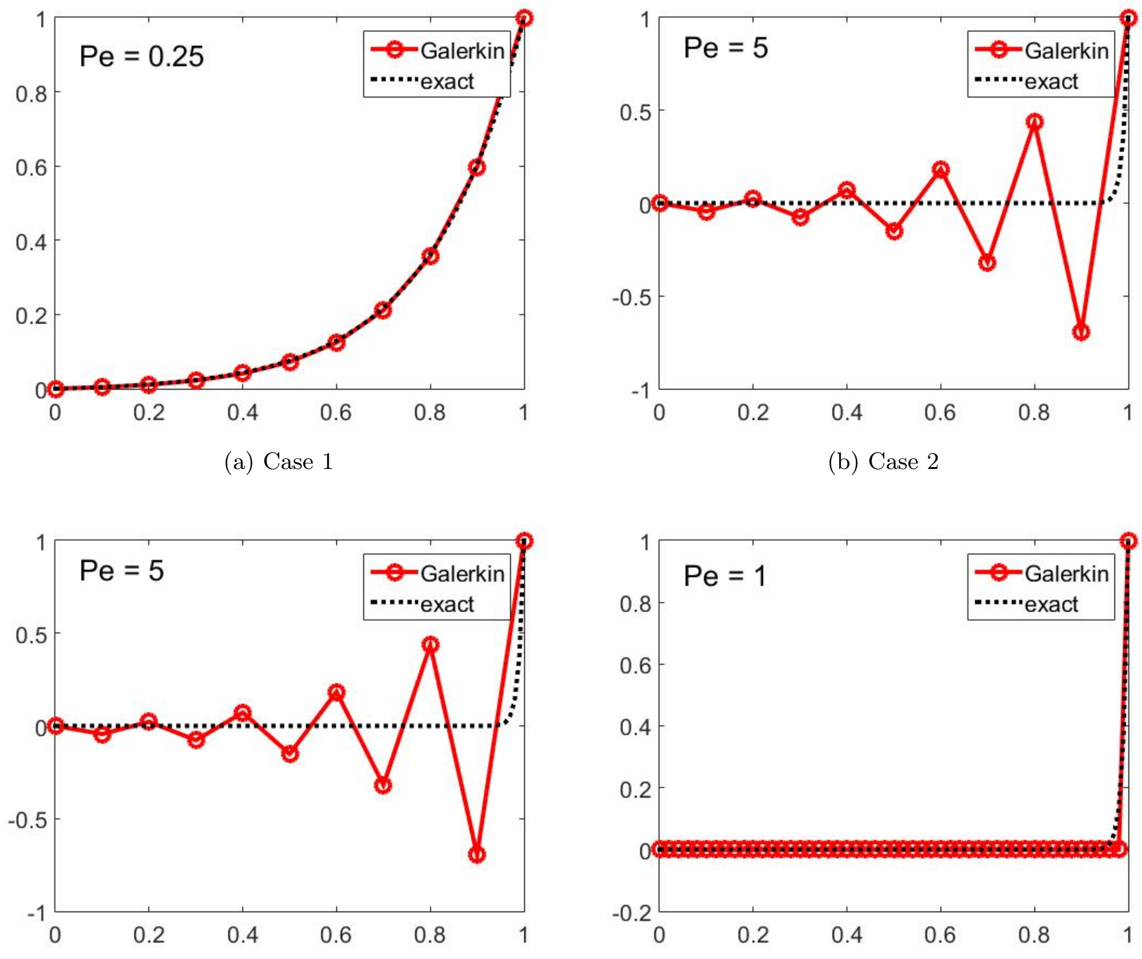
$$u(0) = 0; \qquad u(1) = 1 \tag{2}$$

1.1 GALERKIN'S METHOD

First we compare different solutions using Galerking's method for different cases as follows

1. $a = 1$, $\nu = 0.2$, 10 linear elements
2. $a = 20$, $\nu = 0.2$, 10 linear elements
3. $a = 1$, $\nu = 0.01$, 10 linear elements
4. $a = 1$, $\nu = 0.01$, 50 linear elements

The solutions are plotted in Figure 1. Case 3 and 4 are the same problem, but the discretization is smaller in case 3 than case 4. The results show that the accuracy of the numerical solution depends on the Péclet number ($Pe = \dfrac{ah}{2\nu}$). The case 1 and 2 shows the oscillation problem when convection dominates over diffusion. The case 3 and 4 compares how a smaller discretization improves the numerical solution as the case 4.



(a) Case 1

(b) Case 2

(c) Case 3

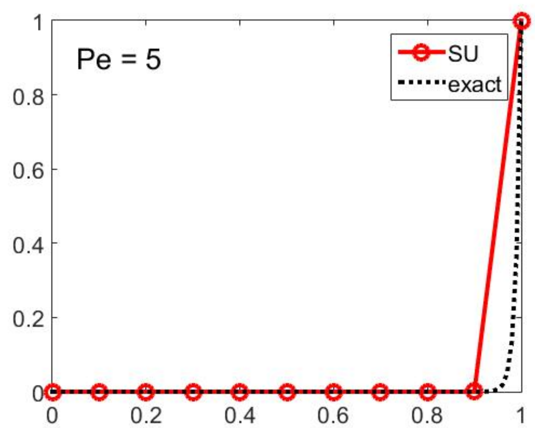
(d) Case 4

Figure 1: Galerkin's method (a) $a = 1$, $\nu = 0.2$, 10 linear elemnts. (b) $a = 20$, $\nu = 0.2$, 10 linear elemnts. (c) $a = 1$, $\nu = 0.01$, 10 linear elemnts.(d)$a = 1$, $\nu = 0.01$, 50 linear elemnts.

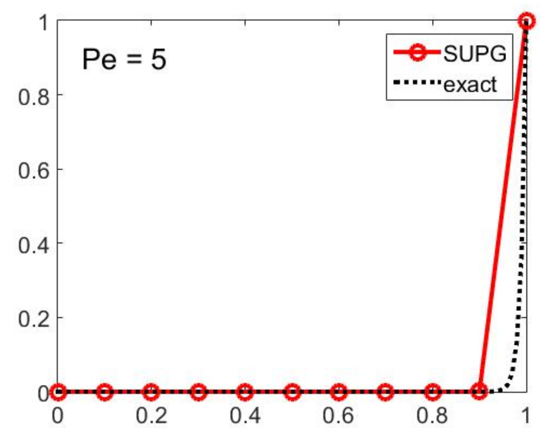
1.2 OTHER METHODS

In this section we compare the case 3 solved with Streamline Upwind (SU), Streamline Upwind Petrov-Galerkin (SUPG) and Galerkin Least-Squares (GLS). The original code includes Galerkin and SU methods in function scripts named as *Galerkin_system.m* and *SU_system.m*, respectively. We added two script files named *SUPG_system.m* and *GLS_system.m* to add the additional terms for consistency.

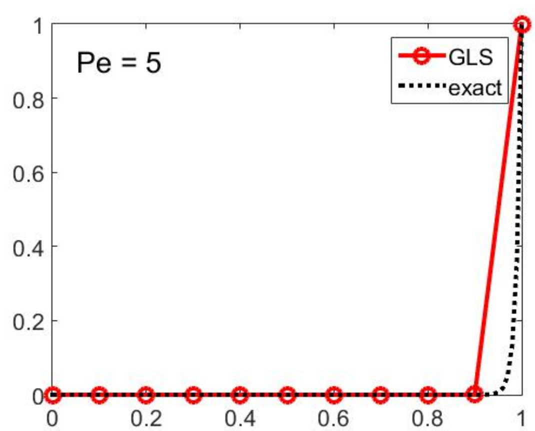
Comparison between the numerical solutions and the analytical solution are shown in Figure 2. The numerical solution is better than Galerkin's method; however, solution between SU, SUPG and GLS has not significant differences.



(a) SU



(b) SUPG



(c) GLS

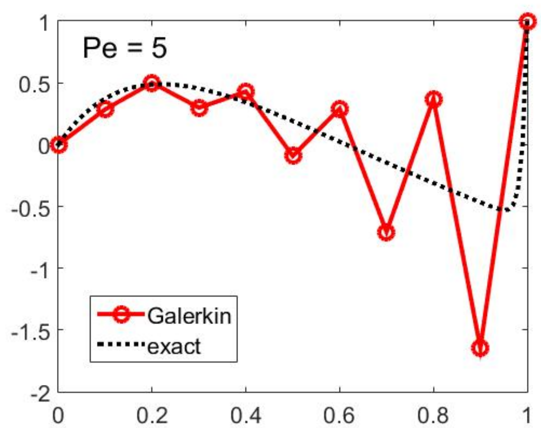
Figure 2: Case 3 (a) SU method. (b) SUPG method (c) GLS method.

1.3 DIFFERENT SOURCE TERM

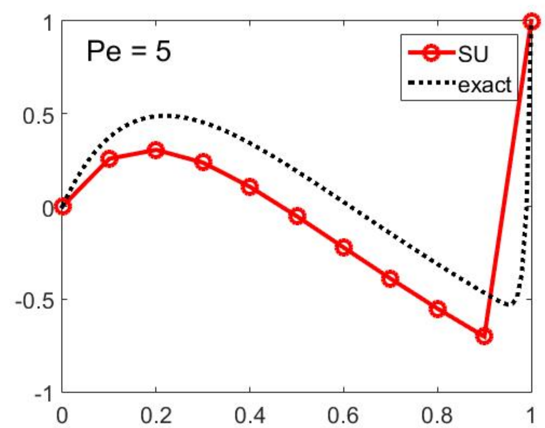
We include another example different than the three original ones that has the code. The source term was modified as

$$s = 10e^{-5x} - 4e^{-x} \tag{3}$$

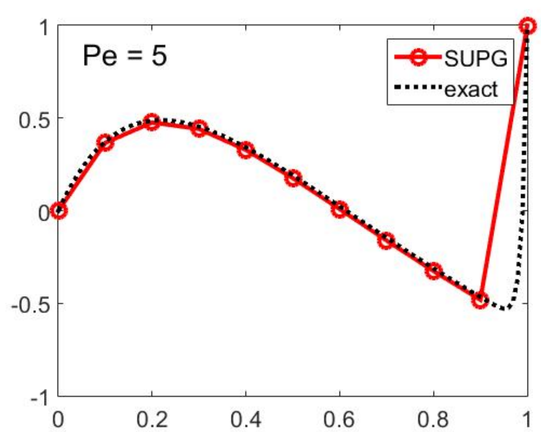
To include the new problem we modified the script *SourceTerm.m* and added the new source term. Also, we modified the the script *ExactSol.m* to compare graphically the numerical solutions with the exact solution. We solve this problem with $a = 1$, $\nu = 0.01$ and 10 linear elements, the same as case 3 of the previous problem.



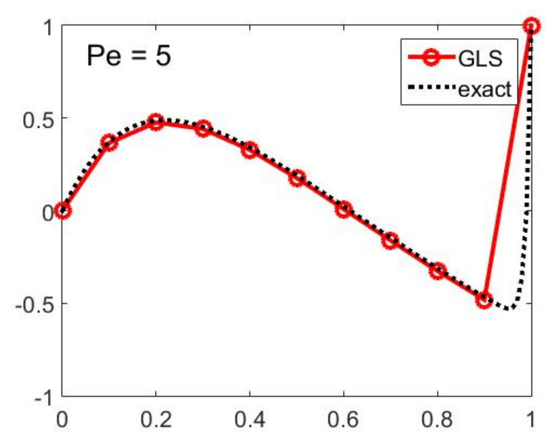
(a) Galerkin



(b) SU



(c) SUPG



(d) GLS

Figure 3: Source term $s = 10e^{-5x} - 4e^{-x}$, $a = 1$, $\nu = 0.01$, 10 linear elements. (a) Galerkin Method. (b) SU method. (c) SUPG method. (d)GLS method.

The Figure 3 compares the result with different methods. The Galerking method has oscillation problems. The SU, SUPG and GLS methods do not show oscillation problems. However, the SU method goes further than the exact solution due to it is a no consistent method. This agrees that SU formulation does not perform well for non-constant source terms.

1.4 QUADRATIC ELEMENTS

The quadratic elements are included in the script *SetRefereceElement.m*. However, we had to modified the script *main.m* to include the matrix of element connectivities T and matrix of nodal coordinates X for quadratic elements.

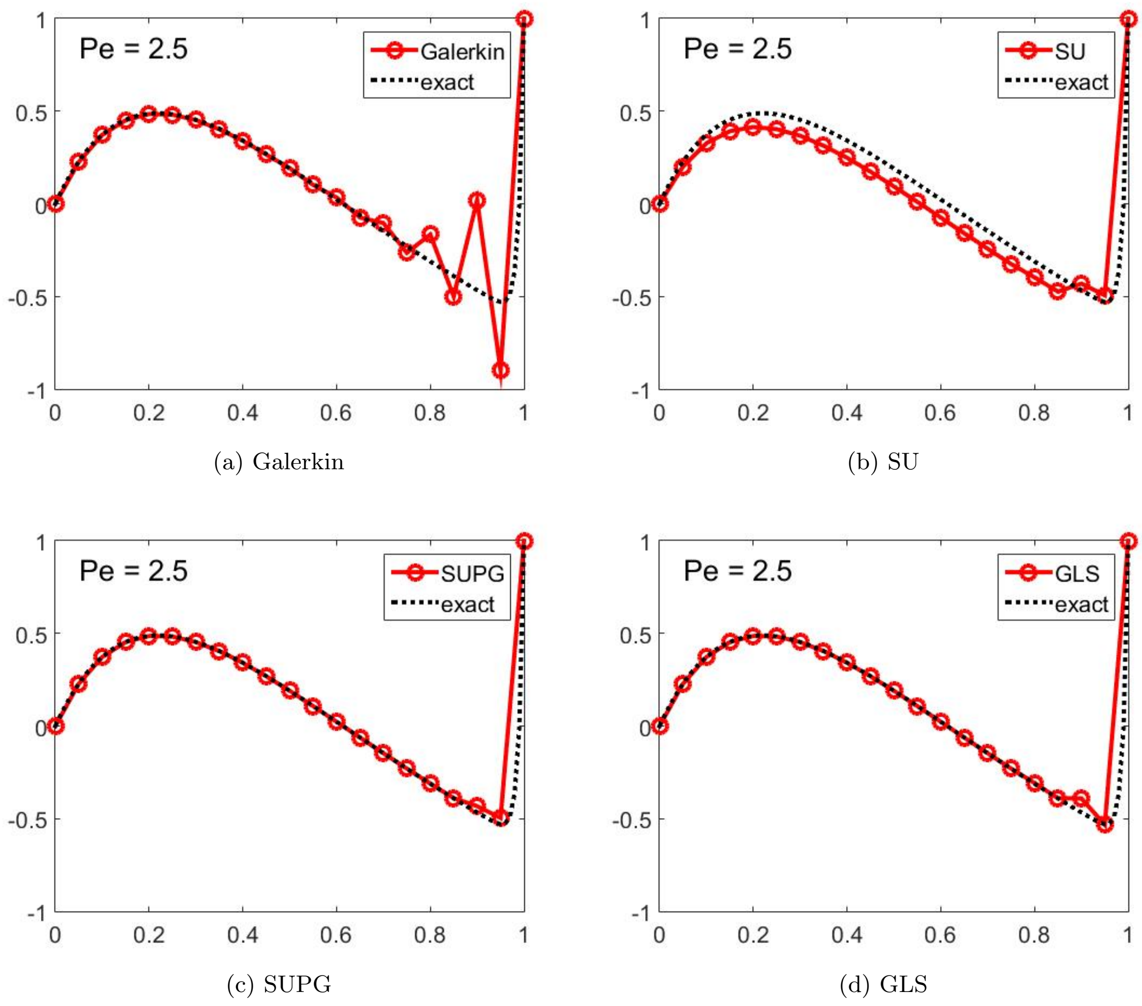


Figure 4: Source term $s = 10e^{-5x} - 4e^{-x}$, $a = 1$, $\nu = 0.01$, 10 quadratic elements. (a) Galerkin Method. (b) SU method. (c) SUPG method. (d)GLS method.

Quadratic elements have 3 nodes, therefore the number of nodes will increase if the number of elements is maintained (10 elements). The numerical results are plotted in Figure 4. The accuracy improves considerably with each method. However, the Galerkin method still has oscillations and SU method does not coincides with the exact solution. The methods SUPG and GLS have a better performance as well as the previous problem with linear elements. However, the difference between SUPG and GLS with quadratic elements is slight more notorious than with linear elements.

1.5 Conclusions

Péclet number is an indicator of the stability of the solution. When convection dominates over diffusion the solution tend to oscillate. Decreasing the size of the elements is a way to improve the stability of the solution. The SU method does not oscillates in the numerical experiments performed. However, we confirm that the SU method does not perform well for non-constant source terms. SUPG and GLS methods have a better performance and stability. Quadratic elements have a better performance than linear elements. However, the number of nodes is higher in quadratic elements.